Encrypted images in "The Prophecies" of Nostradamus

Dr. Ilya K. Razumov
https://orcid.org/0000-0002-7277-2638 *E-mail:* rik@imp.uran.ru

Abstract. Graphic cipher was first discovered by the author in "The Prophecies" of Nostradamus, and approximate methods for recovering encrypted images have been proposed. The encryption method of Nostradamus is essentially "scytale" cipher, which was adapted to transmit images instead of text. This method could have scientific value in the 16th century, without reference to the meaning of the hidden images, which is currently unclear. Given the rather high complexity of such a cipher (especially for the 16th century), it is possible that these images carry the main substantive content of the texts under consideration. Problems such as improving methods for pattern processing and identifying new images, mathematical criteria for distinguishing encrypted images from spillover pareidolia effects, and interpreting the images in a historical context remain relevant. The search for more adequate methods for recovering encrypted images is a technically creative challenge. The proposed original approach can serve as the beginning of a conceptual shift in the study of the prophecies of Nostradamus – from interpretations of foggy texts to the image recovery and recognition. The obtained results are essential for the history of steganography and shed a fundamentally new light on the work of Nostradamus.

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1. Introduction.

"The Prophecies" of Nostradamus were published in 1555–1568 in the form of 10 blocks (so called "centuries") of 100 quatrains in each (with the exception of the 7th century, which remained incomplete and contains only 42 quatrains), and two prosaic prefaces to them, dedicated to the seer's son Caesar and the king of France Henry II [1]. Although the bibliography of books and articles devoted to prophecies of Nostradamus includes thousands of sources [2], a scientific study of his life and works began relatively recently. At the moment, it includes a detailed biography of the predictor [3,4] and historical and philological commentaries on the quatrains [5,6]. An important result of these studies was the discovery of the fact that many quatrains describe events preceding 1555, that is, instead of predicting the future, Nostradamus describes the past. Perhaps, Nostradamus believes in the repeatability of history, but in the absence of dating of such quatrains, they do not seem prophetic, and even give the impression of mystification. In addition, many quatrains describe banal everyday situations, or, on the contrary, have too unclear surreal plot. Herewith, the arrangement of quatrains is usually considered by researchers as random. Attempts to restore the correct order of quatrains on the basis of astronomical data contained in some of them, failed [7], as it can be seen from subsequent studies, astronomical data in quatrains were misunderstood [5]. Thus, at the moment, the supposed predictive motivation of "The Prophecies" eludes researchers. According to Prof. D.Crouzet, *"Nostradamus wrote without knowing what quatrains mean. Even if we assume that the prophetic text was endowed with meaning, it loses it and fills the reader with a sense of his own powerlessness and sinfulness [...] Nostradamus makes it clear to his reader that you need to boldly look beyond those words"* [8].

The author of this article noticed that patterns composed, for example, of the same words in "The Prophecies" plotted on a plane in the coordinates "quatrain number – century number", often show signs of a reasonable arrangement: symmetry elements, clustering, equidistant points. Comparison of this data with random numbers generation suggests the existence of a graphic cipher. From a mathematical point of view, such a cipher is similar to the well-known "scytale cipher" (text in the form of a tape is wound on a cylinder and then cut along the cylinder axis to obtain a rectangular picture), wherein the length of the circle is the quantity of quatrains in the "century". A similar procedure also applies to two prosaic prefaces, the text of which can be presented in the form of a tape, and the diameter of the cylinder in this case is determined by guessing. Further analysis of the cipher is carried out in two directions: (1) the rules for forming patterns and (2) the methods for processing them to improve visual perception. In particular, it was found that images, apparently, can be formed from words beginning with given letters of the alphabet. Herewith, to improve the visual perception, known methods of smoothing, noise filtering, image deconvolution, light engineering formulas and automatic coloring based on machine learning algorithms can be used. Since the original method of constructing images by points is unknown, the obtained images are an approximation of the original. The search for more adequate methods for recovering encrypted images is a creative challenge.

Section 2 describes pattern processing methods for transitioning from discrete to continuous images. Section 3 gives examples of detected images, both for prosaic prefaces and quatrains. Section 4 discusses the most controversial issue: the possibility of random realization of images and the mathematical criteria for the difference between the encrypted images and the effects of pareidolia. The probability estimates indicate the presence of ordering in the patterns, which distinguishes them from the random generation of points. Lastly, in the Section 5, we discuss the obtained results in a historical context, considering the possible motivation for creating such a cipher by Nostradamus. In the Appendix, a collection of additional images is given.

The approach discussed can serve as the beginning of a conceptual shift in the study of the prophecies of Nostradamus – from the interpretation of foggy texts to the recovery and recognition of encrypted images. Given the rather high complexity of such a cipher (especially for the 16th century), it is possible that these images carry the main content of the texts under consideration.

2. Pattern processing methods.

As a result of displaying of some words or letters from the source text as points on the plane, we obtain a discrete image, the visual perception of which is difficult. To facilitate visual perception, it is necessary to connect the points with lines, smooth the transition between them, and possibly colorize the picture.

Filtering of noise and smoothing.

The simplest way to improve the image, presented as a set of points, is to replace these points with spots. Herewith, summation of light intensity in the area of overlapping spots leads to a continuous image in shades of gray. The significant problem is a light intensity gradient at the border of the spot, which is essentially a defect in the image. To combat image defects, you can use the standard methods of image processing, implemented for example in the technical computing system “Wolfram Mathematica”, such as MeanFilter, Blur, TotalVariationFilter. Among these methods, our attention was attracted by FourierDCTFilter, which reduces noise in image by the discrete cosine transforms [9]. Since the elimination of defects is always accompanied by blurring of the image as a whole, our attention was also drawn to the image deconvolution method [10], which restores the sharpness of the image, and implemented in “Wolfram Mathematica” as the “ImageDeconvolve” function. When processing some images, we used two of these methods together.

Model of point light sources.

In addition to the standard methods of image processing, in some cases we used a quasi physical model based on light engineering formulas [11]. The motivation for developing this method is due to the specifics of our task, which requires image reconstruction rather than noise filtering. Imagine that the pattern points are point sources of light that falls on a screen located parallel to the plane of the pattern. In this case a continuous image appears on the screen, and the degree of light scattering is determined by the distance h from the plane of the pattern to the screen. It is easy to show that the illumination of the elementary area (pixel) on the screen with the coordinates $\mathbf{r}$ is determined by the formula

$$S(\mathbf{r}) = \sum_i \sigma_i \Big/ \left(h^2 + R_i^2(\mathbf{r})\right)^{3/2} \qquad (1)$$

where σ_i are the powers of point sources, $R_i(\mathbf{r})$ is the distance between the pixel coordinate $\mathbf{r}$ and the projection of the source onto the screen. This formula means that the points of the original pattern are replaced not with uniformly illuminated spots, but with extended areas in which the light intensity decreases with distance from the center of the spot. We also found that often the image quality increases if an iterative procedure is used to calculate the power of point sources $\sigma_i^{(n+1)} = \sum_i \sigma_i^{(n)} \Big/ \left(h^2 + R_i^2(\mathbf{r})\right)^p$, where $\sigma_i^{(1)}=1$, $p=3/2$.

Reconstruction of image by line segments.

Connecting the points of a pattern by line segments seems to be the most natural operation. However, it is difficult to specify a strict mathematical criterion, the fulfillment of which requires drawing a line through two points. The search for such rules involves a nontrivial analysis of some neighborhood of a selected pair of points, which would shift the focus of our research to the pattern recognition area [12]. Another possibility is to construct an image grid by connecting each point of the pattern with several of its nearest neighbors, for example, with four neighboring points. However, our study showed that the resulting images in this case are of poor quality due to the irregularity of such a grid. At the same time, the connecting of each point of the pattern with *all* points of its environment within a given sufficiently large radius led to positive results. Note, when many lines are superimposed on each other, we get the final picture close to the continuum image.

In a first step of this method, each point of the pattern is connected by lines of a width d to all its neighbors within a radius R_1; at the intersection points of the lines, their brightness is summed and the resulting brightness is normalized between 0 and 1. For the commonality of the method, we assume that the brightness of the line connecting the two points depends on the distance r between these points, according to the law ~ $1/r^m$, where m is the parameter of the method. The choice of $m>0$ means that the brightness of a point with a given coordinate is determined primarily by the points of its immediate environment, and the contribution of other points decreases with increasing distance to them. In the second step, smoothing of the obtained picture is performed. For this purpose, you can use the built-in filters of "Wolfram Mathematica", for example, MeanFilter, Blur, TotalVariationFilter and others. However, when using this method, we prefer to perform smoothing using a procedure similar to the first step. Namely, each pixel of image with brightness higher than critical value s_1 is connected by lines of width d to all pixels with brightness higher than s_2, located from it no more than radius of R_2 (herewith, brightness of intermediate points of line changes according to linear law).

