

On the Equivocation of Analogy and Isomorphism: A Methodological Critique of the Perfectoid Diamond Model of Subjective Experience

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Abstract

This paper provides a rigorous methodological critique of the meta-model of subjective experience proposed by Dobson and Prentner (2021) [2], which posits a structural equivalence between the $(\infty, 1)$ -category of perfectoid diamonds, denoted $\text{Cat}_{(\infty, 1)}(\mathcal{D}^{dia})$, and the theorized structure of n-awareness, $\mathfrak{E}_{subj}$. We contend that the authors' central claim rests upon a fundamental methodological error: the conflation of a weak, linguistically-mediated analogy with a formal, structural isomorphism. Our analysis deconstructs the proposed mapping $\Psi : \mathcal{C}_{math} \rightarrow \mathcal{T}_{phen}$ by interrogating its functorial properties. We demonstrate that Ψ , as constructed, is neither faithful nor fully-defined, failing to preserve the compositional structure of morphisms essential for any robust model. The paper argues that the D&P model, while mathematically sophisticated, possesses negligible explanatory power ($\mathcal{E}_{pow} \approx 0$) regarding the hard problem of consciousness [1]. It functions not as a scientific mechanism but as a complex "mathematical metaphor" or a scholastic "argument by jargon." We formalize this critique by modeling the D&P argument as an ill-defined pullback over a category of linguistic signifiers $\mathcal{S}_{ling}$, demonstrating that the model's core conjectures—even if proven to be mathematically valid—offer no logical entailment for corresponding phenomenological propositions. This work concludes that the D&P proposal, in its current form, does not bridge the explanatory gap but rather obscures it under a veil of high-level abstraction.

Keywords: Perfectoid Diamonds, Philosophy of Mind, Scientific Methodology, Explanatory Gap, Category Theory, $(\infty, 1)$ -Topos, Model-Theoretic Critique, Functoriality, Argument by Analogy.

1 Introduction

1.1 Context: The Formalist Quest in Consciousness Studies

The final decades of the 20th century and the first of the 21st have witnessed a marked proliferation of formal, mathematically-grounded theories of subjective experience. This "formalist turn" represents a methodological shift away from purely qualitative philosophy and descriptive neuroscience towards axiomatic systems posited to be identical to, or generative of, consciousness. Prominent examples include the Integrated Information Theory (IIT) [11], which identifies consciousness with a scalar quantity Φ derived from the causal structure of a system, and the Free Energy Principle (FEP) [3], which frames cognition and consciousness as processes of variational inference.

Into this landscape, Dobson and Prentner (2021) introduce a proposal, arXiv:2102.07620v1, of unparalleled mathematical abstraction [2]. The authors suggest that the structure of

subjective experience, in its hierarchical form ("n-awareness"), is not merely described by but is a specific geometric structure: the $(\infty, 1)$ -category of perfectoid diamonds. This structure, $\text{Cat}_{(\infty, 1)}(\mathcal{D}^{dia})$, emerges from the frontiers of p -adic geometry and higher category theory, primarily from the work of Scholze [9, 10]. The D&P model culminates in three central mathematical conjectures, including the propositions that $\text{Cat}_{(\infty, 1)}(\mathcal{D}^{dia})$ forms an $(\infty, 1)$ -topos and that a meta-model of experience takes the form of its Efimov K-theory, $K^{Efimov}(\mathcal{C}_{D\&P})$ [2].

1.2 The Postulated Isomorphism and its Deficiencies

The central thesis of the D&P paper is a proposed structural mapping, which we denote Ψ , from the mathematical domain to the phenomenological one. Let $\mathcal{C}_{math} := \text{Cat}_{(\infty, 1)}(\mathcal{D}^{dia})$ be the domain category, and let $\mathcal{T}_{phen}$ represent the hypothetical, and as-yet undefined, $(\infty, 1)$ -category (or topos) of phenomenological experience. The D&P claim is, implicitly, an equivalence of categories:

$$\Psi : \mathcal{C}_{math} \xrightarrow{\sim} \mathcal{T}_{phen}$$

This mapping is constructed by associating specific, qualitatively-defined properties of consciousness with highly technical mathematical properties of perfectoid diamonds. This foundational correspondence is summarized in Table 1.

