Muradyants Vagarshak Khachaturovich.
630079 Novosibirsk - 79,
Nemirovicha - Danchenko, 30/1, sq. 31.
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В.Х. Мурадьянц

A **solution of the P versus NP problem .**

(On graphs with coalitions of vertices and their isomorphism .)

It is proved in this paper that there does not exist a polynomial algorithm and a polynomial P task for the graph isomorphism problem. As a consequence, there does not exist a polynomial algorithm and a polynomial P task for any problem in the class NPC. In particular, there does not exist a polynomial algorithm and a polynomial P task for the traveling salesman problem.

Definitions and notations .

$\rightarrow$ - if then .

$\leftrightarrow$ - if and only if .

$\vec{G}$(V,U) is the notation of an oriented graph.

G(V,U) is the notation of an undirected graph without loops and multiple edges , where |V| = n, V is the set of vertices,
U is the set of edges if the graph is undirected ,
U - set of arcs if the graph is oriented .
The degree of a vertex is a number equal to the number of edges incident to this vertex. We denote the degree of a vertex by Deg(v_i) , where i $\in \overline{1,n}$ and $v_i \in$ V.

Let us define a function on the set of vertices of the graph.
Let a graph G(V,U) , $v_l,v_p \in$ V, where l,p$\in \overline{1,n}$ and l ≠p.
(v_l,v_p) $\in$U $\rightarrow$ f(v_l,v_p) =1, (v_l,v_p) $\overline{\in}$ U $\rightarrow$f(v_l,v_p) =0.

Let $v_i,v_j,v_k \in$ V, where i,j,k $\in \overline{1,n}$ and i ≠j , j≠k , i≠k.
We will say that v_i and v_j have equal links to a vertex v_k if f(v_i,v_k)=f(v_j,v_k).
Let $\vec{G}$(V,U) , $v_i \in$ V , where i $\in \overline{1,n}$. If there exists a loop at a vertex , i.e., (v_i,v_i) $\in$ U , then let f(v_i,v_i) =1. That is, (v_i,v_i) $\in$U $\rightarrow$f(v_i,v_i) =1 and (v_i,v_i) $\overline{\in}$ U $\rightarrow$f(v_i,v_i) =0 . For $\forall$ $v_l,v_p \in$ V , where l,p $\in \overline{1,n}$ and l ≠p ,
(v_l,v_p) $\in$ U & (v_p,v_l) $\overline{\in}$U $\rightarrow$ f(v_l,v_p) =1 ,
(v_l,v_p) $\overline{\in}$ U & (v_p,v_l) $\in$ U $\rightarrow$ f(v_l,v_p) = -1 .
(v_l,v_p) $\in$ U & (v_p,v_l) $\in$ U $\rightarrow$ f(v_l,v_p) = 2 ,
(v_l,v_p) $\overline{\in}$ U & (v_p,v_l) $\overline{\in}$ U $\rightarrow$ f(v_l,v_p) = 0 .

Let $v_i,v_j,v_k \in$ V , where i,j,k $\in \overline{1,n}$ and i ≠j , j≠k , i≠k . We will say that v_i and v_j have equal links to vertex v_k ,
if f(v_i,v_k) = f(v_j,v_k) and f(v_i,v_i) = f(v_j,v_j) .

If f(v_l,v_p) = 0 , then we will say that vertices v_l,v_p are not connected to each other.

Let two vertices of different graphs be given . A two-sided arrow placed between the labels of these vertices means that the vertices correspond to each other .$\dot{G}_1(\dot{V},\dot{U})$, $\ddot{G}_2(\ddot{V},\ddot{U})$, $\dot{v}_{i1},\dot{v}_{i2} \in \dot{V}$, $\ddot{v}_{j1},\ddot{v}_{j2} \in \ddot{V}$ be given . It is said that a one-to-one correspondence between the vertices of $\dot{v}_{i1} \leftrightarrow \ddot{v}_{j1}$,$\dot{v}_{i2} \leftrightarrow \ddot{v}_{j2}$ preserves adjacency if $(\dot{v}_{i1},\dot{v}_{i2}) \in \dot{U}$ $\leftrightarrow$ $(\ddot{v}_{j1},\ddot{v}_{j2}) \in \ddot{U}$.

if the graphs are

oriented , i.e., $\vec{G}_1(\dot{V},\dot{U})$, $\vec{G}_2(\ddot{V},\ddot{U})$ and $\dot{v}_{i1}$,$\dot{v}_{i2} \in \dot{V}$, $\ddot{v}_{j1}$,$\ddot{v}_{j2} \in \ddot{V}$.
It is said to be a one-to-one correspondence
between vertices $\dot{v}_{i1} \leftrightarrow \ddot{v}_{j1}$, $\dot{v}_{i2} \leftrightarrow \ddot{v}_{j2}$ preserves adjacency if
$((\dot{v}_{i1},\dot{v}_{i2}) \in \dot{U} \leftrightarrow (\ddot{v}_{j1},\ddot{v}_{j2}) \in \ddot{U})$ &
& $((\dot{v}_{i1},\dot{v}_{i1}) \in \dot{U} \leftrightarrow (\ddot{v}_{j1},\ddot{v}_{j1}) \in \ddot{U})$ &
& $((\dot{v}_{i2},\dot{v}_{i2}) \in \dot{U} \leftrightarrow (\ddot{v}_{j2},\ddot{v}_{j2}) \in \ddot{U})$ &
& $((\dot{v}_{i2},\dot{v}_{i1}) \in \dot{U} \leftrightarrow (\ddot{v}_{j2},\ddot{v}_{j1}) \in \ddot{U})$.

We will denote by β or $\beta(\dot{V},\ddot{V})$, or $\beta(\dot{G}_1,\ddot{G}_2)$ the mutually unambiguous correspondence between vertices $\dot{V}$ and $\ddot{V}$. If the graphs are oriented , then $\beta(\vec{G}_1,\vec{G}_2)$.

A mutually unambiguous correspondence between vertices $\beta(\dot{G}_1,\ddot{G}_2)$ preserves adjacency if for $\forall \dot{v}_{ik} \leftrightarrow \ddot{v}_{jk} \in \beta(\dot{G}_1,\ddot{G}_2)$ and $\forall \dot{v}_{ir} \leftrightarrow \ddot{v}_{jr} \in \beta(\dot{G}_1,\ddot{G}_2)$, the mutually one-to-one correspondences $\dot{v}_{ik} \leftrightarrow \ddot{v}_{jk}$, $\dot{v}_{ir} \leftrightarrow \ddot{v}_{jr}$ preserves adjacency . Similarly, a mutually one-to-one correspondence $\beta(\vec{G}_1,\vec{G}_2)$ preserves contiguity if for
$\forall \dot{v}_{i2} \leftrightarrow \ddot{v}_{j2} \in \beta(\vec{G}_1,\vec{G}_2)$ and $\forall \dot{v}_{i2} \leftrightarrow \ddot{v}_{j2} \in \beta(\vec{G}_1,\vec{G}_2)$,
the mutual one-to-one correspondences $\dot{v}_{i1} \leftrightarrow \ddot{v}_{j1}$, $\dot{v}_{i2} \leftrightarrow \ddot{v}_{j2}$
preserves adjacency .

A mutually unambiguous correspondence between vertices of graphs preserving adjacency will be denoted by $\overline{\beta}(\dot{G}_1,\ddot{G}_2)$.
For oriented graphs, we denote $\overline{\beta}(\vec{G}_1,\vec{G}_2)$. A mutually unambiguous correspondence $\beta(\dot{G}_1,\ddot{G}_2)$ that preserves adjacency will be denoted also by the equality $\beta(\dot{G}_1,\ddot{G}_2) = \overline{\beta}(\dot{G}_1,\ddot{G}_2)$. For oriented graphs,
$\beta(\vec{G}_1,\vec{G}_2) = \overline{\beta}(\vec{G}_1,\vec{G}_2)$.

If $v_{ik} \leftrightarrow v_{jk} \in \beta$, then we denote this by the equality
$\beta(\dot{v}_{ik}) = \ddot{v}_{jk}$ or $\beta(\ddot{v}_{jk}) = \dot{v}_{ik}$.

We will denote by $\beta(\dot{V}_i,\ddot{V}_j)$ the mutual-ambiguous correspondence between subsets of vertices $\dot{V}_i \in \dot{V}_1$, $\ddot{V}_j \in \ddot{V}_2$.
Let $\dot{G}_1(\dot{V},\dot{U})$,$\ddot{G}_2(\ddot{V},\ddot{U})$, $|\dot{V}| = n$, $|\ddot{V}| = n$ be given .
A graph $\dot{G}_1(\dot{V},\dot{U})$ is said to be isomorphic to a graph $\ddot{G}_2(\ddot{V},\ddot{U})$ and denoted by $\dot{G}_1(\dot{V},\dot{U}) \sim \ddot{G}_2(\ddot{V},\ddot{U})$ if $\exists \overline{\beta}(\dot{G}_1,\ddot{G}_2)$. Similarly for oriented graphs ,$\vec{G}_1 \sim \vec{G}_2$ if $\exists \overline{\beta}(\vec{G}_1,\vec{G}_2)$.

2. Related vertices of a graph and their properties .

Definition 1. Let a graph G(V,U) be given, $|V| = n$. Two vertices $v_i,v_j \in V$, where $v_i,v_j \in V$, where $i,j \in \overline{1,n}$ and $i \neq j$, we will call them related if they are not connected by an edge and have equal connections to any vertex different from them and denote by $v_i \equiv v_j$.
That is, $f(v_i,v_j) = 0$ and for $\forall\ v_k \in V$,
where $k \in \overline{1,n}$ and $k \neq i$, $k \neq j$, $f(v_i , v_k) = f(v_j , v_k)$.
Assertion 1. Given graphs , $G_1(V_1 , U_1)$ and $G_2(V_2 , U_2)$, where $|V_1| = n$, $|V_2| = n$ and $G_1 \sim G_2$. $\dot{G}_1(\dot{V}_1,\dot{U}_1)$, $\dot{G}_2(\dot{V}_2,\dot{U}_2)$ are subgraphs respectively of graphs G_1 and G_2 .$\exists\overline{\beta}(G_1 , G_2)$ and $\exists\beta(\dot{G}_1 , \dot{G}_2)$ such that $\beta(\dot{G}_1 , \dot{G}_2) \in \overline{\beta}(G_1 , G_2)$. Then $\dot{G}_1 \sim \dot{G}_2$ and $\beta(\dot{G}_1 , \dot{G}_2)$ preserves adjacency , i.e. $\beta(\dot{G}_1 , \dot{G}_2) = \overline{\beta}(\dot{G}_1 , \dot{G}_2)$.
That is, the subgraphs $\dot{G}_1,\dot{G}_2$ of the isomorphic graphs G_1 , G_2 constructed on the vertices which are in mutual-ambiguous correspondence ,that retain adjacency are put in correspondence with each other are isomorphic .
Proof .
Let $\dot{G}_1(\dot{V}_1,\dot{U}_1)$ and $\dot{G}_2(\dot{V}_2,\dot{U}_2)$ be subgraphs respectively of graphs $G_1(V_1 ,U_1)$ and $G_2(V_2 ,U_2)$ satisfying the condition of the statement .
That is, $\dot{V}_1 = \{\dot{v}_{i1} ,\dot{v}_{i2} , \dots , \dot{v}_{ik}\}$, $\dot{V}_2 = \{\dot{v}_{j1} ,\dot{v}_{j2} , \dots , \dot{v}_{jk}\}$.
And the one-to-one correspondence
$\beta(\dot{G}_1 ,\dot{G}_2) = \{\dot{v}_{i1} \leftrightarrow \dot{v}_{j1} , \dot{v}_{i2} \leftrightarrow \dot{v}_{j2} , \dots , \dot{v}_{ik} \leftrightarrow \dot{v}_{jk}\}$ (1)
is a subset of $\overline{\beta}(G_1 , G_2) = \{\dot{v}_{i1} \leftrightarrow \dot{v}_{j1} , \dot{v}_{i2} \leftrightarrow \dot{v}_{j2}, \dots , \dot{v}_{in} \leftrightarrow \dot{v}_{jn}\}$ (2)
That is, $\beta(\dot{G}_1 ,\dot{G}_2) \in \overline{\beta}(G_1 , G_2)$. (3)
Suppose the contrary $\beta(\dot{G}_1 ,\dot{G}_2)$ does not preserve adjointness .
I.e. $\exists\ \{\dot{v}_{ip} \leftrightarrow \dot{v}_{jp} ,\dot{v}_{il} \leftrightarrow \dot{v}_{jl}\} \in \beta(\dot{G}_1 ,\dot{G}_2)$ and
$f(\dot{v}_{ip} ,\dot{v}_{il}) \neq f(\dot{v}_{jp},\dot{v}_{jl})$. (4)
On the other hand, it follows from (3) that
$\dot{v}_{ip} \leftrightarrow \dot{v}_{jp}$, $\dot{v}_{il} \leftrightarrow \dot{v}_{jl} \in \overline{\beta}(G_1 , G_2)$ .Hence $f(\dot{v}_{ip} , \dot{v}_{il}) = f(\dot{v}_{jp}, \dot{v}_{jl})$. (5)
But (5) contradicts (4) . The obtained contradiction proves approval .
Assertion 2 . Given $G_1(V_1 , U_1)$, $G_2(V_2 , U_2)$, where $|V_1| = n$,