Pseudo-colorization.

In some cases, we used automatic coloring of monochrome images using the service https://algorithmia.com/ based on machine learning algorithms. Although the color of the images obtained in this case is very arbitrary, the multi-colored segmentation of the image facilitates visual perception. We emphasize that the coloring process fully preserves the structure of the image, which can be converted back to monochrome using any available method.

In conclusion of this section, we note that the images discussed below demonstrate sufficient stability with respect to change the processing method. To achieve the best quality, we use various methods from the ones mentioned above, specifying the performed actions in the comments to the corresponding Figures. For the analysis of the source text of "The Prophecies", and the formation and processing of patterns, we used the programming language "Fortran90" and the technical computing system "Wolfram Mathematica".

3. Examples of images encrypted in "The Prophecies".

Let us first consider the images obtained in the analysis of quatrains. Quatrain number N_k and century number N_c serve as natural coordinates indicating the position of the selected word or letter in "The Prophecies". However, in this simplest case, we have too low a resolution (10 possible positions) along one of the coordinate axes, because "The Prophecies" contain only 10 centuries. The author found that the resolution can be increased by taking into account the quatrain string number N_s (it can take integer values from 1 to 4), as well as the relative position of the word in the string, $0<q<1$. Then the position of the selected word on the plane (X,Y) can be defined by formulas

$$X=2N_k\,, \qquad Y=20(N_c-1)+5(N_s-1)+\mathrm{Int}(1+5q), \quad (2)$$

where the operation "Int" means discarding the fractional part of a number; the grid size is 200x200. By default, we assume that the images are plotted white on black, the opposite situation corresponds to color inversion.

The Figure 1 shows the image formed by all the capital letters in the quatrains present in the name "NOSTRADAMVS" (the letter "V" is always used instead of "U" in facsimile of "The Prophecies"). Typically, these letters are located at the beginning of lines, because all lines begin with capital letters. When constructing this image, we used the method of connecting points with line segments. We can see the face of a bearded man with a mark (the third eye?) on his forehead and a supposedly six-pointed star in the lower left corner of the square. Note, in Fig.1 we filled with randomly generated points the strip corresponding to the missing quatrains of the seventh century (a similar procedure will also be used in some other figures without new explanations). Other details of the figure require careful interpretation and are not discussed here.

The Figure 2 presents the image recovered from *all* words in quatrains by the method of point light sources. In this case gradients of light density due to the fact that different words have different lengths. We can see the human face and probably animal's face superimposed on each other. In the upper left corner, in human hair there is probably monogram “NM”. Most surprisingly, this rather complex image appears as a result of overlapping several others images, corresponding to groups of words starting with separate letters of the alphabet. We have found at least three groups of words beginning with the adjacent letters of the alphabet O(o) or P(p), R(r) or S(s), S(s) or T(t) leading to different distinct images which are shown below.

The Figure 3 presents the image obtained from words beginning with adjacent letters of the alphabet R(r) or S(s). The initial pattern is shown in Fig.3a, and the results of processing this pattern by line segments and the method of point light sources with pseudo-coloring are presented in the Fig.3b and Fig.3c respectively. We can see the face of a man close-up, and another face to the right of the previous, with only one open eye. The lower part of the second face is perceived also as a separate object, but unclear: it can be a rose, cat's face, letters R, H, or a complex ligature. In the upper part of the image we can see an illegible inscription, possibly the word "plote", and a cat's face on the right. The author does not insist that the small elements were correctly recognized, but large details such as human faces are not in doubt.

The Figure 4 shows an image recovered from words beginning with adjacent letters of the alphabet S(s) or T(t). We can see the cheerful male face in glasses with curly hair, and the outlines of the tower to the left of this person. We should not be surprised at the appearance of glasses, since the first glasses were invented in Italy in the 13th century.

The Figure 5 presents an image constructed from words beginning with adjacent letters of the alphabet O(o) or P(p). Here we see two faces superimposed on each other, with one eye in common. The letter "O" is probably displayed in the lower-right corner, and the letter "P" is displayed in the upper-left corner of the image. We also see a supposedly goose feather near the character’s lips on the right. The author does not insist that the small elements were correctly recognized.

The author also believes that separate images are formed from words starting with the letters C(c), F(f), G(g) (see Fig.1A in the Appendix), so perhaps *all* letters of the alphabet are involved in the construction of these “second-level” images. Most likely, the methods we have proposed are insufficient for detecting all images and require improvement.

The Figure 6 presents the images obtained from “unique” words (that is, the words found only once in the text, such as “absolution”, “Achilles”, “Aconile”, etc). The first 170 “unique” words from a list sorted alphabetically form the outlines of a winged lion with a cross on his hip (Fig.6a). A similar emblem is widely known as the symbol of Venice (the lion of St. Mark), although in the context of “The Prophecies” it may be a griffin denoting the Christian “king of Europe” of the apocalyptic period in quatrain 10-86: *“Like a griffin will come the King of Europe…”*. A formal sign of the presence of an image in this case is the fact that most of the points that form it are located on segments of smooth curves, forming a closed loop, which would be extremely unlikely if the points were randomly generated. This image is gradually rebuilt with increasing number of points so there is some bizarre ornament *at each stage*. Finally, all unique words form several superimposed human faces represented schematically (Fig.6b). When the picture is rotated 90 degrees, another image appears: a small tower, the head of a beast with open mouth and the falling human head with a star on his forehead (Fig.6c).

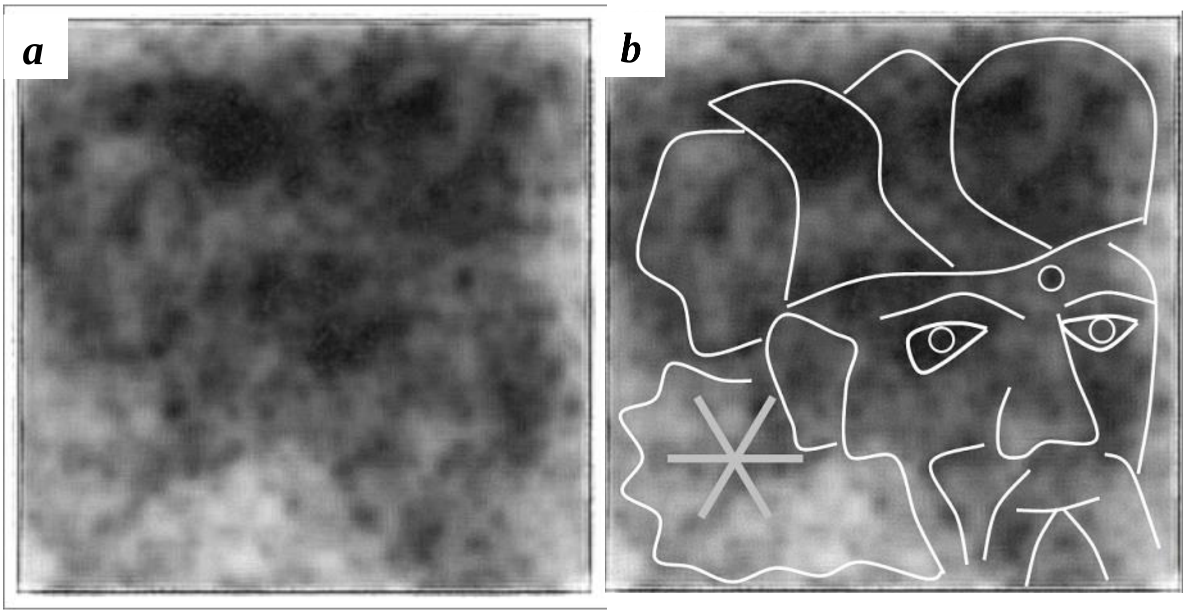

Fig.1. The image recovered from all capital letters in quatrains present in the name "NOSTRADAMVS" using method of line segments with d=2.5, m=1, s_1=s_2=10^{-2}, R_1=90, R_2=7 and subsequent color inversion ***(a)***, and manual markup of the main fragments ***(b)***. Smoothing with FourierDCTFilter and deconvolution were also performed in "Wolfram Mathematica".