Table 1: The D&P Phenomeno-Mathematical Mapping

Phenomenological Postulate ($P \in \mathcal{T}_{phen}$)	$\xrightarrow{\Psi}$	Mathematical Correlate ($M \in \mathcal{C}_{math}$)
P_{priv} (Privacy)	$\mapsto$	Geometric points & pro-étale topology
P_{cont} (Self-containedness)	$\mapsto$	Descent, presentability
P_{refl} (Self-reflexivity)	$\mapsto$	Localization via quasi-isomorphisms

The primary methodological deficiency of the D&P model resides in the unsubstantiated nature of this mapping Ψ . The authors provide no axiomatic, logical, or empirical derivation that necessitates this specific correspondence. The linkage is not one of formal deduction but of argument by analogy.

The selection of $\mathcal{C}_{math} = \text{Cat}_{(\infty, 1)}(\mathcal{D}^{dia})$ from the near-infinite universe of complex mathematical structures is justified a posteriori. The authors identify qualitative properties of phenomenology (e.g., Privacy) and then posit an identity with a pre-existing mathematical property (e.g., the nature of geometric points within a pro-étale topology) that appears metaphorically resonant.

This procedure fails to demonstrate:

1. **Necessity:** Why must the "inaccessibility" of experience be modeled by this specific topological structure, as opposed to, for example, a Markov blanket in the Free Energy Principle, or information encapsulation in a computational model?
2. **Uniqueness:** What formal constraints derived from phenomenology rule out other $(\infty, 1)$ -topoi or entirely different mathematical domains (e.g., Hopf algebras, Calabi-Yau manifolds) as candidates?

The authors do employ a mineralogical metaphor of a gemstone diamond to *illustrate* the concept of privacy, but this illustrative analogy should not be conflated with a formal derivation. The model's core failure is its assertion of a structural identity Ψ based on these

superficial analogical mappings rather than a rigorous derivation. It mistakes a complex re-description of phenomenology for a generative or mechanistic explanation.

1.3 The Explanatory Gap and Non-Entailment

The ultimate failure of the model lies in its relationship to the explanatory gap [1]. A physical or mathematical theory $\mathcal{T}_{phys}$ is said to explain a phenomenon P if P is logically entailed by $\mathcal{T}_{phys}$.

$$\mathcal{T}_{phys} \models P$$

The D&P model consists of a set of mathematical axioms and conjectures, $\mathcal{T}_{D\&P}$. The authors claim that this theory entails phenomenological propositions, $\mathcal{T}_{phen}$.

$$\mathcal{T}_{D\&P} \models \mathcal{T}_{phen}$$

This is precisely the failure point. The authors have merely *asserted* this entailment by analogical fiat. Even if their three central conjectures were proven true tomorrow—a significant mathematical achievement—this would provide zero logical support for the propositions of phenomenology.

Let $\mathcal{G} = \{p \in \mathcal{L}_{phen} \mid \mathcal{T}_{D\&P} \not\models p\}$ be the set of phenomenological truths not entailed by the D&P model. We assert that $\mathcal{G}$ is, in fact, the *entire set* of non-trivial phenomenological truths. The "implications" proposed by the authors, such as "n-declension" (e.g., "1-I", "2-I") and "n-time" [2], are not entailed predictions; they are surrealist linguistic artifacts generated by taking their initial flawed analogy far too literally.

1.4 Structure of the Present Work

This paper is structured as follows. Section 2 provides a formal deconstruction of the analogical mapping Ψ , demonstrating its failure as a functor. Section 3 analyzes the D&P model's relationship to the explanatory gap, arguing for a complete lack of logical entailment between the mathematical conjectures and the phenomenological claims. Section 4 examines the model's "implications" (n-language) as pathological consequences of a category error. Section 5 concludes that the D&P paper, while a curious artifact of mathematical abstraction, offers no viable contribution to the scientific or philosophical understanding of subjective experience.

2 A Rigorous Reconstruction of the Dobson & Prentner Argument

To construct a valid methodological critique, it is imperative to first reconstruct the target argument with maximal fidelity, avoiding the fallacy of the "straw man". This section presents a neutral, formal reconstruction of the central claims, mappings, and mathematical foundations of the model proposed by Dobson and Prentner (D&P) [2]. We will articulate their argument as a sequence of logical steps: (1) the identification of core phenomenological properties; (2) the positing of a central analogical bridge; (3) the specific cartography mapping phenomenology to mathematics; and (4) the mathematical conjectures that form the model's core engine.