$|V_2| = n$,$V_1 = \{\dot{v}_1 ,\dot{v}_2 , \dots , \dot{v}_n)$, $V_2 = \{\ddot{v}_1 ,\ddot{v}_2 , \dots , \ddot{v}_n\}$. G_1 and G_2 have k related vertices $M_1 = \{\dot{v}_{i1} ,\dot{v}_{i2} , \dots , \dot{v}_{ik}\}$, $M_2 = \{\ddot{v}_{j1} ,\ddot{v}_{j2} , \dots , \ddot{v}_{jk}\}$.
$G_1 \sim G_2$. If $\exists\overline{\beta}_1(G_1 , G_2)$ which contains $\beta_1(M_1, M_2)$, then taking in

$\bar{\beta}_1(G_1, G_2)$ instead of $\beta_1(M_1, M_2)$ any $\beta_2(M_1, M_2)$ different from $\beta_1(M_1, M_2)$ we obtain $\bar{\beta}_2(G_1, G_2)$.

Proof .

From the condition of the statement $\bar{\beta}_1(G_1, G_2) =$
$= \beta_1(M_1, M_2) \cup \beta_1(V_1 \setminus M_1, V_2 \setminus M_2)$ .The subgraphs $G_{11}(V_1 \setminus M_1, U_{11})$, $G_{22}(V_2 \setminus M_2, U_{22})$ of graphs G_1 and G_2 . It follows from statement 1 that $G_{11} \sim G_{22}$ and $\bar{\beta}(G_{11}, G_{22}) = \bar{\beta}_1(V_1 \setminus M_1, V_2 \setminus M_2)$. Let

$\bar{\beta}_1(V_1 \setminus M_1, V_2 \setminus M_2) = \{\dot{v}_{ik+1} \leftrightarrow \ddot{v}_{jk+1}, \dot{v}_{ik+2} \leftrightarrow \ddot{v}_{jk+2}, \ldots, \dot{v}_{i(n-k)} \leftrightarrow \ddot{v}_{j(n-k)}\}$. (1)

Let us construct $\beta_2(G_1, G_2)$ by joining $\bar{\beta}_1(V_1 \setminus M_1, V_2 \setminus M_2)$ one-to-one correspondence of the vertices

$\beta_2(M_1, M_2) = \{\dot{v}_{p1} \leftrightarrow \ddot{v}_{l1}, \dot{v}_{p2} \leftrightarrow \ddot{v}_{l2}, \ldots, \dot{v}_{pk} \leftrightarrow \ddot{v}_{lk}\}$. (2)

We prove that $\beta_2(G_1, G_2)$ preserves adjacency, i.e.
$\beta_2(G_1, G_2) = \bar{\beta}_2(G_1, G_2)$. Suppose the contrary $\exists\ \dot{v}_{pr} \leftrightarrow \ddot{v}_{lr}$, $r \in \overline{1,k}$, and $\exists \dot{v}_i \leftrightarrow \ddot{v}_j \in \beta_1(V_1 \setminus M_1, V_2 \setminus M_2)$ such that

$f(\dot{v}_{pr}, \dot{v}_i) \neq f(\ddot{v}_{lr}, \ddot{v}_j)$. (3)

On the other hand, let us take $\forall \dot{v}_{iq} \leftrightarrow \ddot{v}_{jq} \in \beta_1(M_1, M_2)$since

$f(\dot{v}_{iq}, \dot{v}_i) = f(\ddot{v}_{jq}, \ddot{v}_j)$. (4)

M1,M2 are sets of related vertices

$f(\dot{v}_{iq}, \dot{v}_i) = f(\dot{v}_{pr}, \dot{v}_i)$,

$f(\ddot{v}_{jq}, \ddot{v}_j) = f(\ddot{v}_{lr}, \ddot{v}_j)$.

Hence and from (4) it follows

$f(\dot{v}_{pr}, \dot{v}_i)$ ¿ $f(\ddot{v}_{lr}, \ddot{v}_j)$. (5)

But (5) contradicts (3) , Hence the assumption is incorrect .
The related vertices are not adjacent . Hence for $\forall\ q \in \overline{1,k}$ and $\forall\ \gamma \in \overline{1,k}$, where $q \neq \gamma$, $\{\dot{v}_{pq} \leftrightarrow \ddot{v}_{lq}, \dot{v}_{p\gamma} \leftrightarrow \ddot{v}_{l\gamma} \in \beta_2$, then the following is true the equality $f(\dot{v}_{pq}, \dot{v}_{p\gamma}) = f(\ddot{v}_{lq}, \ddot{v}_{l\gamma})$. Hence :

$\bar{\beta}_2(G_1, G_2) = \beta_1(V_1 \setminus M_1, V_2 \setminus M_2) \cup \beta_2(M_1, M_2)$.

The assertion is proved .

Assertion 3. Given graphs $G_1(V_1, U_1)$, $G_2(V_2, U_2)$, where $|V_1|=n$,$|V_2|=n$. $M_1 = \{\dot{v}_{i1}, \dot{v}_{i2}, \ldots, \dot{v}_{ir}\}$, $M_2 = \{\ddot{v}_{j1}, \ddot{v}_{j2}, \ldots, \ddot{v}_{jk}\}$, $M_1 \in V_1$, $M_2 \in V_2$.

M_1,M_2 are related vertices respectively of graphs G_1,G_2 .
If $G_1 \sim G_2$ and $\exists \bar{\beta}(G_1, G_2)$ such that $\dot{v}_i \leftrightarrow \ddot{v}_j \in \bar{\beta}(G_1, G_2)$, where $\dot{v}_i \in M_1$ and $\ddot{v}_j \in M_2$, then all other related vertices $M_1 \setminus \dot{v}_i$ and $M_2 \setminus \ddot{v}_j$ are put in correspondence to each other in $\bar{\beta}(G_1, G_2)$ and $|M_1|=|M_2|$,

i.e., r = k .

Proof .

Let $\dot{v}_i \in M_1$,$\ddot{v}_j \in M_2$, and
$\overline{\beta}(G_1 , G_2) = \{\dot{v}_i \leftrightarrow \ddot{v}_j , \dot{v}_{ip1} \leftrightarrow \ddot{v}_{jl1} , \dot{v}_{ip2} \leftrightarrow \ddot{v}_{jl2}, \ldots , \dot{v}_{ip(n-1)} \leftrightarrow \ddot{v}_{jl(n-1)}\}$.

Suppose the contrary . Without violating generality suppose , that there exists $\dot{v}_{ipq} \leftrightarrow \ddot{v}_{jlq} \in \overline{\beta}(G_1 , G_2)$, where $q \in \overline{1, n-1}$, and $\dot{v}_{ipq} \equiv \dot{v}_i$, and $\ddot{v}_{jlq}$ and $\ddot{v}_j$ are not related vertices .

Hence $\exists \ddot{v}_{jlm}$, where $m \in \overline{1, n-1}$ and

$$f(\ddot{v}_j , \ddot{v}_{jlm}) \neq f(\ddot{v}_{jlq}, \ddot{v}_{jlm}) . \qquad (1)$$

On the other hand $\dot{v}_{ipq} \leftrightarrow \ddot{v}_{jlq} \in \overline{\beta}(G_1 , G_2)$ and $\dot{v}_{ipq} \equiv \dot{v}_i$. Hence and since in $\overline{\beta}(G_1 , G_2)$ there exist

$$\dot{v}_{ipm} \leftrightarrow \ddot{v}_{jlm} , \qquad (a)$$

$$\dot{v}_{ipq} \leftrightarrow \ddot{v}_{jlq} , \qquad (b)$$

$$\dot{v}_i \leftrightarrow \ddot{v}_j \qquad (c)$$

should :

$$f(\dot{v}_i , \dot{v}_{ipm}) = f(\dot{v}_{ipq} , \dot{v}_{ipm}) , \qquad (2)$$

$$f(\dot{v}_i , \dot{v}_{ipm}) = f(\ddot{v}_j , \ddot{v}_{jlm}) , \qquad (3)$$

$$f(\dot{v}_{ipq} , \dot{v}_{ipm}) = f(\ddot{v}_{jlq} , \ddot{v}_{jlm}) . \qquad (4)$$

It follows from (2) that the left parts of (3) and (4) are equal, and from (1) it follows that the right parts of (3) and (4) are not equal .

The obtained contradiction refutes the assumption .

Let us prove that $|M_1| = |M_2|$. Suppose the contrary .Without breaking generality, suppose that $|M_1| > |M_2|$ and $\exists \overline{\beta}(G_1 , G_2)$, such that $\dot{v}_i \leftrightarrow \ddot{v}_j \in \overline{\beta}(G_1 , G_2)$, where $\dot{v}_i \in M_1$, $\ddot{v}_j \in M_2$.

Hence $\exists\ \dot{v}_r \leftrightarrow \ddot{v}_p \in \overline{\beta}(G_1 , G_2)$, where $\dot{v}_r \in M_1$, $\ddot{v}_p \overline{\in} M_2$.

As shown above such an assumption leads to a contradiction . The assertion is completely proved .

3. About a graph with five types of vertices . About a V-graph . (About a graph with coalitions of vertices.)

3.1.Notations .

For convenience of reasoning we introduce the notation

Let two ordered sets $M_1 = (a_1 , a_2 , a_3 , a_4)$, $M_2 = (b_1 , b_2 , b_3 , b_4)$. Mutually unambiguous correspondence between elements of the sets M_1 and M_2 , in which the elements occupying equal positions are put in correspondence , we denote by $M_1 \leftrightarrow M_2$ or $(a_1 , a_2 , a_3 , a_4) \leftrightarrow (b_1 , b_2 , b_3 , b_4)$.