Fig.2. The image recovered from all words in quatrains by the method of point light sources with h=2.5, n=20, and subsequent color inversion and pseudo-colorization ***(a)***, and manual markup of the main fragments ***(b)***. Smoothing with FourierDCTFilter and deconvolution were also performed in "Wolfram Mathematica".

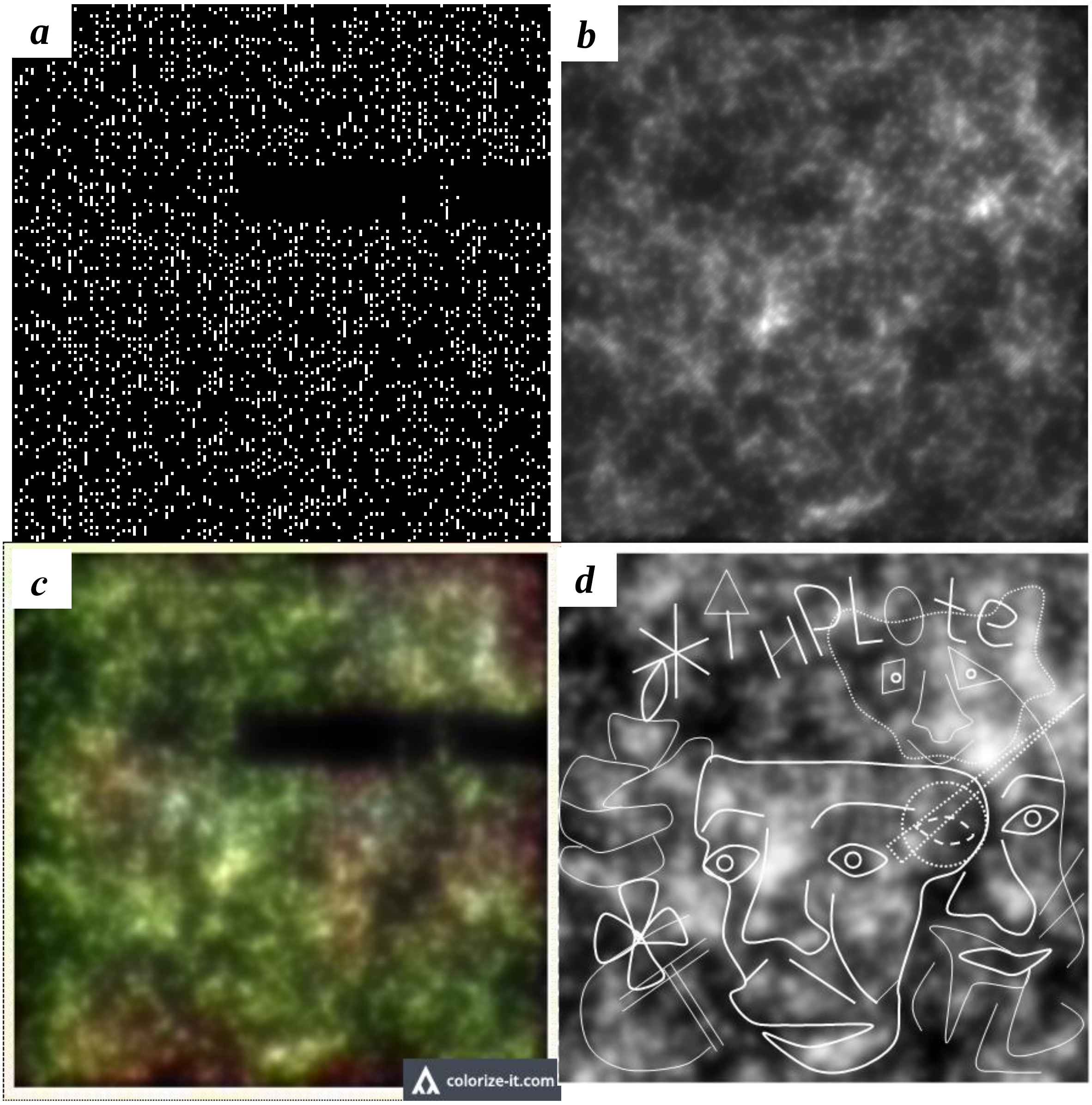

Fig.3. The pattern corresponding to words beginning with the letters R(r) or S(s) in quatrains ***(a)*** and the image reconstruction by line segments with d=1, R_1=20 ***(b)***, by the method of point light sources with h = 3.3, n=2 and subsequent pseudo-colorization ***(c)***, and manual markup of the main fragments ***(d)***.

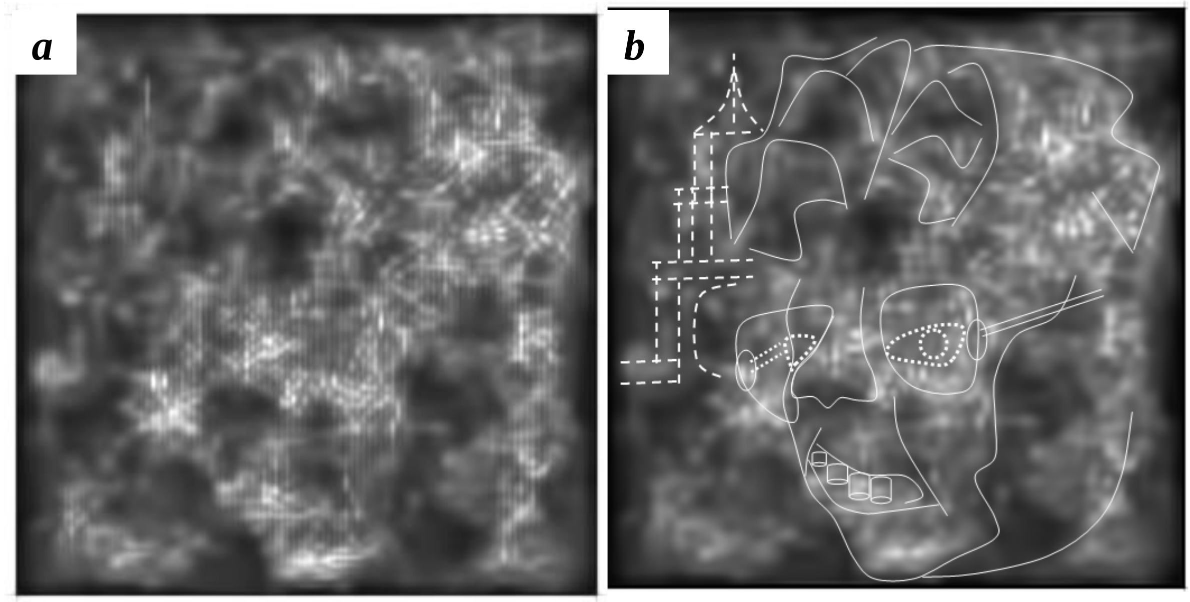

Fig.4. The image recovered from words beginning with the letters S(s) or T(t) in quatrains using method of line segments at d=1, m=1, s_1=s_2=10^{-2}, R_1=20, R_2=10 ***(a)***; and manual markup of the main fragments ***(b)***. Smoothing with FourierDCTFilter and deconvolution were also performed in "Wolfram Mathematica".