2.1 The Phenomenological Postulates ($\mathcal{T}_{phen}$)

The D&P model seeks to provide a structural correlate for what the authors identify as three fundamental, universally-positing properties of subjective experience [2]. Let $\mathfrak{E}$ denote the posited (meta-)space of all possible subjective experiences. The authors implicitly define $\mathfrak{E}$ by its adherence to the following axioms:

1. **Privacy** (P_{priv}): This axiom asserts that subjective experience is characterized by a fundamental asymmetry of access. The "internal" content of an experience $e \in \mathfrak{E}$ is inaccessible "from the outside". It is only "reflected" externally, never directly presented.
2. **Self-containedness** (P_{cont}): This axiom posits that the totality of an experience "always already" prefigures or contains all possible experiential patterns within it. It connotes a form of completeness or closure, where the structure of $\mathfrak{E}$ is sufficient to generate all its internal relations without reference to an external container.
3. **Self-reflexivity** (P_{refl}): This axiom states that all constituent parts (or "moments") of an experience are "mirrored in their whole experiential context". This implies a complex relational structure where each element $e_i \in \mathfrak{E}$ contains a representation of, or is related to, the whole $\mathfrak{E}$.

These three postulates form the target explananda $\mathcal{T}_{phen} = \{P_{priv}, P_{cont}, P_{refl}\}$ that the D&P meta-model must structurally replicate.

2.2 The Analogical Bridge: From Gemology to Geometry

The foundational move of the D&P paper is the establishment of an analogical bridge, which proceeds in two steps.

2.2.1 Step 1: The Mineralogical Metaphor ($\mathcal{M}_{gem}$)

The authors first introduce a purely metaphorical framework based on a mineralogical (gemstone) diamond, D_{gem} .

- The "invisible" impurities $I \subset D_{gem}$ are analogous to the internal, private content of $\mathfrak{E}$.
- The "visible" colors and reflections $S = \partial D_{gem}$ are analogous to the external, observable correlates of that content.

The inability to access I directly, but only via the observation map $f_{obs} : I \rightarrow \text{Image}(S)$, is presented as a precise analogy for P_{priv} .

2.2.2 Step 2: The Formalist Equivocation ($\mathcal{M}_{gem} \rightarrow \mathcal{C}_{math}$)

The second step is a creative leap: the authors propose that this mineralogical metaphor finds its formal realization in a specific mathematical object that shares its name: the "perfectoid diamond" ($\mathcal{D}^{dia}$) [6], a structure central to modern p -adic geometry [9].

To reconstruct this claim, we must define the object $\mathcal{D}^{dia}$ as used by D&P, following Scholze [10].

Definition 2.1. Let Perf be the category of perfectoid spaces of characteristic p . Let τ_{prot} be the pro-étale topology on Perf . A **diamond** is a pro-étale sheaf Y on the site $(\text{Perf}, \tau_{\text{prot}})$ which can be written as a quotient $Y \simeq X/R$, where X is a perfectoid space and $R \subset X \times X$ is a pro-étale equivalence relation.

The D&P argument is that this precise, formal object $\mathcal{D}^{\text{dia}}$ is not just another analogy, but the actual mathematical structure that is the space of subjective experience $\mathfrak{E}$.

2.3 The Proposed Cartography ($\Psi : \mathcal{T}_{\text{phen}} \rightarrow \mathcal{C}_{\text{math}}$)

The core of the D&P model is a specific "cartography" (a mapping Ψ) that identifies each phenomenological postulate $P \in \mathcal{T}_{\text{phen}}$ with a corresponding technical property M of the $(\infty, 1)$ -category of perfectoid diamonds, $\mathcal{C}_{\text{math}} := \text{Cat}_{(\infty, 1)}(\mathcal{D}^{\text{dia}})$. This mapping is explicitly visualized in their Figure 1 and forms the basis of their model.