That is, ((a_1 , a_2 , a_3 , a_4) ↔ (b_1 , b_2 , b_3 , b_4))→ (a_1↔ b_1 , a_{42}↔ b_2 , a_3 ↔b_3 , a_4↔ b_4) . Hence the following relations are true (a_1,a_4) ↔ (b_1,b_4) , (a_2,a_3) ↔ (b_2,b_3) since they establish the same and the same correspondences between the elements .

If we consider any one-to-one correspondence between elements of two sets M_1 and M_2 . The sets M_1 and M_2 let us denote as unordered: M_1 = {a_1 , a_2 , a_3 , a_4} , M_2 = {b_1 , b_2 , b_3 , b_4} . That is, {a_1 , a_2 , a_3 , a_4} ↔ {b_1 , b_2 , b_3 , b_{43}} is any mutually unambiguous correspondence between elements of sets M_1 , M_2 .

Let us introduce graphs of a special kind .In the indicated graphs, vertices are partitioned into subsets of vertices . Each subset consists of four vertices . Each of the four vertices has a property different from the properties of the other three vertices .
This allows us to introduce the notion of vertex type . Thus, each quadruple vertex consists of four types of vertices

These four vertices will be called a **coalition of vertices** or a **coalition vertex** .The main property of the introduced graphs .
Mutually unambiguous correspondence between vertices of the specified graphs , preserving adjacency, puts **coalition vertices** in mutually unambiguous correspondence . In this case , the mutually ambiguous correspondence puts the vertices of the same type into a mutually-ambiguous correspondence .
The introduced graphs allow us to reduce the problem of isomorphism of oriented graphs (Berge graphs) to the problem of isomorphism of undirected graphs without loops and multiple edges . And establish the relation between the P task
(the P task answers the question existence of a solution: Yes or No) with the algorithm for solving the isomorphism problem of graphs without loops and multiple edges .

3.2 Constructing a V - graph .

Definition 2 . Consider the graphical representation of the Latin letter V . It consists of two line segments forming an angle .
Let's transform the image of the letter V into a graph with three vertices .

At the vertex of the angle we place the first vertex of the graph .
At the free end of the left segment we place the second vertex .

At the free end of the right segment we place the third vertex .
The vertex located at the vertex of the corner will be called vertex of type **y** .
The vertex located at the free end of the left segment will be called a vertex of type **x** .
The vertex located at the end of the right segment will be called a vertex of type **z** .
Let's transform the constructed graph into a graph with four vertices.
The fourth vertex will be connected by an edge only to a vertex of type **x** .This fourth vertex will be called a vertex of type **w** .
Let's construct a graph which consists of n graphs constructed above with four vertices . For this purpose , the image of the constructed graph with four vertices, we repeat it n times .
Let us denote and number the vertices . The letter denoting the type of a vertex is at the same time its denotation .
Let's number the vertices as follows .Vertices belonging to the same graph image with four vertices will have equal numbers .
We assign an index to each vertex designation . The value index is equal to the vertex number . Vertices of the same type we number them sequentially from 1 to n.
The vertices of four types forming a quadruple vertex with the same numbers will be called a **coalition of vertices** or **vertices of the same name** .
We denote a coalition of vertices by $\mathbf{s_i}$, where the value i is equal to number assigned to vertices forming this coalition and call a coalition of vertices a **coalition vertex** .
The set of all coalition vertices will be denoted by **S** .
In a coalition vertex we define the following ordering of vertex types: **w,x,z,y** . That is, a coalition vertex is an ordered set containing four vertices . Thus $s_i = (w_i,x_i,z_i,y_i)$, where $i \in \overline{1,n}$, $S = \cup_1^n \mathbf{s_i}$.

We connect vertices of type **z** by edges . That is, the vertices of type **z** form a complete graph . Let us introduce three more related vertices . Let us call them vertices of type **p** . We denote the set of vertices of type **p** by $P=\{p_1 , p_2 , p_3\}$. Let us connect each vertex from P by an edge to each vertex of type **z** .

The edges in the graph described above will be in every graph containing the same number of coalition vertices. Let us denote the set of these edges by U_c .

In the described graph there may be edges connecting vertices of type **x** with vertices of type **z** . We will denote the set of these edges 1by U_{xz}. Also, for vertices of type **w**,

we may introduce related vertices. For a vertex w_i ,we denote by W_i , where i $\in\overline{1,n}$.

U_i is the set of edges connecting vertices from W_i to vertex x_i , where i$\in\overline{1,n}$.

A graph with five types of vertices **x,y,z,w,p** will be called a V-graph and denote by $V_{gr}((S,P,\cup_1^n W_i),(U_c,U_{xz}\cup_1^n U_i))$, where $|S| = n$.

If $|U_{xz}| = 0$, then the V-graph will be called unloaded. If $|U_{xz}| \neq 0$, then the V-graph is called loaded.

The number of coalition vertices$|S|$ will be called the dimension of the V-graph and denoted by $R(V_{gr})$.

If vertices related to one of the vertices of type **w** are added to the V-graph, we will say that this vertex is labeled or has a label .

The set of vertices W_i related to a vertex w_i , where i $\in\overline{1,n}$, we will call the label of vertex w_i . $|W_i|$ - the number of vertices related to vertex w_i we will call the label value .

3.3 Properties of vertices of a V-graph .

The necessary conditions for vertices $\dot{v}_i\in\dot{V}$, where i $\in\overline{1,n}$, , $\ddot{v}_j \in\ddot{V}$, j$\in\overline{1,n}$,such that $\exists\overline{\beta}(G_1(\dot{V},\dot{U}) , G_2(\ddot{V},\ddot{U})) \ni \dot{v}_i\leftrightarrow\ddot{v}_j$.

Assertion 4. Let $G_1(\dot{V},\dot{U})$, $G_2(\ddot{V},\ddot{U})$, where $\dot{V} = \{\dot{v}_1 , \dot{v}_2 ,..., \dot{v}_n\}$, $\ddot{V} = \{\ddot{v}_1 , \ddot{v}_2 ,...,\ddot{v}_n\}$. If $G_1(\dot{V},\dot{U})$ $G_2(\ddot{V},\ddot{U})$ and for a vertex $\dot{v}_i \in \dot{V}$, $\ddot{v}_j \in\ddot{V}\exists\overline{\beta}(G_1 , G_2)$ such that $\dot{v}_i\leftrightarrow\ddot{v}_j \in \overline{\beta}(G_1 , G_2)$. Then the degrees of the specified vertices are equal to .

Proof .

Suppose the contrary $Deg(\dot{v}_i) \neq Deg(\ddot{v}_j)$. Without violating generality

let $Deg(\dot{v}_i) > Deg(\ddot{v}_j)$.

Let $\{\dot{v}_{i1}, \dot{v}_{i2},...,\dot{v}_{ik}\} \in\dot{V}$

be vertices adjacent to vertex $\dot{v}_i$ .I.e. $Deg(\dot{v}_i) = k$.

On the other hand, for $\forall$ l $\in\overline{1,k}$, $f(\dot{v}_i, \dot{v}_{il}) = 1$. Since $\dot{v}_i\leftrightarrow\ddot{v}_j \in \overline{\beta}(G_1 , G_2)$, then $\exists\ddot{v}_{jl}\in\ddot{V}$, such that $\dot{v}_{il}\leftrightarrow\ddot{v}_{jl}\in\overline{\beta}(G_1 , G_2)$.Hence $f(\ddot{v}_j , \ddot{v}_{jl}) = 1$. Hence $Deg(\ddot{v}_j) = k$, and by assumption $Deg(\dot{v}_i) > Deg(\ddot{v}_j)$, i.e. k ¿ k . The resulting contradiction proves the statement .

Assertion 5. Let $G_1(\dot{V}, \dot{U})$,$G_2(\ddot{V}, \ddot{U})$,where $|\dot{V}|$ = n , $|\ddot{V}|$ = n . $\dot{v}_i \in \dot{V}$, $\ddot{v}_j \in \ddot{V}$ the adjacent sets of vertices, respectively $L_1 = \{\dot{v}_{i1}, \dot{v}_{i2},...,\dot{v}_{ik}\}$, $L_2 = \{\ddot{v}_{j1}, \ddot{v}_{j2},..., \ddot{v}_{jm}\}$ and their corresponding vertex degree sets M_1 = {Deg($\dot{v}_{i1}$),Deg($\dot{v}_{i2}$,..., Deg($\dot{v}_{ik}$)} , M_2 = {Deg($\ddot{v}_{j1}$),Deg($\ddot{v}_{j2}$),..., Deg($\ddot{v}_{jm}$)} .

If G_1~ G_2 and $\exists\overline{\beta}(G_1, G_2)$ is such that $\dot{v}_i \leftrightarrow \ddot{v}_j \in \overline{\beta}(G_1, G_2)$, then $|M_1| = |M_2|$, i.e., m = k , the sets of degrees of vertices M_1 , M_2 , adjacent to vertices $\dot{v}_i$, $\ddot{v}_j$, respectively, are equal to each other , i.e., $M_1 = M_2$.

Proof .

By the condition of statement G_1~ G_2 and $\dot{v}_i \leftrightarrow \ddot{v}_j \in \overline{\beta}(G_1, G_2)$, hence and statement 4 follows $|M_1| = |M_2|$, i.e. k = m .

Suppose the contrary $M_1 \neq M_2$. Then any a one-to-one correspondence between vertices from L_1 and L_2 will assign to each other $\dot{v}_{i\beta} \in L_1$ and$\ddot{v}_{j\gamma} \in L_2$, where γ,μ $\in \overline{1,k}$, such that Deg($\dot{v}_{i\beta}$) ≠ Deg($\ddot{v}_{j\gamma}$) . That is, from the assumption that $\exists\overline{\beta}(G_1, G_2)$ and $\dot{v}_i \leftrightarrow \ddot{v}_j \in \overline{\beta}(G_1, G_2)$, it follows that $\exists$ $\dot{v}_{i\beta} \leftrightarrow \ddot{v}_{j\gamma} \in \overline{\beta}(G_{1,}G_2)$ and Deg($\dot{v}_{i\beta}$) ≠ Deg($\ddot{v}_{j\gamma}$) . And this contradicts statement 4 . The resulting contradiction proves the statement .

Definition 3 . Let given $V_{gr}((S,P,\cup_1^n W_i),(U_c,U_{xz},\cup_1^n U_i))$, $W_i = \{w_{i1}, w_{i2},...,w_{ik1}\}$ is the label of vertex w_i , $W_j = \{w_{j1}, w_{j2},..., w_{jk2}\}$ is the label of vertex w_j , where $w_i \in s_i$, $w_j \in s_j$, $s_i,s_j \in S$. We will say that vertices w_i ,w_j , where i ≠ j , have different labels if k1 ≠ k2 . If k1 = k2 we will say that vertices w_i , w_j have equal labels . Similarly, we define labels if W_i ,W_j belong to different V-graphs .

We call $k_i = |W_i|$, $k_j = |W_j|$ the label value .

Definition 4. Let $V_{gr}((S,P,\cup_1^n W_i),(U_c,U_{xz},\cup_1^n U_i))$. We will

say that the vertices of the V-graph are labeled correctly if any two labeled vertices have different labels .

Ranges of degree values of vertices of each type in the V-graph .

Deg(**x**) - denotation of the degree of vertices of type **x** .