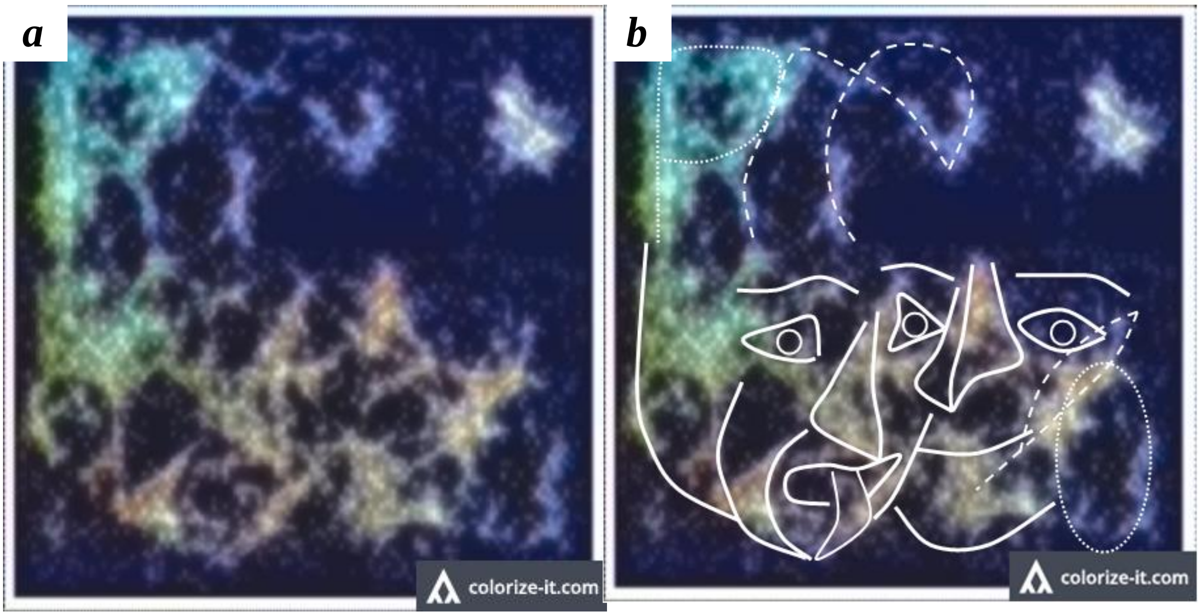

Fig.5. The image recovered from words beginning with the letters O(o) or P(p) in quatrains using method of line segments with d=1, m=1, R_1=20, R_2=4, s_1=0.25, s_2=10^{-2} ***(a)***; and manual markup of the main fragments ***(b)***. Smoothing with FourierDCTFilter and deconvolution were also performed in "Wolfram Mathematica", and automatic pseudo-coloring on the site colorize-it.com.

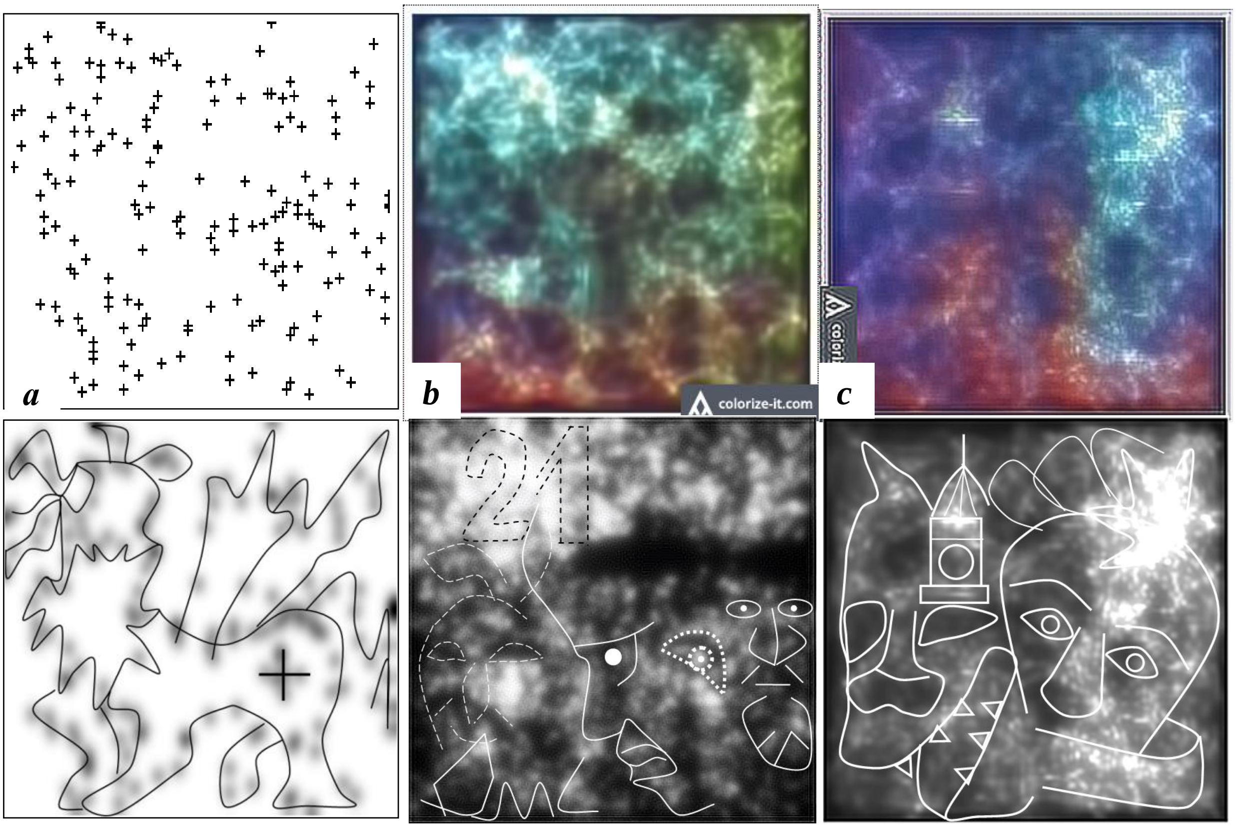

Fig.6. Images obtained from unique words: ***(a)*** 170 first unique words from a list sorted alphabetically; ***(b)*** all unique word, using method of line segments, d=1, m=1, s_1=s_2=10^{-2}, R_1=20, R_2=10, with a subsequent deconvolution in "Wolfram Mathematica" and pseudo-coloring; ***(c)*** the result of rotation of the Fig.6b by 90^0 with a new pseudo-coloring. Manual fragment markup is at the bottom.

Let us now consider the images obtained in the analysis of two prosaic prefaces. We represent each of these texts in the form of a long tape, and perform numbering of the characters. The character with the number i will be denoted by a point on the plane (X,Y) with coordinates

$$X_i = (i \text{ Div } N)+1, \quad Y_i = i \text{ Mod } N \qquad (3)$$

(operations Div and Mod mean the integer division and the remainder after integer division, respectively). Note that here the number N has a meaning similar to the circumference in the well-known "scytale" cipher [13]. As a result of variation N, encrypted images were found in the prefaces to "The Prophecies".

In the preface to Caesar, we display with dots on the plane all the letters of the alphabet from A(a) to O(o) with N=123; then we use the model of point light sources, and then invert the color. The result is a clear image of the human eye, placed in the center of the rectangular computational domain (Fig.7). Note that we used about half of all letters, which results in an informative image, while using all letters, on the contrary, would lead to uniform illumination of the square.

In the preface to King Henry, we display with dots on the plane all the words beginning with the letters of the alphabet from A(a) to I(i) with N=164; then we use the method of image reconstruction by line segments, and then invert the color. The result is a blurred image of the face of a bearded crowned man (Fig.8). At the bottom of the square, we also probably see the word "Roi" (King).

When constructing the last two figures, difficulties may arise due to the fact that the coordinates of words are determined with an error depending on the edition of «The Prophecies» (we followed the edition of 1568), typos, counting spaces and punctuation marks, and so on. These problems for a specific version of the text are solved, first of all, by calibrating the image by varying the number N, which may differ slightly from the one we specified.

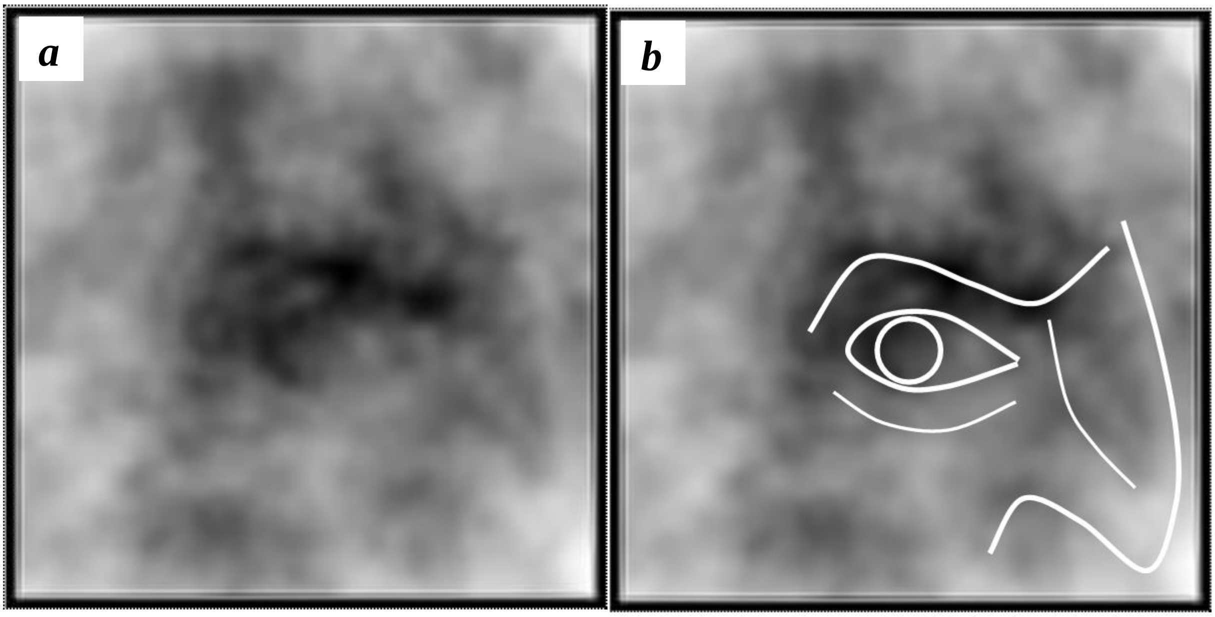

Fig.7. The image obtained from all the letters from A(a) to O(o) in the preface to Caesar ***(a)***, and manual markup of the main fragments ***(b)***. The model of point light sources with h=3, n=10 and subsequent color inversion, and also smoothing with FourierDCTFilter and deconvolution in “Wolfram Mathematica” were used. The grid size is 125x123.