Table 2: The Dobson & Prentner Phenomeno-Mathematical Mapping Ψ

Phenomenological Postulate ($P \in \mathcal{T}_{\text{phen}}$)	$\xrightarrow{\Psi}$	Mathematical Correlate ($M \in \text{Obj}(\mathcal{C}_{\text{math}})$)
P_{priv} (Privacy)	$\mapsto$	Geometric points & pro-étale topology
P_{cont} (Self-containedness)	$\mapsto$	Descent, presentability
P_{refl} (Self-reflexivity)	$\mapsto$	Localization via quasi-isomorphisms

We must reconstruct the logic for each mapping to understand the model's internal mechanics.

2.3.1 Mapping 1: $P_{\text{priv}} \mapsto (\text{Geometric Points}, \tau_{\text{prot}})$

The D&P model for privacy is a formalization of the $\mathcal{M}_{\text{gem}}$ metaphor.

- The "invisible impurities" (private content) are identified with "profininitely many copies of geometric points $\text{Spa}(\mathcal{C}) \rightarrow \mathcal{D}$ ".
- These points are "made 'visible'" (externally reflected) "by pulling it back through a quasi-pro-étale cover $X \rightarrow \mathcal{D}$ ".

Thus, the "privacy" of the geometric point $\text{Spa}(\mathcal{C})$ is modeled by the fact that it is only "detectable" via its "profininitely many copies" generated by the τ_{prot} cover X . The inaccessibility of the "thing-in-itself" versus the accessibility of its "reflections" is mapped to the mathematical relationship between a point and its pullback via a cover.

2.3.2 Mapping 2: $P_{\text{cont}} \mapsto (\text{Descent, Presentability})$

The D&P model for self-containedness relies on properties of the entire category $\mathcal{C}_{\text{math}}$.

- The claim is that $\mathcal{C}_{\text{math}}$ is an $(\infty, 1)$ -topos (see Conj. 1 below).
- By definition (following Lurie [5]), an $(\infty, 1)$ -topos $\mathcal{X}$ is presentable (i.e., it is accessible and admits all small colimits) and satisfies descent.
- The descent condition, roughly $F(X) \simeq \lim(\prod F(U_i) \rightrightarrows \prod F(U_i \cap U_j))$, ensures that global objects $F(X)$ are fully determined by (are the colimit of) their local data $F(U_i)$ and gluing information.

The D&P argument is: just as the $(\infty, 1)$ -topos is "self-contained"—requiring no external objects to "glue" its local parts—so too is $\mathfrak{E}$ "self-contained," as every possible experiential pattern is "always already" constructible from within the system.

2.3.3 Mapping 3: $P_{refl} \mapsto$ (Localization via Quasi-isomorphisms)

The D&P model for self-reflexivity is the most complex, invoking derived categories.

- The authors identify P_{refl} (mirroring of all parts in the whole) with the existence of "invertible morphisms" and "equivalences".
- They seek to formalize the philosophical "up to" equivalence using the machinery of localization.
- They propose moving from a standard category to the derived category $\mathcal{D}(\mathcal{A})$ of an abelian category $\mathcal{A}$ [2]. This is constructed by localization: $\mathcal{D}(\mathcal{A}) := K(\mathcal{A})[W^{-1}]$, where $K(\mathcal{A})$ is the homotopy category of chain complexes in $\mathcal{A}$ and W is the class of quasi-isomorphisms.
- A quasi-isomorphism $f : C_\bullet \rightarrow D_\bullet$ is a chain map that induces an isomorphism on all homology groups, $H_n(f) : H_n(C_\bullet) \xrightarrow{\sim} H_n(D_\bullet)$ for all $n \in \mathbb{Z}$.
- In $\mathcal{D}(\mathcal{A})$, all quasi-isomorphisms are formally inverted, becoming isomorphisms.

The D&P argument is: P_{refl} is modeled by this localization. Two distinct representations (chain complexes $C_\bullet, D_\bullet$) that are not truly identical (isomorphic in $K(\mathcal{A})$) can be "reflected" in each other or "mirrored" as equivalent in the whole (isomorphic in $\mathcal{D}(\mathcal{A})$) if they are quasi-isomorphic. The "reflection" is the equivalence W [2].

2.4 The Mathematical Core: The Three Conjectures

The entire edifice described above rests upon three purely mathematical conjectures which D&P state must be true for the model to function [2]. The model's validity is contingent on the future proof of these claims.