Deg(**y**) - designation of the degree of vertices of type **y** .

Deg(**w**) - denotation of the degree of vertices of type **w** .

Deg(**z**) - denotation of the degree of vertices of type **z** .
Deg(**p**) - denote the degree of vertices of type **p** .

Given $V_{gr}((S,P,\cup_1^n W_i),(U_c,U_{xz},\cup_1^n U_i))$,. Only vertices of type **x** adjacent to one or more vertices that have degree equal to 1 since Deg(**w**) = 1 .

Deg(**x**) = 2 if the V-graph is not loaded and vertices of type **w** are not labeled .
Deg(**x**) ≤ n + 2 if the V-graph is loaded and vertices of type **w** are not labeled .
Deg(**y**) =2 always .
Deg(**w**) = 1 always .
Deg(**z**) = n + 3 if the V-graph is unloaded, since vertices of type **z** are connected by edges and to one vertex of type **y** , each of them is adjacent to each of three vertices of type **p** .
Deg(**z**) ≤ 2n + 3 if the V-graph is loaded , since each vertex of type **z** can be adjacent to each vertex of type **x** .
Deg(**p**) = n since vertices of type **p** are always adjacent only to each vertex of type **z** .

Hence :
2 ≤ Deg(**x**) ≤ n + 2
Deg(**y**) = 2 always .
Deg(**w**) = 1 always .
n + 3 ≤ Deg(**z**) ≤ 2n + 3 .
Deg(**p**) = n always .

Assertion 6. Given $V_{gr1}((\dot{S},\dot{P},\cup_1^n \dot{W}_i),(\dot{U}_c,\dot{U}_{xz},\cup_1^n \dot{U}_i))$, $V_{gr2}((\ddot{S},\ddot{P},\cup_1^n \ddot{W}_i),(\ddot{U}_c,\ddot{U}_{xz},\cup_1^n \ddot{U}_i))$. If V_{gr1} V_{gr2}, then any mutual-ambiguous correspondence between vertices V_{gr1}, V_{gr2},

preserving contiguity , puts vertices of the same type in correspondence with each other .

Proof .

Let V_{gr1} V_{gr2}. Consider each vertex type in separately .

1)Deg(**w**) = 1 . The degree of the other vertex types have degree greater than 1 . Hence from statement 4 follows $\exists\overline{\beta}(V_{gr1},V_{gr2})$ in which a vertex of type **w** is put in correspondence to a vertex of another type .

2) $2 \le Deg(\mathbf{x}) \le n + 2$. A vertex of type **x** is the only vertex type adjacent to a vertex having degree equal to 1 . It follows from statement 5 that $\overline{\exists}\overline{\beta}(V_{gr1},V_{gr2})$ in which a vertex of type **x** corresponds to a vertex of another type .

3) $Deg(\mathbf{y}) = 2$. A vertex of type **y** is not adjacent to a vertex whose degree is 1 . It follows from statement 5 that $\overline{\exists}\overline{\beta}(V_{gr1},V_{gr2})$ in which a vertex of type **y** corresponds to a vertex of type **x** . The degrees of vertices of types **w,z,p** cannot be equal to Deg(**y**) . It follows from statement 4 that $\overline{\exists}\overline{\beta}(V_{gr1},V_{gr2})$ in which a vertex of type **y** corresponds to a vertex of another type

4) $n + 3 \le Deg(\mathbf{z}) \le 2n + 3$. That is, the degree of any vertex of type **z** cannot be equal to the degree of a vertex of another type .
From here and statement 4, it follows that $\overline{\exists}\overline{\beta}(V_{gr1},V_{gr2})$ in which a vertex of type **z** is put in correspondence to a vertex of another type.

5) $Deg(\mathbf{p}) = n$. A vertex of type **p** is not adjacent to a vertex whose degree is 1 . It follows from statement 5 that $\overline{\exists}\overline{\beta}(V_{gr1},V_{gr2})$ in which a vertex of type **p** corresponds to a vertex of type **x** .
The degrees of the vertices of types **w,z,y** cannot be equal to Deg(**p**) . Hence and statement 4 it follows that $\overline{\exists}\overline{\beta}(V_{gr1},V_{gr2})$ in which a vertex of type **P** is put in correspondence to a vertex of a different type . The assertion is completely proved .

Assertion 7. Given $V_{gr1}((\dot{S},\dot{P},\cup_1^n \dot{W}_i),(\dot{U}_c,\dot{U}_{xz},\cup_1^n \dot{U}_i))$, $V_{gr2}((\ddot{S},\ddot{P},\cup_1^n \ddot{W}_i),(\ddot{U}_c,\ddot{U}_{xz},\cup_1^n \ddot{U}_i))$, where $|\dot{S}| = n$,$|\ddot{S}| = n$, $|\dot{P}| = 3$, $|\ddot{P}| = 3$. If $V_{gr1} \sim V_{gr2}$, then $\forall\overline{\beta}(V_{gr1},V_{gr2})$ puts the same-named vertices of one graph in correspondence with the same-named vertices

of the same type. That is, $(V_{gr1} \sim V_{gr2}) \rightarrow (\forall\overline{\beta}(V_{gr1},V_{gr2}) =$
$= \{\dot{s}_{i1} \leftrightarrow \ddot{s}_{j1}, \dot{s}_{i2} \leftrightarrow \ddot{s}_{j2},\ldots,\dot{s}_{in} \leftrightarrow \ddot{s}_{jn}\} \cup \dot{P} \leftrightarrow \ddot{P})$.

Proof .

It follows from Assertion 6 that $\forall\overline{\beta}(V_{gr1},V_{gr2})$ puts vertices of the same type in correspondence to each other . It remains to prove that if $\dot{s}_l \in \dot{S}$,$\ddot{s}_j \in \ddot{S}$, where $\dot{s}_i = \{\dot{w}_i,\dot{x}_i,\dot{z}_i,\dot{y}_i\}$,$\ddot{s}_j = \{\ddot{w}_j,\ddot{x}_j,\ddot{z}_j,\ddot{y}_j\}$, $i,j \in \overline{1,n}$ and at least one of the relations $\dot{x}_l \leftrightarrow \ddot{x}_j$,$\dot{y}_i \leftrightarrow \ddot{y}_j$, $\dot{z}_i \leftrightarrow \ddot{z}_j$, $\dot{w}_i \leftrightarrow \ddot{w}_j$ belongs to $\overline{\beta}(V_{gr1},V_{gr2})$, then each of them belongs to $\overline{\beta}(V_{gr1},V_{gr2})$.

The proof will be carried out in 4 steps .For each relation, we prove that if it belongs to $\overline{\beta}(V_{gr1},V_{gr2})$, then the
other three relations also belong to $\overline{\beta}(V_{gr1},V_{gr2})$.

1)Let $\dot{x}_i \leftrightarrow \ddot{x}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, we need to prove that $\{\dot{y}_i \leftrightarrow \ddot{y}_j , \dot{z}_i \leftrightarrow \ddot{z}_j , \dot{w}_i \leftrightarrow \ddot{w}_j\} \in \overline{\beta}(V_{gr1},V_{gr2})$. Suppose the contrary $\dot{x}_l \leftrightarrow \ddot{x}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, and at least one of the relations : $\dot{y}_i \leftrightarrow \ddot{y}_j$, $\dot{z}_i \leftrightarrow \ddot{z}_j$, $\dot{w}_i \leftrightarrow \ddot{w}_j$
does not belong to $\overline{\beta}(V_{gr1},V_{gr2})$.

a) Let $\exists$ be such a $\overline{\beta}(V_{gr1},V_{gr2})$ such that $\dot{x}_i \leftrightarrow \ddot{x}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, but $\dot{w}_i \leftrightarrow \ddot{w}_j \in \overline{\beta}(V_{gr1},V_{gr2})$. That is, $\dot{x}_i \leftrightarrow \ddot{x}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, $\overline{\beta}(\dot{w}_i) \neq \ddot{w}_j$,$\overline{\beta}(\dot{w}_i) = \ddot{w}_{j1}$, where j ≠ j1 . Hence $f(\dot{x}_i,\dot{w}_i) = f(\ddot{x}_j,\ddot{w}_{j1})$.

On the other hand $f(\dot{x}_i,\dot{w}_i) \neq 0$, and $f(\ddot{x}_j,\ddot{w}_{j1}) = 0$, since any vertex of type **x** is adjacent only to a vertex of the same name of type **w** . Hence $f(\dot{x}_i,\dot{w}_i) \neq f(\ddot{x}_j,\ddot{w}_{j1})$. The resulting contradiction refutes assumption .

b) Let $\exists\ \overline{\beta}(V_{gr1},V_{gr2})$ such that $\overline{\beta}(\dot{x}_i) = \ddot{x}_j$, but $\overline{\beta}(\dot{y}_i) = \ddot{y}_{j1}$, where j≠ j1 . Hence $f(\dot{x}_i,\dot{y}_i) = f(\ddot{x}_j,\ddot{y}_{j1})$. On the other hand a vertex of type **x** is adjacent only to a vertex of the same name of type **y** . Hence
$f(\dot{x}_l,\dot{y}_i) \neq 0$, and $f(\ddot{x}_j,\ddot{y}_{j1}) = 0$, i.e., $f(\dot{x}_i,\dot{y}_i) \neq f(\ddot{x}_j,\ddot{y}_{j1})$. The resulting contradiction refutes the assumption .

c) Let $\exists\ \overline{\beta}(V_{gr1},V_{gr2})$ such that $\overline{\beta}(\dot{x}_i) = \ddot{x}_j$, but $\overline{\beta}(\dot{z}_i) = \ddot{z}_{j1}$, where j ≠ j1 . From (b) it follows that $\overline{\beta}(\dot{y}_i) = \ddot{y}_j$. Hence $f(\dot{y}_i,\dot{z}_i) = f(\ddot{y}_j,\ddot{z}_{j1})$.

On the other hand , since $\ddot{y}_j$ is adjacent only to a vertex of type **z** of the same name, $f(\dot{y}_i,\dot{z}_i) \neq 0$, $f(\ddot{y}_j,\ddot{z}_{j1}) = 0$, i.e., $f(\dot{y}_i ,\dot{z}_i) \neq f(\ddot{y}_j,\ddot{z}_{j1})$.
The resulting contradiction refutes the assumption .