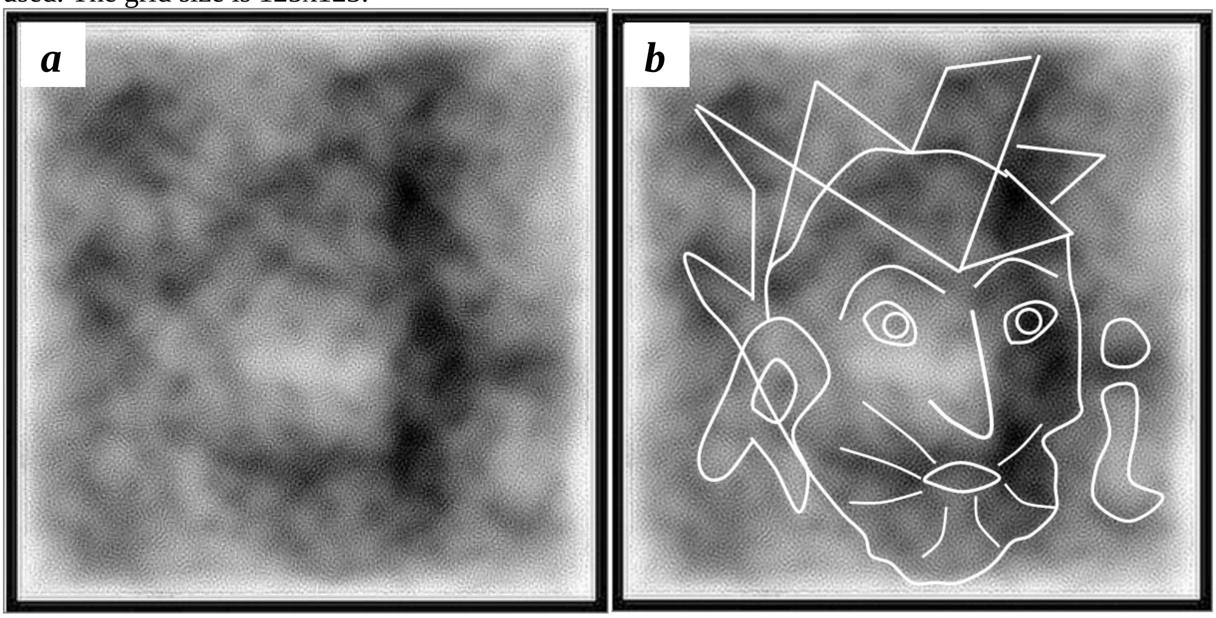

Fig.8. The image obtained from all the words beginning with the letters from A(a) to I(i) in the preface to King Henry ***(a)***, and manual markup of the main fragments ***(b)***. The method of line segments with d=1, m=0, $s_1=s_2=10^{-2}$, R_1=30, R_2=10 and subsequent color inversion, and also smoothing with FourierDCTFilter and deconvolution in “Wolfram Mathematica” were used. The grid size is 167x164.

4. Discussion.

4.1. Graphic cipher or pareidolia? Probability of random realization of images.

It is well known that the human mind has a propensity to detect the outlines of faces among the random noise [14]. This effect, called pareidolia, indicates the high importance of face recognition in the life of a human population. Apparently, the reason of pareidolia is the ability of the brain to quickly analyze a large amount of information, highlighting the contours of significant objects, and neglecting defects. Herewith, the contours of significant objects are realized in large volumes of noise due to natural random factors. Therefore, the problem is not to find a visible or mathematical difference between a real image with defects and a pareidolic image, but to evaluate the probability of random realization of image contours in random noise.

A famous example of pareidolia is the face of the sphinx on Mars [15]. This example demonstrates two differences from the cipher we are considering. Firstly, the signs of the image for the “Martian sphinx” are less pronounced than in our images. It is no coincidence that it is called the “sphinx”, because in fact, this face is not very human. Secondly, the “sphinx” is perhaps the single pareidolic effect in the infinite variety of the Martian landscape. Therefore, it is not surprising that among a wide variety of forms, contours resembling a fantastic face are found. In contrast, in “The Prophecies” of Nostradamus an ornament with signs of artificial origin is observed almost everywhere. The author has placed in the article only those images that seem most distinct, although the total number of obtained images is much larger (see the Appendix for a collection of some additional images). Thus, the author is inclined to think that the main arguments for proving the existence of the graphic cipher of Nostradamus should be (1) the large number of images and (2) the evidence of their artificial origin. Therefore, first of all, it is necessary to obtain a sufficient number of high-quality images. However, this does not negate the usefulness of mathematical evaluations.

First, we examined 100 images obtained in a similar manner from control texts. The first text was composed of small poems by French poets (Francois Villon, Jean Molinet, Clement Marot, Pierre de Ronsard). The first 4000 lines of the poem “Lancelot ou le Chevalier de la Charrette” by Chrétien de Troyes (modern french version), grouped as quatrains, were used as the second text. In these cases, we did not visually find any qualitatively similar images.

Let us now proceed to the mathematical estimation of probabilities. The Figure 9 shows examples of patterns formed from the some words in quatrains of Nostradamus These patterns demonstrate the following features of the regular arrangement of points: compactness (clustering), presence of chains of points, symmetry elements. To evaluate the probability of random implementation of such patterns we calculated the average number of points ($N_{pattern}$) in the vicinity of radius R of the selected pattern point. We also calculated the average length ($C_{pattern}$) of the chain formed by the nearest neighbors, spaced no more than R. Then, 10^6 random patterns containing the same number of points were generated, and for each of them the corresponding N_{random}, C_{random} were calculated. The fraction of random patterns for which the conditions $N_{random}>N_{pattern}$ and $C_{random}>C_{pattern}$ are simultaneously satisfied, is an estimate of the probability of a random realization $P(R)$ of the original pattern according to the indicated criteria (compactness, the presence of chains), for a given value of R. It turned out that for many patterns (“arab”, “contre”, “ciel”, “grand”, etc), this value falls to $10^{-4}\ldots10^{-5}$ at a certain R (see Fig.10a, solid curves).

Unfortunately, these results are not enough to confirm the presence of a graphic cipher, since they only indicate that the words in the text are not placed randomly. For comparison, we chose the poem “Lancelot ou le Chevalier de la Charrette”, and grouped the first 4000 lines of this text in the form of quatrains. In this text, words with the roots “arab” and “ciel” were not found, while for words with the roots “grand” and “contre” one can compare the calculation of $P(R)$ with the above. The results of this comparison are ambiguous (see Fig.10a, dashed curves 2’,4’): the words with the root “grand” in the poem “Lancelot” are placed quite randomly, however, for words with the root “contre” the order is unexpectedly found. The reason for this is the fact that poetic speech of Chrétien de Troyes tends to repeat some words in one line or adjacent lines.

On the other hand, the considered method of estimating probabilities does not take into account a number of characteristic signs of “reasonableness” of images, primarily the presence of symmetry elements. For example, the pattern obtained for the french word “bras” in quatrains of Nostradamus (Fig.9a) is estimated as completely random by the criterion $P(R)$, although we see a figure with almost perfect symmetry, the random appearance of which seems unlikely. The pattern obtained for a group of words with the root “arab” (Fig.9b) also shows signs of symmetry. Finally, in the pattern of words with the root “contre” (Fig.9c), the ordering of the points is most pronounced vertically, not horizontally, that can not be explained by the proximity of the words in the source text.