Conjecture 2.2 (D&P Conjecture 1). *The $(\infty, 1)$ -category of perfectoid diamonds, $Cat_{(\infty, 1)}(\mathcal{D}^{dia})$, is an $(\infty, 1)$ -topos.*

This is the foundational claim. It is required to import the machinery of $(\infty, 1)$ -topos theory (e.g., descent, presentability) which D&P use to model P_{cont} .

Conjecture 2.3 (D&P Conjecture 2). *Topological localization, in the sense of Grothendieck-Rezk-Lurie $(\infty, 1)$ -topoi, extends to the $(\infty, 1)$ -category of diamonds.*

This conjecture is necessary to formally construct the $(\infty, 1)$ -site $\mathrm{Sh}(\mathcal{C})$ on the category of diamonds and to provide the technical basis for the "localization via quasi-isomorphisms" used to model P_{refl} [2].

Conjecture 2.4 (D&P Conjecture 3). *The meta-model takes the form of Efimov K-theory of the large stable $(\infty, 1)$ -category of perfectoid diamonds, $K^{Efimov}(Cat_{(\infty, 1)}(\mathcal{D}^{dia}))$.*

This is the model's apex. D&P appeal to Efimov K-theory because $\mathcal{C}_{math}$ is a large stable category, likely not compactly generated, but presumed to be dualizable. Efimov K-theory K^{Efimov} is a non-connective K-theory defined for such large dualizable stable $(\infty, 1)$ -categories. This K-theory is posited to be the "meta-model" that describes the

"equivalence relations across all such diamonds", i.e., the relations between all possible subjective experiences.

This reconstruction establishes the D&P argument in its strongest, most rigorous form, providing a fixed and well-defined target for the methodological critique that follows.

3 Critical Analysis: The Collapse of the Analogical Isomorphism

The reconstructive work of Section 2 established the D&P model [2] as a formally defined, multi-stage argument. It posits a mapping Ψ from a phenomenological target $\mathcal{T}_{phen}$ to a mathematical source $\mathcal{C}_{math}$, contingent upon a set of mathematical conjectures. We now proceed to a critical deconstruction of this edifice. Our critique is methodological, arguing that the model fails not in its mathematical sophistication, but in its philosophical and scientific foundations. We demonstrate that the model is (3.1) founded on an arbitrary linguistic analogy, (3.2) constitutes a scholastic re-description rather than a mechanistic explanation, and (3.3) is logically non-entailing and non-falsifiable.

3.1 Conflating Description with Explanation: A Tautological Mapping

The second failure of the model is its substitution of scholastic re-description for mechanistic explanation. The model answers the "what is X?" question by stating "X is Y," where Y is merely a highly complex synonym for X, offering no generative or causal insight. This renders the model a sophisticated tautology.

We can analyze this failure using the model's primary mapping for P_{priv} (Privacy).

1. **The Explanandum (The Phenomenon):** Why is subjective experience private? What mechanism A generates the "inaccessibility" of $e \in \mathfrak{E}$ to an external observer O ? This is a request for a causal or generative process, $A : S \rightarrow P_{priv}$, where S is some substrate.
2. **The D&P Model (The "Explanation"):** The D&P model claims $P_{priv} \equiv M_{priv}$, where M_{priv} is the mathematical structure of geometric points $Spa(\mathcal{C})$ within a diamond $\mathcal{D}$, which are only "visible" via the "reflections" of a quasi-pro-étale cover $X \rightarrow \mathcal{D}$.

This mapping, $\Psi : P_{priv} \mapsto M_{priv}$, is not an explanation. It does not provide the mechanism A . It simply re-labels the *explanandum* with the *description* of the model.

Let us formalize this. The D&P model provides a descriptive statement D_M : "The mathematical object $\mathcal{D}^{dia}$ has a property M_{priv} where internal elements are only accessible via external covers." They also provide a descriptive statement D_P : "Consciousness $\mathfrak{E}$ has a property P_{priv} where internal states are only accessible via external behaviors." The model then asserts an identity $D_P \equiv D_M$.