2) Let $\dot{y}_i \leftrightarrow \ddot{y}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, it is required to prove that $\{\dot{x}_i \leftrightarrow \ddot{x}_j , \dot{z}_i \leftrightarrow \ddot{z}_j , \dot{w}_i \leftrightarrow \ddot{w}_j\} \in \overline{\beta}(V_{gr1},V_{gr2})$. Suppose the contrary ,$\dot{y}_i \leftrightarrow \ddot{y}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, and at least one of the relations $\dot{x}_i \leftrightarrow \ddot{x}_j$, $\dot{z}_i \leftrightarrow \ddot{z}_j$, $\dot{w}_i \leftrightarrow \ddot{w}_j$ does not belong to $\overline{\beta}(V_{gr1},V_{gr2})$.

a) Let $\exists\ \overline{\beta}(V_{gr1},V_{gr2})$) such that ,$\dot{y}_i \leftrightarrow \ddot{y}_j \in \overline{\beta}(V_{gr1},V_{gr2})$ but $\dot{x}_i \leftrightarrow \ddot{x}_j \in \overline{\beta}(V_{gr1},V_{gr2})$. That is $\overline{\beta}(\dot{y}_i) = \ddot{y}_j$,$\overline{\beta}(\dot{x}_i) = \ddot{x}_{j1}$,where j≠ j1 . Hence $f(\dot{y}_i,\dot{x}_i) = f(\ddot{y}_j,\ddot{x}_{j1})$. On the other hand , $f(\dot{y}_i,\dot{x}_i) \neq 0$, $f(\ddot{y}_j,\ddot{x}_{j1}) = 0$, since any vertex of type **y** is adjacent only to a vertex of the same name of type **x** . Hence $f(\dot{y}_i,\dot{x}_i) \neq f(\ddot{y}_j,\ddot{x}_{j1})$. The resulting
the contradiction refutes the assumption .

b) Let $\exists\ \overline{\beta}(V_{gr1},V_{gr2})$ such that $\dot{y}_i \leftrightarrow \ddot{y}_j \in \overline{\beta}(V_{gr1},V_{gr2})$,but

$\dot{z}_i \leftrightarrow \ddot{z}_j \overline{\in} \bar{\beta}(V_{gr1},V_{gr2})$ That is $\bar{\beta}(\dot{y}_i) = \ddot{y}_j$, $\bar{\beta}(\dot{z}_i)$ ¿ $\ddot{z}_{j1}$where j ≠ j1 . Hence $f(\dot{y}_i,\dot{z}_i) = f(\ddot{y}_j,\ddot{z}_{j1})$. On the other hand $f(\dot{y}_i,\dot{z}_i) \neq 0$, $f(\ddot{y}_j,\ddot{z}_{j1}) = 0$, since any vertex of type **y** is adjacent only to a vertex of the same name of type **z** .Hence $f(\dot{y}_i,\dot{z}_i) \neq f(\ddot{y}_j,\ddot{z}_{j1})$. The resulting contradiction refutes the assumption .

c) Let $\exists\, \bar{\beta}(V_{gr1},V_{gr2})$ such that $\bar{\beta}(\dot{y}_i) = \ddot{y}_j$, but $\bar{\beta}\dot{w}_i) = \ddot{w}_{j1}$, where j ≠j1 , i.e. $\bar{\beta}(\dot{w}_i) \neq \ddot{w}_j$. But in this case it follows from (a) that $\bar{\beta}(\dot{x}_i) = \ddot{x}_j$. Hence $f(\dot{x}_i,\dot{w}_i) = f(\ddot{x}_j,\ddot{w}_{j1})$. On the other hand, since a vertex of type **x** is adjacent only to a vertex of the same name of type **w** , $f(\dot{x}_i,\dot{w}_i) \neq 0$, $f(\ddot{x}_j,\ddot{w}_{j1}) = 0$. Hence $f(\dot{x}_i,\dot{w}_i) \neq f(\ddot{x}_j,\ddot{w}_{j1})$. The resulting contradiction refutes the assumption .

3) Let $\dot{w}_i \leftrightarrow \ddot{w}_j \in \bar{\beta}(V_{gr1},V_{gr2})$, it is required to prove that $\{\dot{x}_i \leftrightarrow \ddot{x}_j , ,\dot{y}_i \leftrightarrow \ddot{y}_j, \dot{z}_i \leftrightarrow \ddot{z}_j\} \in \bar{\beta}$(Vgr1,Vgr2) . Suppose the contrary $\dot{w}_i \leftrightarrow \ddot{w}_j \in \bar{\beta}(V_{gr1},V_{gr2})$, and at least one of the relations $\dot{x}_i \leftrightarrow \ddot{x}_j$, $\dot{y}_i \leftrightarrow \ddot{y}_j$, $\dot{z}_i \leftrightarrow \ddot{z}_j$ does not belong to $\bar{\beta}(V_{gr1},V_{gr2})$.

(a) Let $\exists\, \bar{\beta}(V_{gr1},V_{gr2})$ such that $\dot{w}_i \leftrightarrow \ddot{w}_j \in \bar{\beta}(V_{gr1},V_{gr2})$, but $\dot{x}_i \leftrightarrow \ddot{x}_j \overline{\in} \bar{\beta}(V_{gr1},V_{gr2})$. That is, $\bar{\beta}(\dot{w}_i) = \ddot{w}_j$, $\bar{\beta}(\dot{x}_i) = \ddot{x}_{j1}$, Where j ≠ j1 . Hence $f(\dot{w}_i,\dot{x}_i) = f(\ddot{w}_j,\ddot{x}_{j1})$. On the other hand $f(\dot{w}_i,\dot{x}_i) \neq 0$,$f(\ddot{w}_j,\ddot{x}_{j1}) = 0$ since any vertex of type **w** is adjacent only to a vertex of the same name of type **x** . Hence $f(\dot{w}_i,\dot{x}_i) \neq f(\ddot{w}_j,\ddot{x}_{j1})$. The resulting contradiction refutes the assumption .

b) Let $\exists\, \bar{\beta}(V_{gr1},V_{gr2})$ such that $\dot{w}_i \leftrightarrow \ddot{w}_j \in \bar{\beta}(V_{gr1},V_{gr2})$, but

$\dot{y}_i \leftrightarrow \ddot{y}_j \overline{\in} \bar{\beta}(V_{gr1},V_{gr2})$. That is, $\bar{\beta}(\dot{w}_i) = \ddot{w}_j$, $\bar{\beta}(\dot{y}_i) = \ddot{y}_{j1}$, where j1 ≠ j . Then from (a) it follows that $\bar{\beta}(\dot{x}_i) = \ddot{x}_j$. Hence $f(\dot{x}_i,\dot{y}_i) = f(\ddot{x}_j,\ddot{y}_{j1})$. On the other hand , since a vertex of type **x** is adjacent only to a vertex of type **y** with the same name, $f(\dot{x}_i,\dot{y}_i) \neq 0$, $f(\ddot{x}_j,\ddot{y}_{j1}) = 0$. Hence $f(\dot{x}_i,\dot{y}_i) \neq f(\ddot{x}_j,\ddot{y}_{j1})$. The resulting contradiction refutes the assumption .

c) Let $\exists\, \bar{\beta}(V_{gr1},V_{gr2})$ such that $\dot{w}_i \leftrightarrow \ddot{w}_j \in \bar{\beta}(V_{gr1},V_{gr2})$, but $\dot{z}_i \leftrightarrow \ddot{z}_j \overline{\in} \bar{\beta}(V_{gr1},V_{gr2})$. That is, $\bar{\beta}(\dot{w}_i) = \ddot{w}_j$,$\bar{\beta}(\dot{z}_i) = \ddot{z}_{j1}$, where j1 ≠ j . Hence and (b) follows $\bar{\beta}(\dot{y}_i) = \ddot{y}_j$. Hence $f(\dot{y}_i,\dot{z}_i)=f(\ddot{y}_j,\ddot{z}_{j1})$. On the other hand , since a vertex of type **y** is adjacent only to a vertex of type **z** with the same name, $f(\dot{y}_i,\dot{z}_i) \neq 0$, $f(\ddot{y}_j,\ddot{z}_{j1}) = 0$. Hence $f(\dot{y}_i,\dot{z}i) \neq f(\ddot{y}_j,\ddot{z}_{j1})$. The resulting contradiction refutes the assumption .

4) Let $\dot{z}_i \leftrightarrow \ddot{z}_j \in \bar{\beta}(V_{gr1},V_{gr2})$, it is required to prove that $\{\dot{x}_i \leftrightarrow \ddot{x}_j , \dot{y}_i \leftrightarrow \ddot{y}_j , \dot{w}_i \leftrightarrow \ddot{w}_j\} \in \bar{\beta}(V_{gr1},V_{gr2})$. Suppose the contrary

$\dot{z}_i \leftrightarrow \ddot{z}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, and at least one of the relations $\dot{x}_i \leftrightarrow \ddot{x}_j$, $\dot{y}_i \leftrightarrow \ddot{y}_j$, $\dot{w}_i \leftrightarrow \dot{w}_i$ does not belong to $\overline{\beta}(V_{gr1},V_{gr2})$.

(a) Let $\exists\ \overline{\beta}(V_{gr1},V_{gr2})$ such that $\dot{z}_i \leftrightarrow \ddot{z}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, but $\dot{y}_i \leftrightarrow \ddot{y}_j \in \overline{\beta}(V_{gr1},V_{gr2})$. That is ,$\overline{\beta}(\dot{z}_i) = \ddot{z}_j$, $\overline{\beta}(\dot{y}_i)$ ¿ $\ddot{y}_{j1}$ where j1 ≠ j . Hence $f(\dot{z}_i,\dot{y}_i) = f(\ddot{z}_j,\ddot{y}_{j1})$. On the other hand $f(\dot{z}_i,\dot{y}_i) \neq 0$, $f(\ddot{z}_j,\ddot{y}_{j1}) = 0$, since any vertex of type **z** is adjacent only to a vertex of the same name of type **y** . Hence $f(\dot{z}_i,\dot{y}_i) \neq f(\ddot{z}_j,\ddot{y}_{j1})$. The resulting contradiction refutes the assumption .

b) Let $\exists\ \overline{\beta}(V_{gr1},V_{gr2})$ such that $\dot{z}_i \leftrightarrow \ddot{z}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, but $\dot{x}_i \leftrightarrow \ddot{x}_j \in \overline{\beta}(V_{gr1},V_{gr2})$. That is $\overline{\beta}(\dot{z}_i) = \ddot{z}_j$, $\overline{\beta}(\dot{x}_i) = \ddot{x}_{j1}$, where j1 ≠ j . Hence and (a) follows $\overline{\beta}(\dot{y}_i) = \ddot{y}_j$. Hence $\dot{x}_i \leftrightarrow \ddot{x}_j \in \overline{\beta}((V_{gr1},V_{gr2})$. That is, $\overline{\beta}(\dot{x}_i) = \ddot{x}_{j1}$, $\overline{\beta}(\dot{z}_i) = \ddot{z}_j$, $f(\dot{y}_i,\dot{x}_i) = f(\ddot{y}_j, \ddot{x}_{j1})$. On the other hand $f(\dot{y}_i,\dot{x}_i) \neq 0$, $f(\ddot{y}_j,\ddot{x}_{j1}) = 0$, since a vertex of type **y** is adjacent only to a vertex of type **x**$\ddot{x}_j$with the same name . Hence $f(\dot{y}_i,\dot{x}_i) \neq f(\ddot{y}_j,\ddot{x}_{j1})$. The resulting contradiction refutes the assumption .

c) Let $\exists$ be such a $\overline{\beta}(V_{gr1},V_{gr2})$ such that $\dot{z}_i \leftrightarrow \ddot{z}_j \in \overline{\beta}(V_{gr1},V_{gr2})$, but $\dot{w}_i \leftrightarrow \ddot{w}_j \in \overline{\beta}$(Vgr1,Vgr2) . That is, $\overline{\beta}(\dot{z}_i) = \ddot{z}_j$,$\overline{\beta}(\dot{w}_i) = \ddot{w}_{j1}$, where j1 ≠ j . Hence and from (b) it follows that $\overline{\beta}(\dot{x}_i) = \ddot{x}_j$. Hence

$f(\dot{x}_i,\dot{w}_i) = f(\ddot{x}_j,\ddot{w}_{j1})$.

On the other hand $f(\dot{x}_i,\dot{w}_i) \neq 0$, $f(\ddot{x}_j,\ddot{w}_{j1}) = 0$, since any vertex of type **x** is adjacent only to a vertex of the same name of type **w** .Hence $f(\dot{x}_i,\dot{w}_i) \neq f(\ddot{x}_j,\ddot{w}_{j1})$. The resulting contradiction refutes the conjecture . The assertion is completely proved .