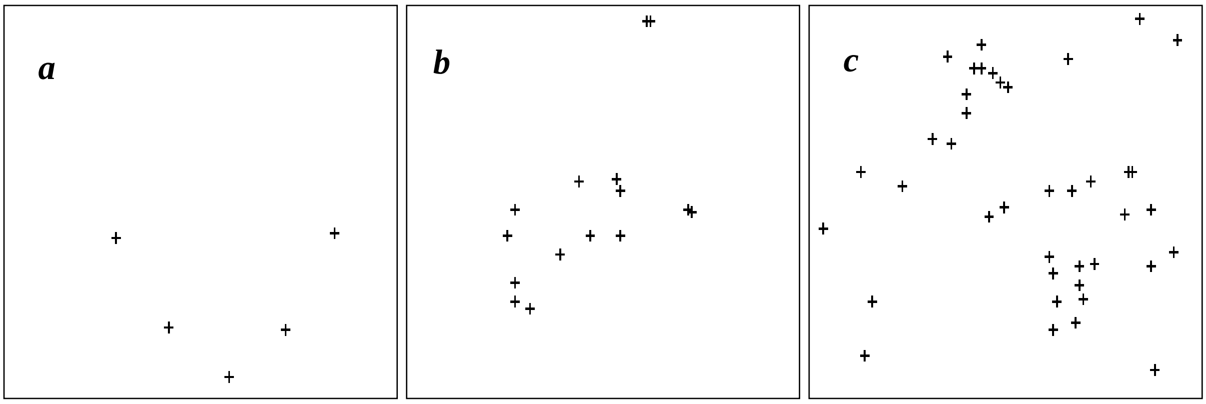

Fig.9. Patterns of points obtained for words with roots “bras” *(a)*, “arab” *(b)* “contre” *(c)* in quatrains of Nostradamus.

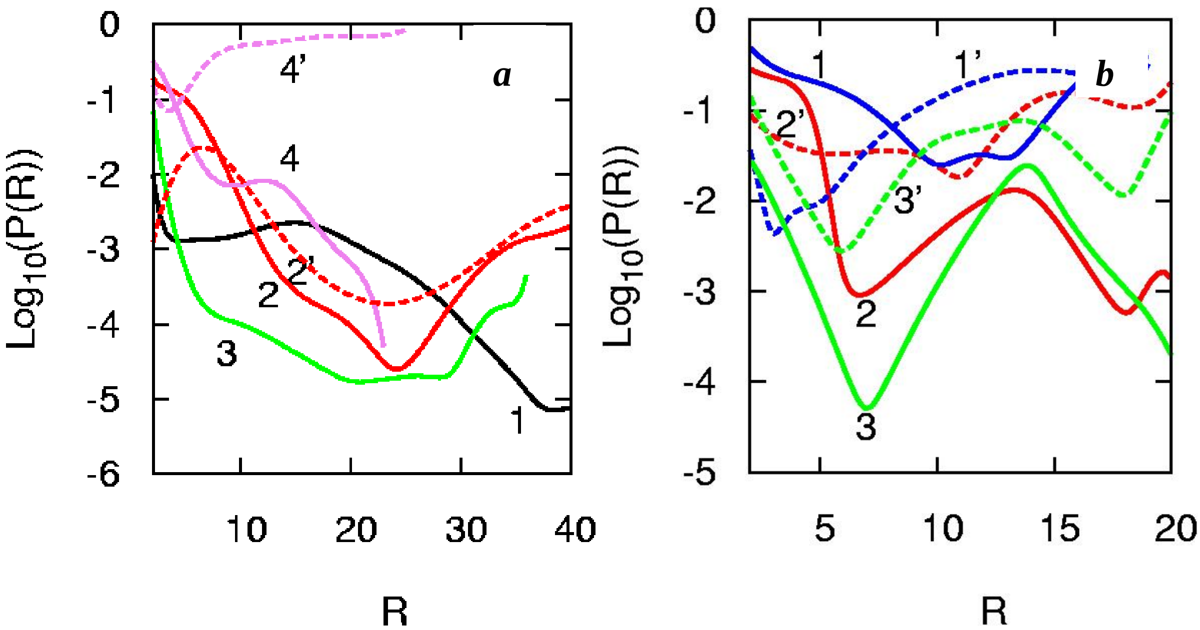

Fig.10. ***(a)*** Probabilities of random realization of patterns depending on the scale considered for words with roots (1) "arab", (2,2’) "contre", (3) "ciel", (4,4’) "grand" in quatrains of Nostradamus (1-4) and in the poem “Lancelot ” by Chrétien de Troyes (2’,4’); the grid size is 200x160. ***(b)*** Probabilities of random realization of patterns composed of words beginning with adjacent letters of the alphabet O(o) or P(p) (1,1’); R(r) or S(s) (2,2’); S(s) or T(t) (3,3’) in quatrains of Nostradamus (1–3) and in the control text (poem “Lancelot” by Chrétien de Troyes) (1’-3’), estimated by the criterion of maximum gradients depending on averaging scale; the grid size is 100x100.

When the pattern contains a large number of points, the use of the above order criteria requires large computational resources. Therefore, it becomes necessary to propose an ordering criterion for a continuous image. Our idea is that a reasonably organized image should be less uniform (which leads to its greater information content), compared with the image resulting from a random generation of points. For example, a line separating image fragments is characterized by a high light intensity gradient in the direction normal to it. Let n_{ij} be the occupation numbers of the grid nodes, n_{ij}=1 if there is a pattern point in the grid node, n_{ij}=0 in the opposite case. For the selected pattern, we perform averaging of the occupation numbers of nodes over a neighborhood of radius R to obtain a continuous image, $m_{ij} = \sum_{k=-R}^{R} \sum_{l=-R}^{R} n_{i+k,j+l}$. Then, for each grid node, we calculate the gradients of this variable in eight directions (using the first and second neighbors of the selected node), and choose the maximum of them, $g_{ij} = \max_s \left\{ \left| \nabla_s m_{ij} \right| \right\}$. Finally, we calculate the average value of the maximum gradient over the computational domain, $G = < g_{ij} >$. We perform the described procedures for 10^5 random generated patterns containing the same number of points. The proportion of random patterns P, for which the calculated value of G is greater than for the original pattern, is an estimate of the probability of a random realization of the original pattern by the criterion of maximum gradients.

The Figure 10b shows the calculation of the probability of random realization of patterns composed of words beginning with neighboring letters of the alphabet O (o) or P (p); R (r) or S (s); S (s) or T (t), for which the most distinct images are observed. The calculation is carried out at various values of the averaging radius *R*. For comparison, dashed lines show the results of a similar calculation for 4000 lines of the poem “Lancelot ou le Chevalier de la Charrette”. It can be seen that at some optimal values of *R*, the probability of random realization of patterns formed for quatrains of Nostradamus is approximately two orders of magnitude lower than for the control text, and reaches $10^{-3}\ldots10^{-4}$. Thus, the order in the patterns corresponding to the quatrains of Nostradamus is much more pronounced compared to the control text.

4.2. The cipher of Nostradamus in a historical context.

By the time “The Prophecies” were published (1555–1568), the most famous encryption method was “scytale cipher” [13], in which the tape is wound on a cylinder, and text is written along the axis of the cylinder. After removing the cylinder, a set of letters remains on the tape, which seems random. The disadvantage of this method is that a meaningless sequence of letters clearly signals the presence of a scytale cipher, for decryption of which it is only necessary to find the correct radius of the cylinder.

In the 16th century, interest in developing more promising encryption methods grew in Europe. So in 1553, that is almost simultaneously with the publication of "The Prophecies" of Nostradamus, the Italian cryptologist Bellaso proposed an important modification of scytale cipher [16]. The text encrypted with the Bellaso cipher still looks like a random sequence of letters, however, to decrypt it, it is not enough to choose the step value (cylinder radius), but you need to know the sequence of steps determined by the keyword. Another line of thought is reflected in the “Steganography” of Trithemius [17]. This manuscript was written in 1499, and distributed in handwritten versions; its first partial publication was made only in 1606. When using steganography techniques, an uninformed reader does not suspect that he is dealing with an encrypted message, because hidden information is embedded into plain text. Some techniques of Trithemius cipher was only recently understood [18].