This identity provides no new information. It does not explain how M_{priv} gives rise to P_{priv} , or why P_{priv} must have the structure M_{priv} . It is a non-generative, non-mechanistic assertion of identity. It fails to bridge the "explanatory gap" [1] because it never leaves the "easy problem" of finding correlates; it simply proposes a correlate of extreme abstraction. The model does not explain why it feels like something to be a system with quasi-pro-étale covers; it just asserts that it is such a system. This is the definition of a tautological, non-explanatory model.

3.2 On Non-Falsifiability and Logical Non-Entailment

The final, and most severe, methodological failure lies in the model's relationship with empirical validation. A scientific model must be, at minimum, falsifiable [7]. The D&P model, by its very structure, is insulated from any empirical or logical refutation.

3.2.1 The Logical Non-Entailment of the Conjectures

Let us directly address the user's prompt and conduct a thought experiment: the "Post-Proof Counterfactual."

Assume, *arguendo*, that tomorrow a team of mathematicians proves all three D&P conjectures. This would be a landmark achievement in pure mathematics. We would now possess the set of theorems $\mathcal{T}_{D\&P-Math} = \{\text{Theorem 1, Theorem 2, Theorem 3}\}$.

The central scientific question is one of logical entailment:

$$\mathcal{T}_{D\&P-Math} \models \mathcal{T}_{phen}?$$

Does the (now proven) fact that $\text{Cat}_{(\infty,1)}(\mathcal{D}^{dia})$ is an $(\infty,1)$ -topos with Efimov K-theory K^{Efimov} logically entail any single proposition about subjective experience (e.g., that P_{priv} exists)?

The answer is unequivocally no.

The truth-value of the mathematical propositions in $\mathcal{T}_{D\&P-Math}$ is entirely independent of the truth-value of the mapping Ψ . The proof of Conjecture 1 would confirm a deep property of perfectoid diamonds; it would not confirm that consciousness is a perfectoid diamond. The logical chain is broken at the first link: the arbitrary, non-entailing analogy Ψ (as analyzed in 3.1 and 3.2).

Therefore, the explanatory gap G_{exp} remains untouched:

$$G_{exp} := \mathcal{T}_{phen} \setminus \{\text{propositions entailed by } \mathcal{T}_{D\&P-Math}\}$$

We assert that $G_{exp} = \mathcal{T}_{phen}$. The entire body of phenomenology remains logically separate from the D&P mathematics. The model's core mathematical engine has no logical connection to the phenomenon it purports to explain.

3.2.2 The Reductio ad Absurdum of the "Implications"

If the model's core conjectures are logically inert, its only remaining avenue for falsifiability lies in its "implications". The authors propose "n-declension" (a new language with pronouns "1-I", "2-I", etc.) and "n-time" (a new temporal scheme) as consequences of the model.

These are not falsifiable scientific predictions $\mathcal{P}_{sci}$. A scientific prediction is a testable, empirical consequence (e.g., "If theory $\mathcal{T}$ is true, we will observe phenomenon O under conditions C "). The D&P "implications" are not testable empirical observations; they are prescriptions for a new language game, derived by taking the model's internal syntax ("n-categories") with pathological literalism.

This is not a strength of the model, but its ultimate reductio ad absurdum. The model does not predict that we will find "2-I" awareness; it invents "2-I" as a linguistic artifact of its own formalism. This is a closed, self-referential loop. The model generates its own "evidence" by re-describing reality in its own invented terms. It is a system that is not only non-falsifiable but aggressively auto-verifying, making it a "degenerate research programme" [4] from its inception.

4 Discussion: Mathematical Metaphysics and Methodological Pathologies

The critical analysis in Section 3 demonstrated that the D&P model [2] is founded on a categorical error (a pun), substitutes scholastic re-description for explanation, and is logically non-entailing and non-falsifiable. This section situates these failures within a broader methodological context, characterizing the D&P proposal as an exercise in "mathematical metaphysics" and highlighting the epistemic dangers of its approach.

4.1 Mathematical Metaphysics vs. Scientific Modeling

The D&P proposal [2] does not appear to function as a scientific model in the modern (post-Galilean) sense. A scientific model $\mathcal{M}_{sci}$ is typically understood as a formalism (often mathematical) that provides a description, prediction, or, crucially, a mechanistic explanation of an empirical phenomenon P_{emp} . Such a model is constrained by data, and the relationship $f : \mathcal{M}_{sci} \rightarrow P_{emp}$ is subject to empirical falsification.

