The real physical meaning of Planck's constant

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It is shown how the operator of kinetic energy of a microparticle proposed by Schrödinger contains the real physical meaning of Planck's constant.

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To make the physical meaning of Planck's constant clear:

$$\hbar = 1{,}05458 \cdot 10^{-34}\ \text{Дж} \cdot \text{с} \tag{1}$$

which suggests itself from the kinetic energy operator proposed by Schrödinger in wave quantum mechanics [1]:

$$\hat{E}_k = \frac{\hbar^2}{2m}\Delta \tag{2}$$

let's use corpuscular kinetic energy from Newton's classical mechanics [2]:

$$E_k = \frac{P^2}{2m} \tag{3}$$

And also de Broglie's formula for momentum [3]:

$$\mathbf{P} = \hbar\mathbf{k} \tag{4}$$

and the momentum operator proposed by Schrödinger:

$$\hat{\mathbf{P}} = -i\hbar\nabla \tag{5}$$

The similarity of the right-hand side of the kinetic energy operator (2) with the right-hand side of corpuscular kinetic energy (3), as well as the connection of Planck's constant (1) in formulas (4) and (5) with corpuscular momentum indicates that the physical meaning of Planck's constant (1) can be similar to corpuscular momentum:

$$\mathbf{P} = m\mathbf{v} \tag{6}$$

The verify this, let's use the following relations:

$$\frac{1}{i\mathbf{r}}(-i\nabla) = \Delta \tag{7}$$

$$\mathbf{k} = -i\nabla \tag{8}$$

here, ***ir*** – linear wavelength of microparticle , ***k*** – wave vector.

The kinetic energy operator (2) proposed by Schrödinger based on (7) takes the form:

$$\hat{E}_k = \frac{\hbar^2}{2mi\mathbf{r}}(-i\nabla) \tag{9}$$

In turn, if we introduce the notation:

$$\mathbf{m}^* = mi\mathbf{r} \tag{10}$$

then, taking into account (10) instead of (9), we obtain the kinetic energy operator (2) in the form:

$$\hat{E}_k = \frac{\hbar^2}{2\mathbf{m}^*}(-i\nabla) \tag{11}$$

Now, if we denote the quantity from the right side of (11) as a separate quantity:

$$\mathbf{E}^*_k = \frac{\hbar^2}{2\mathbf{m}^*} \tag{12}$$

then the value (12) we obtained will turn out to be a complete analogue of corpuscular kinetic energy (3).

As we can see, it has become obvious that the right-hand side of the kinetic energy operator (2) proposed by Schrödinger contains the quantity (12), which is a complete analogue of the classical corpuscular kinetic energy (3). Of course, such complete analogue is a kind of Ariadne's thread, and therefore, we will pay closer attention to the features of the objects of study we have considered.

A material point is an idealized hypothetical object and it does not have its own spatial parameter. At the same time, according to classical Newtonian mechanics, a material point is characterized by corpuscular quantities:

Mass: m (13)

Momentun $\mathbf{P} = m\mathbf{v}$ (14)

Kinetic energy: $E_k = \dfrac{P^2}{2m}$ (15)

A microparticle is a real object of Nature and it simultaneously has both mass *m* and linear wavelength ***ir.*** Therefore, for a microparticle, instead of mass *m*, the initial quantity will be the corpuscular-wave (CW) quantity (10), and the set of CW quantities based on (10) will have the form:

CW mass: $\mathbf{m}^* = mi\mathbf{r}$ (16)

CW momentum: $P^* = \hbar = (\mathbf{m}^*\mathbf{v})$ (17)

CW kinetic energy: $\mathbf{E}_k^* = \frac{\hbar^2}{2\mathbf{m}^*}$ (18)

Now, due to the fact that the corpuscular quantities of a material point (13)...(15) in Newton's classical mechanics itself are defined within the framework of three-dimensional Euclidean space, and therefore, let's find out what the CW quantities of a microparticle (16)…(18) look like within the space. To do this, we transform the CW quantities of the microparticle (16)…(18) using the differential operator (8):

$$-i\mathbf{m}^*\nabla = mir(-i\cdot\nabla) = m \quad (19)$$

$$-i\hbar\nabla = (m\mathbf{v})_{1,2,3} - (mir^*(-iw))_4 = \mathbf{P}_{1,2,3} - \mathbf{P}_4 \quad (20)$$

$$-i\mathbf{E}_k^*\cdot\nabla = E_{k_{1,2,3}} - \left(\frac{\mathbf{m}^*(-i\omega\mathbf{v})}{2}\right)_4 - \left(\frac{-i\hbar\omega}{2}\right)_0 \quad (21)$$

here, ratios used:

$$(ir(-i\nabla)) = 1 \quad (22)$$

$$(\mathbf{v}\cdot(-i\nabla)) = -i\omega \quad (23)$$

and the subscripts 1,2,3,4,0 are symbols of the five dimensions of the five-dimensional Klein-Gordon space:

$$R^2 = (x^2+y^2+z^2)_{1,2,3} - (ct)_4 - \left(\frac{\hbar}{mc}\right)_0 \quad (24)$$

According to transformations(19)…(21), the CW quantities of the microparticle(16)…(18) within the five-dimensional Klein-Gordon space (24) appear in the form of corpuscular, mixed and wave quantities. At the same time, the three-dimensional components that arise on the right sides of transformations (19)…(21) are the corpuscular quantities of the material poin (13)…(15), and they are also applicable in relation to the microparticle, since the three-dimensional Euclidean space is an integral part of the five-dimensional Klein-Gordon space (24). According to transformations (19)…(21), a microparticle within space appears in the form of an object, which is characterized by both corpuscular and wave properties (quantities), historically, precisely because of this, at the beginning of the twentieth century, physicists perceived the microparticle as a real object of Nature, which is characterized by CW dualism.

Now, when it has become obvious that the corpuscular quantities of a material point (13)...(15) from classical Newtonian mechanics are connected with the CW quantities of a microparticle (16)…(18) using transformations (19)...(21), it has become clear that in its real physical meaning Planck’s constant (1) is the CW momentum of a microparticle (17).

At the same time, it became obvious that the newly discovered CW quantities of the microparticle (16)…(18) are extra-spatial in relation to the five-dimensional Klein-Gordon space (24). In this regard, Einstein’s words became relevant: “ I am more and more inclined to think that extra-spatial structures are necessary for the further development of physics “. As we see, the existence of extra-spatial structures (quantities) was suggested by the great Einstein himself, and we encountered them by chance during our research [4].

Literature.

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