The graphic cipher of Nostradamus occupies an intermediate position between the two mentioned directions of thought. Indeed, the cipher technique in prosaic prefaces is mathematically similar (it is described by the same simple formulas) to scytale cipher, where after “winding” the text on a cylinder, you can “cut” it along the cylinder axis to receive an encrypted message on a rectangular sheet. The difference is that in the prophecies of Nostradamus this procedure results in a picture instead of a text message, because the selected letters do not form words, but figures on the plane. At the same time, as in the case of steganography, an uninformed reader doesn’t know he is dealing with an encrypted message. Herewith, the transition from prosaic texts to numbered quatrains is a logical next step in the methodology development, designed to transmit encrypted images in a clearer form due to the permanent refinement of coordinates along the text. It is in this case that you can encrypt a fairly complex and specific image. Note, the presence of a graphic cipher provides a simple explanation of quatrains with a non-prophetic content. Indeed, although such quatrains do not have predictive meaning, they serve as useful building material for images.

Since Nostradamus offers this method for the first time, he is at the forefront of the scientific thought of his time. Therefore, it made sense for him to encrypt in this way some formal message, for example, the image of the human eye. Already this fact would prove the authorship of Nostradamus in the development of a new promising method of steganography. However, this does not exclude that Nostradamus wanted to convey a specific message to posterity. In this case, it is necessary to identify the faces in the figures, but unfortunately, at the moment this is problematic. It seems that the artist’s propensity for surrealism (for example, using overlapping images) conflicts with the idea of transmitting specific information. At the same time, poor image quality may be due to imperfections in our pattern processing methods. Indeed, the initial procedure for constructing images is unknown, and the use of our methods for processing patterns leads only to approximate images. Probably, a search for more adequate methods of image restoration and processing is necessary.

In conclusion, we will discuss the life context that could motivate Nostradamus to create a graphic cipher. Perhaps, this idea came to Nostradamus when working on a translation of “Hieroglyphica” by Horapollo, that is, ten years before the first publication of the prophecies [19].

The content of Gorapollo's text is a very free (and erroneous, as we know today) interpretation of Egyptian hieroglyphs, including numerous images of people, animals and birds. It is easy to imagine that being impressed by the world of mysterious symbols on ancient monuments, Nostradamus decided to create his own system of mysterious graphics. Also, it is well known, Nostradamus sometimes used the first letters of words for a simple coding. So the first letters of the first lines of dedication to the Princess of Navarre, which is placed before the translation of “Hieroglyphica”, form the name “NOSTRADAMVS”. Also, the first letters of the preface to Caesar and the first quatrains (1-1, 2-1, 3-1, 4-1) from the 1555 edition (which contained only 353 quatrains) form the word CAVET, meaning in Latin “[he] comes in”. This indicates the important role of the first letters of words in the steganography system of Nostradamus. Thus, we find the prerequisites for creating a graphic cipher, and for using the selected letters of words for the hidden transmission of information. On the other hand, Nostradamus was not a painter. Only the watercolors of the “papal prophecy" known [20], attributed to Nostradamus, most likely, by mistake. Therefore, it cannot be ruled out that Nostradamus encrypts the drawings of another artist, not his own.

It seems to us noteworthy the quatrain 3-94, often interpreted as Nostradamus prediction of deciphering "The Prophecies" 500 years after publication [19]: *“For five hundred years more one will keep count of him / Who was the ornament of his time: / Then suddenly great light will he give, / He who for this century will render them very satisfied”.* If our assumption is correct, the work of the predictor is likened here to some kind of “ornament”, i.e. a set of patterns on which you need to "shed light" in the right way.

5. Conclusion.

Under the cover of a foggy predictive text, Nostradamus created a large graphic cipher, wherein the position of words and letters carries information about image elements. Approximate methods for recovering encrypted images were proposed, and mostly drawings of human faces, made in a style reminiscent of surrealism, were discovered. The results obtained need to be independently verified. Finding more appropriate methods to recover encrypted images is a non-trivial mathematical task. Clarification of the meaning of the obtained images is an urgent problem.

Appendix. Additional Images.

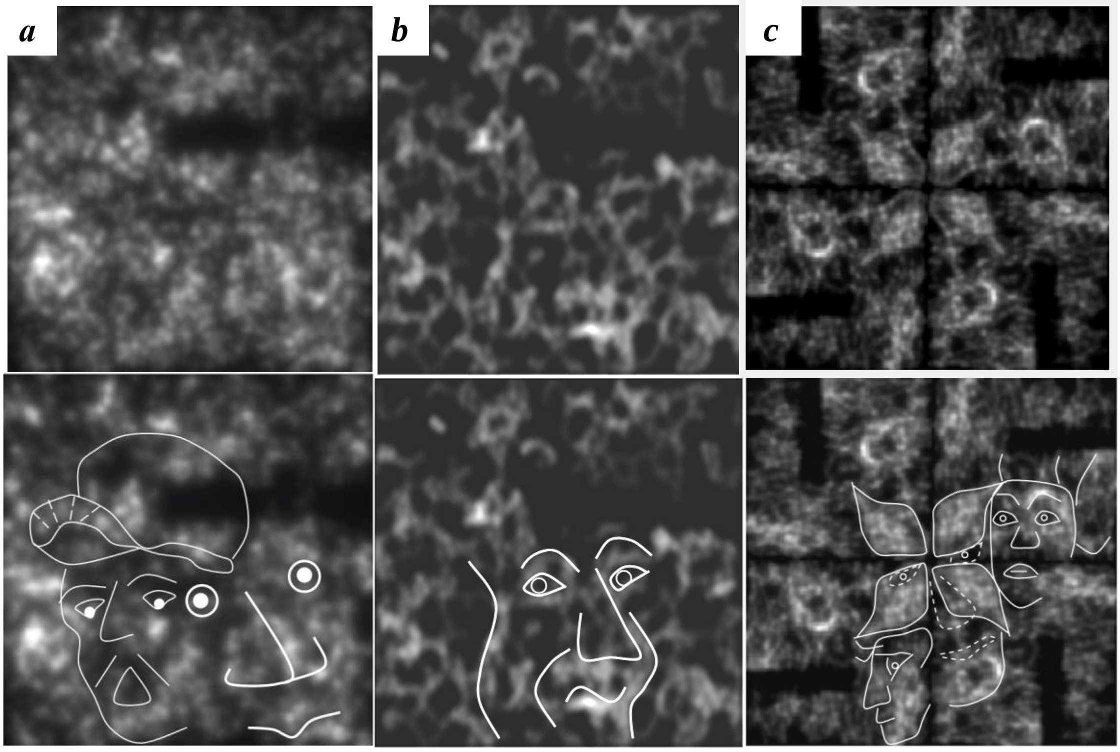

Fig.A1. The images recovered from words beginning with the letter C(c) in quatrains using a model of point light sources with h=3.3, n=1 ***(a)***, from words beginning with the letter G(g) using the method of line segments, d=1, m=1, R_1=8 ***(b)***; mosaic obtained by rotating the pattern composed of words beginning with the letter F(f) using the method of line segments, d=1, m=1, R_1=30 ***(c)***, and manual markup of the main fragments at the bottom.