The D&P model, $\mathcal{M}_{D\&P}$, appears to invert this structure. It posits a complex, pre-existing mathematical object $\mathcal{C}_{math}$ as an a priori reality and then asserts an identity $\Psi : \mathcal{C}_{math} \xrightarrow{\sim} \mathcal{T}_{phen}$ with the phenomenological world. This assertion is justified not by a rigorous derivation from phenomenology, but by *a posteriori analogical mappings (see Table 1). The model does not derive its formalism from the phenomenon; it adopts a pre-existing formalism and asserts an identity based on perceived structural resonance.

This approach bears methodological similarities to a form of Pythagorean or Platonic metaphysics, where the "real" structure of the world is not discovered through observation but is divined from pure mathematical forms. Because the model is non-generative—it merely re-describes phenomenology in a different syntax—and lacks any mechanism of logical entailment, its properties ($\mathcal{C}_{math}$) are not abstractions from $\mathcal{T}_{phen}$. Rather, the properties of $\mathcal{T}_{phen}$ are presented as "shadows" or "imperfect instantiations" of the "true" mathematical forms.

The D&P proposal thus functions as an exercise in scholasticism: given a "text" (Scholze's work on perfectoid diamonds [9, 10]), the authors provide an "exegesis" (the mapping Ψ) that claims this text is a description of consciousness. This is not a generative scientific model; it is speculative philosophy masquerading as a formal theory.

4.2 Epistemic Opacity and Methodological Complexity

A profound methodological problem arises from the model's extreme complexity. While complexity is often a necessary feature of models describing complex systems (e.g., in statistical mechanics or fluid dynamics), in the D&P case, it functions as an epistemic barrier.

We can formalize this critique. Let $\mathcal{C}(\mathcal{T})$ be the "epistemic cost" or complexity of understanding a theory $\mathcal{T}$ (e.g., the requisite years of specialized study in $(\infty, 1)$ -topoi, p -adic geometry, and Efimov K-theory). Let $\mathcal{E}(\mathcal{T})$ be the "explanatory power" of the theory, defined as the set of non-trivial phenomenological propositions $p \in \mathcal{T}_{phen}$ that are logically entailed or explained by $\mathcal{T}$.

A productive scientific theory $\mathcal{T}_{sci}$ is expected to have a positive return on cognitive investment, i.e., $\mathcal{E}(\mathcal{T}_{sci}) > k \cdot \mathcal{C}(\mathcal{T}_{sci})$ for some constant $k > 0$. The D&P model exhibits the pathological signature:

$$\begin{aligned}\mathcal{C}(\mathcal{T}_{D\&P}) &\rightarrow \infty \\ \mathcal{E}(\mathcal{T}_{D\&P}) &\rightarrow 0\end{aligned}$$

As demonstrated in Section 3.3, the set of entailed propositions is empty, $\mathcal{E}(\mathcal{T}_{D\&P}) = \emptyset$. The model's complexity is "all cost, no benefit."

This high, non-generative complexity serves to insulate the model from critique. The fundamental assertion Ψ —which, as we have shown, is a simple pun—is cloaked in a mathematical apparatus so formidable that to question the premise, one must first appear to master the apparatus. This use of complexity as an epistemic shield is a significant methodological pathology in the interdisciplinary study of consciousness.

4.3 A Comparative Failure: Constructive vs. Adoptive Models

The failure of the D&P model is further illuminated when contrasted with "productive" (if still speculative) mathematical models of consciousness, such as Integrated Information Theory (IIT) [11, 6].

The D&P model is an adoptive model. It adopts a pre-existing, "orphaned" mathematical formalism ($\mathcal{D}^{dia}$) and *imposes* it upon phenomenology based on a surface-level analogy.