4. Immersion of an oriented graph (Berge graph) Into an undirected graph without loops and multiple edges V-graph .

Definition 5. Let an unloaded $V_{gr}((S,P,\cup_1^n W_i),(U_c,U_{xz},\cup_1^n U_i))$ with dimension $R(V_{gr}) = n$ and an oriented graph$\vec{G}_1(V_1,U_1)$, where $|V_1| = n$.$\forall\ v_i \in V_1$,where $i \in \overline{1,n}$, with number i , correspond to two vertices of the same name $x_i \in s_i$, $z_i \in s_i$, with the same number i, where $i \in \overline{1,n}$ and $s_i \in S$. A vertex of the graph $\vec{G}_1$ with number i and vertices $(w_i,x_i,z_i,y_i) \in S$ of the V-graph with the same number i will be called homonymous . Let V_{gr} be supplemented with edges so that the following conditions are fulfilled .

For $\forall$ $v_i, v_j \in V$, where $i,j \in \overline{1,n}$, if v_i has a loop, then the vertices with the same name in V_{gr} x_i and z_i are connected by an edge ; if v_j has a loop, then the vertices with the same name in V_{gr} x_j and z_j are connected by an edge .
If it is an arc going from v_i to v_j , i.e.$(v_i,v_j) \in U_1$, then the vertices x_i and z_j with the same name in Vgr are connected by an edge . If it is an arc going from v_j to v_i , i.e.,$(v_j,v_i) \in U_1$, then in V_{gr} the vertices z_i and x_j with the same name are connected by an edge . I.e. for $\forall i,j \in \overline{1,n}$ and $i \neq j$, the relations :

$(v_i,v_j) \in U_1 \leftrightarrow (x_i,z_j) \in U_{xz}$,

$(v_j,v_i) \in U_1 \leftrightarrow (x_j,z_i) \in U_{xz}$,

$(v_i,v_i) \in U_1 \leftrightarrow (x_i,z_i) \in U_{xz}$,

$(v_j,v_j) \in U_1 \leftrightarrow (x_j,z_j) \in U_{xz}$.

The V-graph thus augmented with edges will be called the image of the graph $\vec{G}_1(V_1,U_1)$ or the representation of the oriented graph $\vec{G}_1(V_1,U_1)$ as an undirected graph without loops and multiple edges or loading an oriented graph into a V-graph and denote by $Vgr[\vec{G}_1(V_1,U_1)]$.

Thus the V-graph augmented with edges will be also

be called an immersion of an oriented graph into an undirected graph without loops and multiple edges .

5. On the non-existence of the polynomial P problem and the a polynomial algorithm for the graph isomorphism problem .

Assertion 8. Let oriented graphs $\vec{G}_1(V_1,U_1)$,$\vec{G}_2(V_2,U_2)$ be given , where $|V_1| = n$, $|V_2| = n$. Their corresponding images
$V_{gr1}[\vec{G}_1(V_1,U_1),((\dot{S},\dot{P},\cup_1^n \dot{W}_i),(U_c,U_{xz},\cup_1^n \dot{U}_i))]$,
$V_{gr2}[\vec{G}_2(V_2,U_2),((\ddot{S},\ddot{P},\cup_1^n \ddot{W}_i),(\ddot{U}_c,\ddot{U}_{xz},\cup_1^n \ddot{U}_i))]$, where $R(V_{gr1}) = n$ and $R(V_{gr2}) = n$ .The oriented graphs $\vec{G}_1$, $\vec{G}_2$ are isomorphic if and only if when their images $V_{gr1}[\vec{G}_1]$, $V_{gr2}[\vec{G}_2]$ are isomorphic .
If mutually the one-to-one correspondence between vertices $\vec{G}_1$,$\vec{G}_2$ preserves adjacency , then the one-to-one correspondence between coalitional vertices of V-graphs with the same name as them $V_{gr1}[\vec{G}_1]$, $V_{gr2}[\vec{G}_2]$ also preserves adjacency . Conversely, if mutually unambiguous correspondence between coalitional vertices $V_{gr1}[\vec{G}_1]$, $V_{gr2}[\vec{G}_2]$ preserves adjacency , then a mutually

the one-to-one correspondence between the vertices of graphs $\vec{G}_1$(V$_1$,U$_1$) , $\vec{G}_2$(V$_2$,U$_2$) with the same name as them preserves adjacency . I.e.

$\vec{G}_1$ $\vec{G}_2$ ⇔ V$_{gr1}$[$\vec{G}_1$(V$_1$,U$_1$] V$_{gr2}$[$\vec{G}_2$(V$_2$,U$_2$)] ,

$(\{\dot{v}_{i1} \leftrightarrow \ddot{v}_{j1}, \dot{v}_{i2} \leftrightarrow \ddot{v}_{j2}, \dots, \dot{v}_{in} \leftrightarrow \ddot{v}_{jn}\} = \overline{\beta}(\vec{G}_1, \vec{G}_2)) \Leftrightarrow$

$\Leftrightarrow (\{\dot{s}_{l1} \leftrightarrow \ddot{s}_{j1}, \dot{s}_{l2} \leftrightarrow \ddot{s}_{j2}, \dots, \dot{s}_{in} \leftrightarrow \ddot{s}_{jn}\} \in \overline{\beta}(V_{gr1}[\vec{G}_1], V_{gr2}[\vec{G}_2])$, where

$\{\dot{s}_{i1}, \dot{s}_{i2}, \dots, \dot{s}_{in}\} = \dot{S}$, $\{\ddot{s}_{j1}, \ddot{s}_{j2}, \dots, \ddot{s}_{jn}\} = \ddot{S}$,

$\{\dot{v}_{i1}, \dot{v}_{i2}, \dots, \dot{v}_{in}\} = V_1$, $\{\ddot{v}_{j1}, \ddot{v}_{j2}, \dots, \ddot{v}_{jn}\} = V_2$.

Proof .

Sufficiency .

Given V$_{gr1}$[$\vec{G}_1$(V$_1$,U$_1$] V$_{gr2}$[$\vec{G}_2$(V$_2$,U$_2$)] . It is required to prove that $\vec{G}_1$(V$_1$,U$_1$) $\vec{G}_2$(V$_2$,U$_2$) and if $\overline{\beta}$(V$_{gr1}$ [$\vec{G}_1$(V$_1$,U$_1$)], V$_{gr2}$[$\vec{G}_2$(V$_2$,U$_2$)]) ∋

∋ $(\{\dot{s}_{l1} \leftrightarrow \ddot{s}_{j1}, \dot{s}_{l2} \leftrightarrow \ddot{s}_{j2}, \dots, \dot{s}_{in} \leftrightarrow \ddot{s}_{jn}\}$, then

$\{\dot{v}_{i1} \leftrightarrow \ddot{v}_{j1}, \dot{v}_{i2} \leftrightarrow \ddot{v}_{j2}, \dots, \dot{v}_{in} \leftrightarrow \ddot{v}_{jn}\} = \overline{\beta}(\vec{G}_1(V_1,U_1), \vec{G}_2(V_2,U_2))$.

From V$_{gr1}$[$\vec{G}_1$(V$_1$,U$_1$] V$_{gr2}$[$\vec{G}_2$(V$_2$,U$_2$)] and assertion 7, it follows that

$\forall \overline{\beta}(V_{gr1}[\vec{G}_1], V_{gr2}[\vec{G}_2]) = \{\dot{s}_{l1} \leftrightarrow \ddot{s}_{j1}, \dot{s}_{l2} \leftrightarrow \ddot{s}_{j2}, \dots, \dot{s}_{in} \leftrightarrow \ddot{s}_{jn}\} \cup \dot{P} \leftrightarrow \ddot{P}$.

Hence , let

$(\dot{w}_{i1}, \dot{x}_{i1}, \dot{y}_{i1}, \dot{z}_{i1}) \leftrightarrow (\ddot{w}_{j1}, \ddot{x}_{j1}, \ddot{y}_{j1}, \ddot{z}_{j1})$,

$(\dot{w}_{i2}, \dot{x}_{i2}, \dot{y}_{i2}, \dot{z}_{i2}) \leftrightarrow (\ddot{w}_{j2}, \ddot{x}_{j2}, \ddot{y}_{j2}, \ddot{z}_{j2})$,

…

$(\dot{w}_{in}, \dot{x}_{in}, \dot{y}_{in}, \dot{z}_{in}) \leftrightarrow (\ddot{w}_{jn}, \ddot{x}_{jn}, \ddot{y}_{jn}, \ddot{z}_{jn})$,

$\{\dot{P}_1, \dot{P}_2, \dot{P}_3\} \leftrightarrow \{\ddot{P}_1, \ddot{P}_2, \ddot{P}_3\}$

mutually one-to-one correspondence between vertices V$_{gr1}$[$\vec{G}_1$(V$_1$,U$_1$)], V$_{gr2}$[$\vec{G}_2$(V$_2$,U$_2$)] preserving adjacency .

Hence for ∀ k,l $\in \overline{1,n}$, where k ≠l ,

$\dot{x}_{ik} \leftrightarrow \ddot{x}_{jk}$, $\dot{z}_{ik} \leftrightarrow \ddot{z}_{jk}$,

$\dot{x}_{il} \leftrightarrow \ddot{x}_{jl}$, $\dot{z}_{il} \leftrightarrow \ddot{z}_{jl}$,

preserve adjacency . Hence

$f(\dot{x}_{ik}, \dot{z}_{ik}) = f(\ddot{x}_{jk}, \ddot{z}_{jk})$, (1)

$f(\dot{x}_{ik}, \dot{x}_{il}) = 0$, $f(\ddot{x}_{jk}, \ddot{x}_{jl}) = 0$,

since vertices of type x are not adjacent .

$f(\dot{x}_{ik}, \dot{z}_{il}) = f(\ddot{x}_{jk}, \ddot{z}_{jl})$, (2)

$f(\dot{z}_{ik}, \dot{x}_{il}) = f(\ddot{z}_{jk}, \ddot{x}_{jl})$, (3)

$f(\dot{z}_{ik}, \dot{z}_{il}) = f(\ddot{z}_{jk}, \ddot{z}_{jl})$,

$f(\dot{x}_{il}, \dot{z}_{il}) = f(\ddot{x}_{jl}, \ddot{z}_{jl})$, (4) On the other hand $\dot{v}_{ik}$,$\dot{v}_{il}$ of graph $\vec{G}_1(V_1,U_1)$ and $\ddot{v}_{jk}$, $\ddot{v}_{jl}$ of graph $\vec{G}_2(V_2,U_2)$ are homonymous vertices respectively with $\dot{x}_{ik},\dot{z}_{ik}$; $\dot{x}_{il},\dot{z}_{il}$ and $\ddot{x}_{jk},\ddot{z}_{jk}$; $\ddot{x}_{jl},\ddot{z}_{jl}$.

Consider the relations

$$\dot{v}_{ik} \leftrightarrow \ddot{v}_{jk} \qquad (5)$$

$$\dot{v}_{il} \leftrightarrow \ddot{v}_{jl}. \qquad (6)$$

From relation 1 and the loading of the graphs V_{gr1} and V_{gr2} it follows that $f(\dot{v}_{ik},\dot{v}_{ik}) = f(\ddot{v}_{jk},\ddot{v}_{jk})$.