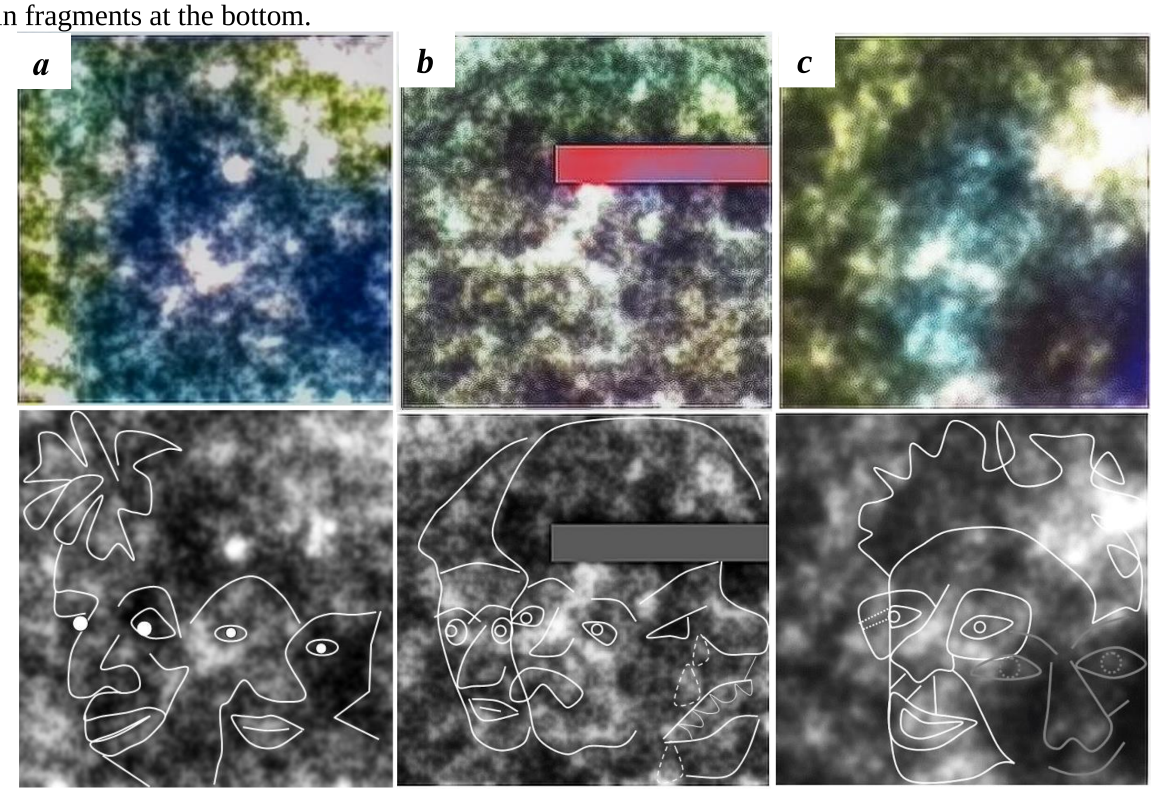

Fig.A2. The images recovered from words containing the first, second or third letters of the alphabet E(e) or F(f) ***(a)***, R(r) or S(s) ***(b)***, S(s) or T(t) ***(c)*** using a model of point light sources at h=2.5, n=10 and subsequent pseudo-colorization, and manual fragment markup at the bottom.

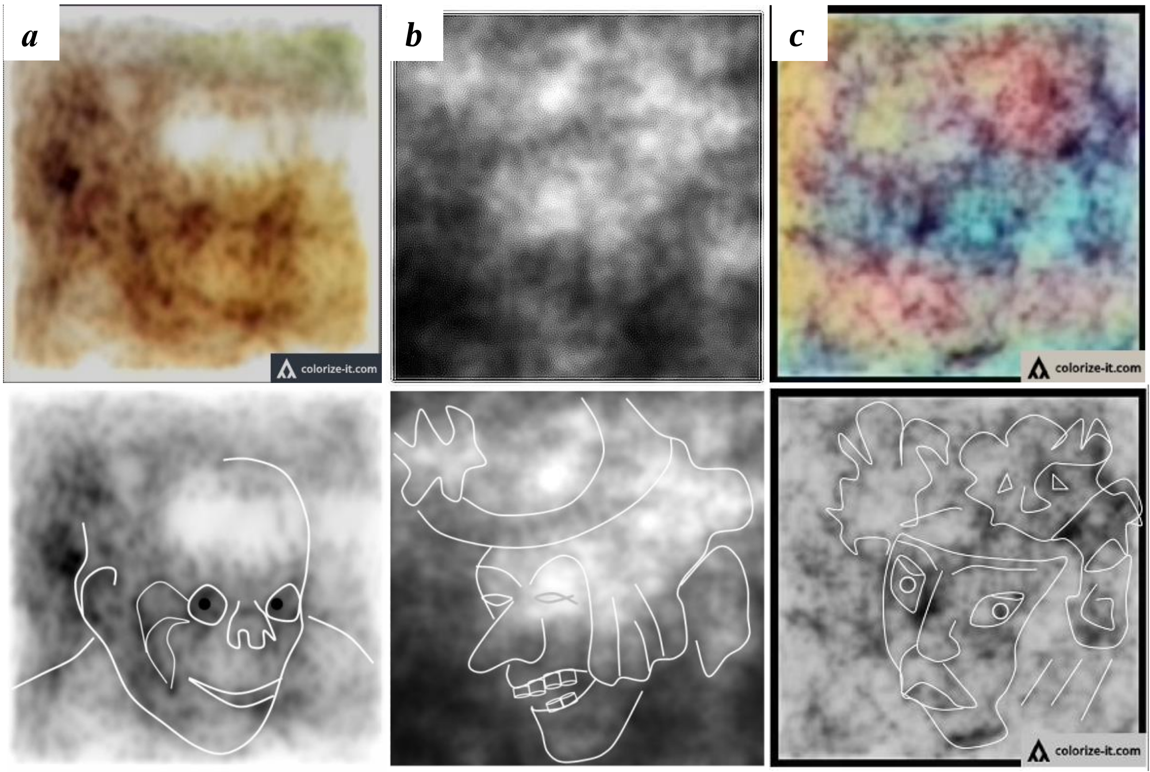

Fig.A3. The images obtained from all definite articles (la, le, les) by lines segments with d=2, m=0, R=40 ***(a)***; from all the words beginning with the letters present in the name "N(n)O(o)S(s)T(t)R(r)A(a)D(d)A(a)M(m)V(u)S(s)" using the model of point light sources with h=3, n=20 ***(b)***; the result of pseudo-coloring and color inversion in Fig.3b ***(c)***, and manual marking of the main fragments at the bottom.

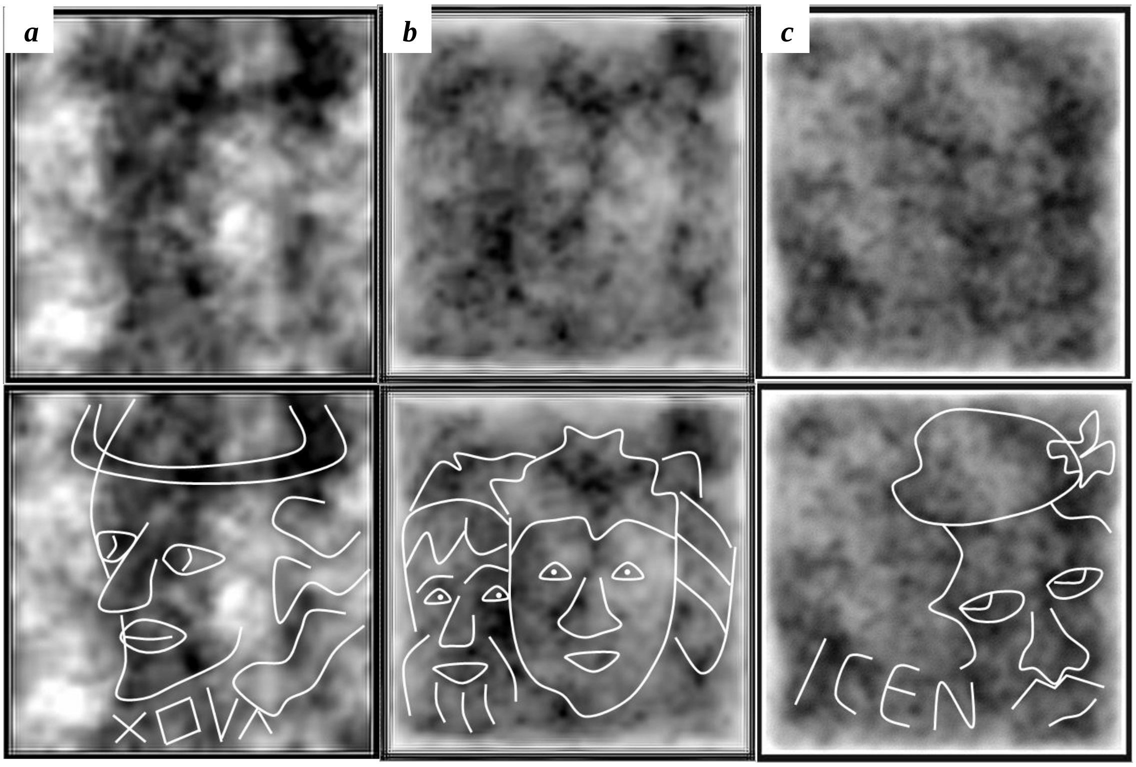

Fig.A4. The images constructed from all the letters **(a)** A(a)…L(l); **(b)** G(g)…M(m); **(c)** P(p)…S(s) in the preface to King Henry, and manual markup of the main fragments at the bottom. The method of line segments at d=1, m=0, $s_1=s_2=10^{-2}$, R_1=30, R_2=10 and subsequent color inversion, smoothing with FourierDCTFilter and deconvolution were used. The grid size is 167x164.

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