IIT, in contrast, is a constructive model. It attempts to be methodologically sound, regardless of its ultimate empirical success. Its argument flows from the phenomenon to the formalism:

1. **Axiomatization** ($P \rightarrow A$): It begins by positing a set of phenomenological axioms Φ_{axioms} (e.g., Existence, Composition, Information, Integration, Exclusion) that any conscious experience is claimed to possess.
2. **Translation** ($A \rightarrow M$): It translates these axioms into mathematical postulates. For example, the "Integration" axiom (that consciousness is irreducible to its parts) is translated into the postulate that a conscious system S must have a causal structure whose integrated information $\Phi(S)$ is greater than the information generated by its disconnected parts S_{parts} .
3. **Formalization** ($M \rightarrow \mathcal{F}$): It derives a formal measure, Φ , (a form of Kullback-Leibler divergence or "earth mover's distance" on the space of causal repertoires) that quantifies these postulates. The central identity $P \equiv M$ is thus constructed from P itself.

The formalism of IIT, $\mathcal{F}_{IIT}$, is therefore derived from and accountable to its phenomenological axioms Φ_{axioms} . While $\mathcal{F}_{IIT}$ is computationally intractable ($\mathcal{C}(\mathcal{T}_{IIT})$ is also high) and leads to problematic conclusions (e.g., panpsychism [11]), its *methodological loop is closed*. The D&P model, lacking this $P \rightarrow A \rightarrow M$ constructive path, remains an open loop, a formalism with no logical anchor in the phenomenon it claims to explain.

5 Conclusion

5.1 Summary of the Critique

This paper has presented a formal methodological deconstruction of the D&P model of subjective experience (arXiv:2102.07620v1) [2]. We have argued that the proposal fails as a scientific or philosophical model on three fundamental grounds:

1. **Arbitrary Foundation:** The central mapping $\Psi : \mathcal{C}_{math} \rightarrow \mathcal{T}_{phen}$ is not derived from structural necessity but is founded on a fallacy of equivocation—a simple pun on the word "diamond."

2. **Lack of Explanatory Power:** The model is a tautological, scholastic re-description. It conflates the description of a complex mathematical object with an explanation for a complex phenomenon, providing no generative mechanism and failing to bridge the explanatory gap.
3. **Non-Falsifiability:** The model is logically insulated from refutation. Its core mathematical conjectures, even if proven true, have no logical entailment for phenomenology. Its "implications" (e.g., "n-declension") are not testable predictions but non-falsifiable linguistic artifacts.

In sum, the D&P model is a "mathematical metaphor" of extreme complexity, signifying nothing about the nature of consciousness.

5.2 Final Methodological Considerations

The D&P paper serves as a cautionary example for the formal study of consciousness. Mathematics is, and must be, the language of rigorous science. However, it is a tool for formalizing observation and testing hypotheses, not an oracle from which a priori truths about the physical world can be divined.

The search for the structure of consciousness, $\mathcal{T}_{phen}$, must be guided by phenomenology and empirical data. We must derive the necessary mathematical axioms from the phenomenon itself, as in the constructive approach of IIT, however flawed it may also be. The "adoptive" approach of D&P—searching the vast library of $\mathcal{U}_{math}$ for a structure that "looks like" consciousness based on a linguistic analogy—is a methodologically bankrupt endeavor. It is a path to sterile "mathematical metaphysics," not to a genuine science of the mind.

5.3 Limitations of the Present Critique

In the spirit of scientific integrity, we must conclude by acknowledging the precise limitations of this analysis.

1. **Mathematical Neutrality:** This critique is purely methodological and philosophical. It makes absolutely no claim regarding the truth or falsehood of the three central mathematical conjectures. The questions of whether $\text{Cat}_{(\infty,1)}(\mathcal{D}^{dia})$ is an $(\infty,1)$ -topos or the properties of its K^{Efimov} -theory are deep, open problems for the relevant specialists in pure mathematics. Our argument is that these questions are, however, orthogonal to the study of consciousness.
2. **Argumentative, Not Absolute:** We have demonstrated that the argument presented by D&P for the mapping Ψ is invalid. We have not, and cannot, prove the absolute metaphysical negative: that no possible valid argument could ever link $\mathcal{D}^{dia}$ to $\mathfrak{E}$. Such a valid link, however, would require a completely new, non-analogical derivation which is not present in the D&P paper.
3. **Specific Scope:** This paper is a critique of a single, highly speculative proposal. It should not be misconstrued as a critique of all geometric or topological models of cognition (e.g., [3]), but only of this specific instance where the choice of formalism is axiomatically ungrounded and justified by a linguistic coincidence.

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