From relations 2 , 3 and the loading of the graphs V_{gr1} and V_{gr2} it follows that $f(\dot{v}_{ik},\dot{v}_{il}) = f(\ddot{v}_{jk},\ddot{v}_{jl})$.

From relation 4 and the loading of the graphs V_{gr1} and V_{gr2} it follows that $f(\dot{v}_{il},\dot{v}_{il}) = f(\ddot{v}_{jl},\ddot{v}_{jl})$.

It follows that relations 5 , 6 preserve adjointness .

By virtue of an arbitrary choice of $\dot{x}_{ik},\dot{z}_{ik}$; $\dot{x}_{il},\dot{z}_{il}$ and $\ddot{x}_{jk},\ddot{z}_{jk}$; $\ddot{x}_{jl},\ddot{z}_{jl}$
It follows that

$\overline{\beta}(\vec{G}_1(V_1,U_1),\vec{G}_2(V_2,U_2)) = \{\dot{v}_{i1} \Leftrightarrow \ddot{v}_{j1}, \dot{v}_{i2} \Leftrightarrow \ddot{v}_{j2}, \dots ,\dot{v}_{in} \Leftrightarrow \ddot{v}_{jn}\}$.

That is, $\vec{G}_1(V_1,U_1)$ $\vec{G}_2(V_2,U_2)$. Sufficiency is proved .

Necessity .

Given $\vec{G}_1(V_1,U_1)$ $\vec{G}_2(V_2,U_2)$ it is required to prove that

$V_{gr1}[\vec{G}_1(V_1,U_1)]$ $V_{gr2}[\vec{G}_2(V_2,U_2)]$ and if

$\overline{\beta}(\vec{G}_1(V_1,U_1),\vec{G}_2(V_2,U_2)) =\{\dot{v}_{i1} \leftrightarrow \ddot{v}_{j1} ,\dot{v}_{i2} \leftrightarrow \ddot{v}_{j2} , \dots , \dot{v}_{in} \leftrightarrow \ddot{v}_{jn}\}$ then

$\{\dot{s}_{l1} \leftrightarrow \ddot{s}_{j1} ,\dot{s}_{l2} \leftrightarrow \ddot{s}_{j2} , \dots , \dot{s}_{in} \leftrightarrow \ddot{s}_{jn}\} \in \overline{\beta}(V_{gr1}[\vec{G}_1(V_1,U_1)] ,V_{gr2}[\vec{G}_2(V_2,U_2)])$.

Let $\overline{\beta}(\vec{G}_1(V_1,U_1),\vec{G}_2(V_2,U_2)) = \{\dot{v}_{i1} \leftrightarrow \ddot{v}_{j1} , \dot{v}_{i1} \leftrightarrow \ddot{v}_{j2} , \dots , \dot{v}_{i1} \leftrightarrow \ddot{v}_{jn}\}$ where $\dot{v}_{ik} \in V_1$ for $\forall$ k $\in \overline{1,n}$ and $\ddot{v}_{jk} \in V_2$ for $\forall$ k $\in \overline{1,n}$. Take $\forall$r,l $\in \overline{1,n}$, where r ≠ l .

$$\dot{v}_{ir} \leftrightarrow \ddot{v}_{jr} , \qquad (8)$$

$$\dot{v}_{il} \leftrightarrow \ddot{v}_{jl}. \qquad (9)$$

Then $f(\dot{v}_{ir},\dot{v}_{il}) = f(\ddot{v}_{jr},\ddot{v}_{jl})$, (10)

$f(\dot{v}_{ir},\dot{v}_{ir}) = f(\ddot{v}_{jr},\ddot{v}_{jr})$,

$f(\dot{v}_{il},\dot{v}_{il}) = f(\ddot{v}_{jl},\ddot{v}_{jl})$.

From 10 and since V_{gr1} ,V_{gr2} are images, respectively, of the graphs of $\vec{G}_1$,$\vec{G}_2$ it follows.

$$f(\dot{x}_{ir},\dot{z}_{il}) = f(\ddot{x}_{jr},\ddot{z}_{jl}) , \qquad (11)$$

$$f(\dot{x}_{il},\dot{z}_{ir}) = f(\ddot{x}_{jl},\ddot{z}_{jr}) , \qquad (12)$$

$$f(\dot{x}_{ir},\dot{z}_{ir}) = f(\ddot{x}_{jr},\ddot{z}_{jr}) , \qquad (13)$$

$$f(\dot{x}_{il},\dot{z}_{il}) = f(\ddot{x}_{jl},\ddot{z}_{jl}) . \qquad (14)$$

In a V-graph, vertices of type **x** are not connected by edges , and vertices of type **z** are connected by edges . Hence

$f(\dot{x}_{ir},\dot{x}_{il}) = f(\ddot{x}_{jr},\ddot{x}_{jl})$, (15)

$f(\dot{z}_{ir},\dot{z}_{il}) = f(\ddot{z}_{jr},\ddot{z}_{jl})$. (16)

From 11,12,13,14,14,15,16 it follows that

$\dot{x}_{ir} \leftrightarrow \ddot{x}_{jr}$, $\dot{z}_{il} \leftrightarrow \ddot{z}_{jl}$, (17)

$\dot{x}_{il} \leftrightarrow \ddot{x}_{jl}$, $\dot{z}_{ir} \leftrightarrow \ddot{z}_{jr}$, (18)

remain contiguous.

On the other hand .

A vertex $\dot{w}_{ir}$- adjacent only to $\dot{x}_{ir}$.

Vertex $\ddot{w}_{jr}$- adjacent only to $\ddot{x}_{jr}$.

Vertex $\dot{w}_{il}$- adjacent only to $\dot{x}_{il}$.

Vertex $\ddot{w}_{jl}$ - adjacent only to $\ddot{x}_{jl}$.

Hence

$\dot{w}_{ir} \leftrightarrow \ddot{w}_{jr}$, $\dot{w}_{il} \leftrightarrow \ddot{w}_{jl}$ (19)

Together with relations 17,18 preserve the adjacency .

Hence the relations

$(\dot{w}_{ir},\dot{x}_{ir},\dot{z}_{ir}) \leftrightarrow (\ddot{w}_{jr},\ddot{x}_{jr},\ddot{z}_{jr})$, (20)

$(\dot{w}_{il},\dot{x}_{il},\dot{z}_{il}) \leftrightarrow (\ddot{w}_{jl},\ddot{x}_{jl},\ddot{z}_{jl})$ (21)

preserve contiguity .

Vertices of type **y** are adjacent only to vertices of the same name of types **x** and **z** . Hence

$\dot{y}_{ir} \leftrightarrow \ddot{y}_{jr}$, $\dot{y}_{il} \leftrightarrow \ddot{y}_{jl}$

together with relations 20 , 21 preserve adjacency.

Hence

$(\dot{w}_{ir},\dot{x}_{ir},\dot{y}_{ir},\dot{z}_{ir}) \leftrightarrow (\ddot{w}_{jr},\ddot{x}_{jr},\ddot{y}_{jr},\ddot{z}_{jr})$,

$(\dot{w}_{il},\dot{x}_{il},\dot{y}_{il},\dot{z}_{il}) \leftrightarrow (\ddot{w}_{jl},\ddot{x}_{jl},\ddot{y}_{jl},\ddot{z}_{jl})$

preserve adjacency .

The vertices placed in correspondence with each other are taken from (1) arbitrarily . Consequently, there is a one-to-one correspondence between coalition vertices V_{gr1} , V_{gr2} constructed from (1) by replacing vertices from $\vec{G}_1$, $\vec{G}_2$ by coalition vertices of the same name from V_{gr1} , V_{gr2} preserves adjacency .

That is, $\dot{v}_{ir} \leftrightarrow \ddot{v}_{jr}$, where r $\in \overline{1,n}$, is replaced by $\dot{s}_{ir} \leftrightarrow \ddot{s}_{jr}$, and $\dot{v}_{il} \leftrightarrow \ddot{v}_{jl}$, where l $\in \overline{1,n}$, is replaced by $\dot{s}_{il} \leftrightarrow \ddot{s}_{jl}$.

Vertices of type **p** are adjacent to each vertex of type **z** . Hence any mutually one-to-one correspondence between vertices of type **p** of graphs V_{gr1} ,V_{gr2} and the specified mutually one-to-one correspondence between their coalitional vertices preserve adjacency .

That is, V_{gr1} V_{gr2}. The assertion is proved .

Notation .

Let $V_{gr1}((S_1,P_1,\cup_1^n \dot{W}_i),(\dot{U}_c,\dot{U}_{xz},\cup_1^n \dot{U}_i))$ be given ,
$V_{gr2}((S_2,P_2,\cup_1^n \ddot{W}_i),(\ddot{U}_c ,\ddot{U}_{xz},\cup_1^n \ddot{U}_i)))$, where $S_1 = \cup_1^n(\dot{w}_i,\dot{x}_i,\dot{y}_i,\dot{z}_i)$,
$S_2 = \cup_1^n(\ddot{w}_j,\ddot{x}_j,\ddot{y}_j,\ddot{z}_j)$.

We label the vertices $\dot{w}_l,\ddot{w}_r$, $|\dot{W}_l|=|\ddot{W}_r|$ and $|\dot{W}_l| \neq |\dot{W}_k|$, for $\forall$ k $\in \overline{1,n}$, where k ≠ l , and $|\ddot{W}_r| \neq |\ddot{W}_k|$, for $\forall$ k $\in \overline{1,n}$, where k ≠ r .

We will denote labeled vertices by $(\dot{w}_l)$, $(\ddot{w}_r)$. V-graphs in which m vertices are labeled we denote by $V_{gr}(m)$.

Mutually one-to-one correspondence between vertices $V_{gr1}(m)$, $V_{gr2}(m)$ will be denoted by β_m .

Assertion 9 . Let loaded V-graphs be given in which

m - 1 vertices are correctly labeled, where m $\in \overline{1,n}$.

$V_{gr1}[(m - 1),((S_1,P_1,\cup_1^n \dot{W}_i),(\dot{U}_c,\dot{U}_{xz},\cup_1^n \dot{U}_i)]$ ~
~ $V_{gr2}[(m - 1),((S_2,P_2,\cup_1^n \ddot{W}_i),(\ddot{U}_c ,\ddot{U}_{xz},\cup_1^n \ddot{U}_i))]$ where $S_1 = \cup_1^n \dot{s}_i$,
$\dot{s}_i= (\dot{w}_i,\dot{x}_i,\dot{y}_i,\dot{z}_i)$, $S_2 = \cup_1^n \ddot{s}_j$, $\ddot{s}_j= (\ddot{w}_j,\ddot{x}_j,\ddot{y}_j,\ddot{z}_j)$.

We label the vertices $\dot{w}_l \in \dot{s}_l$ and $\ddot{w}_r \in \ddot{s}_r$, where l,r $\in \overline{1,n}$, with proper labels that are equal to each other .

If $V_{gr1}(m - 1)$ ~ $V_{gr2}(m - 1)$, then
$\exists\ \bar{\beta}_{m-1}(V_{gr1}(m - 1),V_{gr2}(m - 1)) \ni \dot{w}_l \leftrightarrow \ddot{w}_r$ if and only if
$(\dot{w}_l) \leftrightarrow (\ddot{w}_r) \in \bar{\beta}_m(V_{gr1}(m),V_{gr2}(m))$, where
$\bar{\beta}_m(V_{gr1}(m) ,V_{gr2}(m)) = \bar{\beta}_{m-1}(V_{grr1}(m - 1) ,V_{gr2}(m - 1)) \cup \{\dot{w}_l \leftrightarrow \ddot{w}_r\}$.

That is, $V_{gr1}(m - 1)$ $V_{gr2}(m – 1)$ →
→$[((\exists\ \bar{\beta}_{m-1}(V_{gr1}(m - 1) ,V_{gr2}(m – 1)) \ni$
$\ni \dot{w}_l \leftrightarrow \ddot{w}_r) \Leftrightarrow ((\dot{w}_l) \leftrightarrow (\ddot{w}_r) \in \bar{\beta}_m(V_{gr1}(m) ,V_{gr2}(m)))]$, where
$\bar{\beta}_m(V_{gr1}(m) ,V_{gr2}(m)) =\bar{\beta}_{m-1}(V_{grr1}(m - 1) ,V_{gr2}(m – 1)) \cup \{\dot{W}_l \leftrightarrow \ddot{W}_r\}$.

Proof .

Necessity .

Given : $V_{gr1}(m - 1)$ $V_{gr2}(m – 1)$ and
$\exists\ \bar{\beta}_{m-1} (V_{gr1}(m - 1) ,V_{gr2}(m - 1)) \ni \dot{w}_l \leftrightarrow \ddot{w}_r$. It is required to prove that
$(\dot{w}_l) \leftrightarrow (\ddot{w}_r) \in \bar{\beta}_m(V_{gr1}(m) ,V_{gr2}(m))$, where $\bar{\beta}_m(V_{gr1}(m) ,V_{gr2}(m))$ =

$= \overline{\beta}_{m-1}(V_{grr1}(m-1), V_{gr2}(m-1)) \cup \{\dot{W}_l \leftrightarrow \ddot{W}_r\}$.

From assertion 3 and the condition of this assertion follows proof .

Sufficiency .

Given

$\overline{\beta}_m(V_{gr1}(m), V_{gr2}(m)) = \overline{\beta}_{m-1}(V_{gr1}(m-1), V_{gr2}(m-1)) \cup \{\dot{W}_l \leftrightarrow \ddot{W}_r\}$

It is required to prove that

$\dot{w}_l \leftrightarrow \ddot{w}_r \in \overline{\beta}_{m-1}(V_{grr1}(m-1), V_{gr2}(m-1))$.

From assertion 3 and the condition of our assertion follows

$\{\dot{W}_l \leftrightarrow \ddot{W}_r\} \in \overline{\beta}_m(V_{gr1}(m), V_{gr2}(m))$, where

$\overline{\beta}_m(V_{gr1}(m), V_{gr2}(m)) = \overline{\beta}_{m-1}(V_{grr1}(m-1), V_{gr2}(m-1)) \cup \{\dot{W}_l \leftrightarrow \ddot{W}_r\}$,

it follows that $\dot{w}_l \leftrightarrow \ddot{w}_r \in \overline{\beta}_{m-1}(V_{gr1}(m-1), V_{gr2}(m-1))$.

The assertion is proved .

Assertion 10 . For the isomorphism problem of graphs without loops

and multiple edges :

a) there does not exist a polynomial solution algorithm ,

b) there does not exist a polynomial P of the task .

Proof .

a) Let $\vec{G}_1(\dot{V}_1, \dot{U}_1)$ and $\vec{G}_2(\ddot{V}_2, \ddot{U}_2)$, where $|\dot{V}_1| = n$, $|\ddot{V}_2| = n$, be given oriented graphs (Berge graphs) . Their corresponding images are

$V_{gr1}[\vec{G}_1(\dot{V}_1, \dot{U}_1), ((\dot{S}, \dot{P}, \cup_1^n \dot{W}_i), (\dot{U}_c, \dot{U}_{xz}, \cup_1^n \dot{U}_i))]$,

$V_{gr2}[\vec{G}_2(\ddot{V}_2, \ddot{U}_2), ((\ddot{S}, \ddot{P}, \cup_1^n \ddot{W}_i), (\ddot{U}_c, \ddot{U}_{xz}, \cup_1^n \ddot{U}_i)]$.

Suppose the contrary . There exists a polynomial algorithm computing isomorphism of graphs without loops and multiple edges . Let us compute $\overline{\beta}$ $(V_{gr1}(\vec{G}_1), V_{gr2}(\vec{G}_2))$. It follows from assertion 7 that

$\overline{\beta}(V_{gr1}(\vec{G}_1), V_{gr2}(\vec{G}_2)) \ni \cup_1^n((\dot{w}_{ik}, \dot{x}_{ik}, \dot{y}_{ik}, \dot{z}_{ik}) \leftrightarrow ((\ddot{w}_{jk}, \ddot{x}_{jK}, \ddot{y}_{jk}, \ddot{z}_{jk}))$.

It follows from hence and assertion 8 that $\overline{\beta}(\vec{G}_1, \vec{G}_2) = \cup_1^n(\dot{v}_{ik} \leftrightarrow \ddot{v}_{jk})$.

I.e., it follows from our assumption that there exists a polynomial algorithm for the Berge graph isomorphism problem . And this contradicts the statement proved by S.V. Yablonsky [1] that there exist Berge graphs for which there does not exist a polynomial algorithm for computing their isomorphism .

The obtained contradiction proves the point a of the assertion .

b) Suppose the contrary . There exists a polynomial P task for the isomorphism problem of graphs without loops and multiple edges .

Let us start labeling with correct labels the vertices of type **w** of graphs V_{gr1},V_{gr2} . Consider a sequentially numbered series of vertices $\dot{w}_1,\dot{w}_2, \dots , \dot{w}_n$ of graph V_{gr1}.
For vertex $\dot{w}_1$ we assign label $k_1 = |\dot{W}_1|$, where $|\dot{W}_1| = 1$.
In the graph V_{gr2} , compute a vertex $\ddot{W}_{j1}$ such that after assigning to it the label $k_{j1} = |\ddot{W}_{j1}|$, where $|\ddot{W}_{j1}| = 1$, P the task establishes that $V_{gr1}(1) \sim V_{gr2}(1)$. In this case, it follows from statements 3 and 2 that $\exists\ \overline{\beta}\ (V_{gr1}(1), V_{gr2}(1))$ such that
$\{\dot{W}_1 \leftrightarrow \ddot{W}_{j1} , \dot{w}_1 \leftrightarrow \ddot{w}_{j1}\} \in \overline{\beta}\ (V_{gr1}(1), V_{gr2}(1))$.

For vertex $\dot{w}_2$ of graph $V_{gr1}(1)$, we assign a label with value $K_2 = |\dot{W}_2|$, where $|\dot{W}_2|= 2$. In the graph $V_{gr2}(1)$, compute a vertex $\ddot{w}_{j2}$ such that after labeling it $k_{j2} = |\ddot{W}_{j2}|$, where $|\ddot{W}_{j2}|= 2$,

P task will establish $V_{gr1}(2) \sim V_{gr2}(2)$. In this case, from statements 3 and 2, it follows $\exists\ \overline{\beta}\ (V_{gr1}(2), V_{gr2}(2))$ such that
$\{\dot{W}_1 \leftrightarrow \ddot{W}_{j1} , \dot{w}_1 \leftrightarrow \ddot{w}_{j1} , \dot{W}_2 \leftrightarrow \ddot{W}_{j2} , \dot{w}_2 \leftrightarrow \ddot{w}_{j2}\} \in \overline{\beta}\ (V_{gr1}(2) , V_{gr2}(2))$.

Let us continue the above process . For each vertex in the set $\{\dot{w}_1,\dot{w}_2,\dot{w}_3, \dots , \dot{w}_n\}$, we compute their corresponding vertices $\{\ddot{w}_{j1},\ddot{w}_{j2},\ddot{w}_{j3},\dots,\ddot{w}_{jn}\}$. That is, let us compute
$\overline{\beta}\ (V_{gr1}(n) , V_{gr2}(n)) \ni \{ \dot{W}_1 \leftrightarrow \ddot{W}_{j1} , \dot{w}_1 \leftrightarrow \ddot{w}_{j1} , \dot{W}_2 \leftrightarrow \ddot{W}_{j2} , \dot{w}_2 \leftrightarrow \ddot{w}_{j2} ,$
$\dots , \dot{W}_n \leftrightarrow \ddot{W}_{jn} , \dot{w}_n \leftrightarrow \ddot{w}_{jn} \}$. (22)

The graphs V_{gr1} , V_{gr2} are subgraphs, respectively, of the graphs $V_{gr1}(n)$, $V_{gr2}(n)$. It follows from 22 that
$\beta(V_{gr1},V_{gr2}) = \overline{\beta}\ (V_{gr1}(n) , V_{gr2}(n)) \setminus \{ \dot{W}_1 \leftrightarrow \ddot{W}_{j1} , \dot{W}_2 \leftrightarrow \ddot{W}_{j2}, \dots ,$
$\dot{W}_n \leftrightarrow \ddot{W}_{jn}\}$. From assertion 1, it follows that $V_{gr1} \sim V_{gr2}$ and $\beta(V_{gr1},V_{gr2})$ preserves adjacency i.e. $\beta(V_{gr1},V_{gr2}) = \overline{\beta}\ (V_{gr1} , V_{gr2})$ and
$\{ \dot{w}_1 \leftrightarrow \ddot{w}_{j1}, \dot{w}_2 \leftrightarrow \ddot{w}_{j2}, \dots , \dot{w}_n \leftrightarrow \ddot{w}_{jn}\} \in \overline{\beta}\ (V_{gr1} , V_{gr2})$.
It follows from hence and assertion 7 that
$\overline{\beta}\ (V_{gr1} , V_{gr2}) =\{ \dot{s}_1 \leftrightarrow \ddot{s}_{j1} ,\dot{s}_2 \leftrightarrow \ddot{s}_{j2} , \dots , \dot{s}_n \leftrightarrow \ddot{s}_{jn} \} \cup \{\dot{P} \leftrightarrow \ddot{P}\}$. Hence and assertion 8 follows
$\{\dot{v}_1 \leftrightarrow \ddot{v}_{j1} ,\dot{v}_2 \leftrightarrow \ddot{v}_{j2}, \dots , \dot{v}_n \leftrightarrow \ddot{v}_{jn} \} = \overline{\beta}(\vec{G}_1,\vec{G}_2)$. That is, we applied the polynomial P task n times and
computed the isomorphism of Berge graphs . We obtained a contradiction .
S.V. Yablonsky [1] proved that there exist oriented
graphs for which there does not exist a polynomial algorithm

for computing their isomorphism . Hence our assumption is incorrect . There is no polynomial P of the task and there is no polynomial algorithm for solving .

Assertion 11. For NP tasks belonging to the NPC class there are no polynomial P problems and polynomial solution algorithms.

Proof .

The polynomial solution to any problem from the NPC class gives polynomial algorithm to solve each problem belonging to the NPC class.

The graph isomorphism problem is a special case of the problem isomorphism to a subgraph belonging to the class NPC. From here

a) if there is a polynomial P task for at least one problems from the NPC class, then there is a polynomial P problem for the graph isomorphism problem,

b) if there is a polynomial algorithm for solving at least for one of the problems of the NPC class, then there is a polynomial algorithm for solving the graph isomorphism problem.

From here and assertion 10 it follows that for any problem of the class NPC there is no polynomial P task and there is no polynomial solution algorithm.

Literature .

1. С.В. Яблонский .
“Об алгоритмических трудностях синтеза минимальных
контактных схем//Проблемы кибернетики . М., 1959. с. 75 – 121”