Hidden Geometries in Maxwell's Equations and a Force–Flux Route to Unified Static Fields

Ioannis Xydous [1]

[1] BSc. E. Engineering, Piraeus, Greece, E-mail: ioannisxydous@gmail.com

This work reveals previously hidden structural features of Maxwell's equations that emerge when the Lorentz force is embedded into their flux formulation, exposing motional and rotational contributions to the field of an elementary charge. These modifications extend the field–source relationship to encompass phenomena beyond the standard static Coulomb case. We demonstrate this by deriving the magnetic field of a rotating spherical charge distribution, where magnetic charge appears not as a monopole but as an emergent quantity producing a dipole-like field. The same configuration generates a distinct component opposing the Coulomb field, arising purely from rotation. Building on these results, we formulate a force–flux law analogous to Gauss's electric flux relation, but with force replacing field. Owing to its units, this law permits direct replacement of electric charge and constants with other charge types—magnetic, gravitational, or inertial—while preserving the force magnitude for a given separation. The result is a compact, nonrelativistic framework that retains Maxwell's geometric elegance while extending its scope to unified static-field interactions.

DOI 10.5281/zenodo.16877284

I. INTRODUCTION

The quest to unify nature's fundamental forces has long been guided by the interplay of experiment and theory. Faraday's investigations into electromagnetic induction [1] laid the groundwork for linking electricity and magnetism, driven by his conviction in the unity of physical phenomena. He also speculated on possible ties between electromagnetism and gravity—early, though inconclusive, explorations that foreshadowed later unification attempts [2]. Maxwell transformed this vision into a coherent framework [3], uniting electricity and magnetism, predicting the wave nature of light, and establishing a paradigm central to classical field theory. His concise, geometric formulation continues to inspire extensions from fluid dynamics to gravitation. Adaptations of Maxwellian structure to gravitational theory include gravito-electromagnetism [4, 9, 12, 18] and models from linearized general relativity. Classical analogies, such as the Heaviside equations [4] and scalar–vector–tensor models [6, 11], capture aspects of this correspondence, while historical surveys [15] and modern proposals [14] explore deeper structural links. Most, however, rely on relativistic assumptions, limiting their scope in static or purely classical regimes. Here we present a classical, nonrelativistic framework unifying electrostatic, magnetostatic, gravitational, and inertial interactions in a common form. Embedding the Lorentz force into Maxwell's flux representation introduces *structural extensions* that integrate motional effects into the field–source relationship. Applied to a rotating spherical charge, the method yields a dipole-like emergent magnetic charge and a secondary field opposing the Coulomb term—both set by geometry and motion. Building on this, we employ a force–flux relation, analogous to Gauss's law but expressed in terms of force, whose dimensional form (N·m^2) enables substitution of magnetic, gravitational, or inertial charges while preserving force magnitude. This dual development—uncovering hidden geometries and establishing a force–flux route—offers a compact, nonrelativistic approach that retains Maxwell's elegance while extending its scope to unified static-field interactions.

The search for such extensions parallels the historical pursuit of magnetic monopoles as theoretical constructs and unifying elements. Dirac's quantization condition [5] inspired grand unified monopole solutions [7, 8] and modern duality-based approaches [10], while reviews [16, 17, 19] summarize theory and experiments. Laboratory analogues have demonstrated monopole-like configurations in synthetic magnetic fields [13]. The framework developed here remains strictly classical, avoids quantum or relativistic premises, and reveals latent symmetries linking electromagnetic, gravitational, and inertial phenomena.

II. MAXWELL EQUATIONS AND STRUCTURAL EXTENSIONS

Maxwell's equations form the foundation of classical field theory, providing a unified framework that connects fields to their sources—charge and current distributions—and governs how time-varying fields induce one another. In integral form, they reveal the geometric and physical relationships among fields, fluxes, and sources. With appropriate substitutions of constants and source variables, they can be generalized to describe arbitrary charge types q_T and $q_{T'}$, each associated with characteristic permittivity ε_{T_0} and permeability $\mu_{T'_0}$. The fields $\vec{E}_T$ and $\vec{B}_{T'}$ are then interpreted as those generated by such sources—electric, magnetic, gravitational, or inertial, depending on context:

$$\oint_{\partial V} \vec{E}_T \cdot d\vec{A} = \frac{1}{\varepsilon_{T_0}} \int_V \rho_T \, dV, \tag{1}$$

$$\oint_{\partial V} \vec{B}_{T'} \cdot d\vec{A} = 0, \tag{2}$$

$$\oint_{\partial S} \vec{E}_T \cdot d\vec{\ell} = -\frac{d}{dt} \int_S \vec{B}_{T'} \cdot d\vec{A}, \tag{3}$$

$$\oint_{\partial S} \vec{B}_{T'} \cdot d\vec{\ell} = \mu_{T'_0} \int_S \vec{J}_T \cdot d\vec{A} + \mu_{T'_0}\varepsilon_{T_0} \frac{d}{dt} \int_S \vec{E}_T \cdot d\vec{A}. \tag{4}$$

These relations capture the coupling between fields and sources for both static and dynamic conditions. However, when charges or media move—especially with intrinsic rotation—additional structural terms are required to represent physical effects absent in the standard formulation.

A classic case is the Faraday disc (homopolar generator), where motion of charges through a static magnetic field produces an electromotive force (emf) not accounted for by $\partial\vec{B}/\partial t$ alone in Eq. (3). Such behavior is more completely described by a Lorentz-type force on a charge q_T:

$$\vec{F}_{TT'} = q_T\big(\vec{E}_T + \vec{u}\times\vec{B}_{T'}\big), \tag{5}$$

where $\vec{u}$ is the velocity of the charge relative to $\vec{B}_{T'}$. The induced circulation along a closed contour is then

$$\mathcal{E}_{TT'} = \oint_{\partial S} \big(\vec{E}_T + \vec{u}\times\vec{B}_{T'}\big)\cdot d\vec{\ell}, \tag{6}$$

leading to the generalized Faraday induction law:

$$\oint_{\partial S} \big(\vec{E}_T + \vec{u}\times\vec{B}_{T'}\big)\cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B}_{T'}\cdot d\vec{A}. \tag{7}$$

The right-hand side represents the total time derivative of the generalized magnetic flux $\vec{B}_{T'}$ through a time-dependent surface $S(t)$, encompassing both intrinsic field variation (transformer effect) and flux change due to surface motion (motional induction). The term $\vec{u}\times\vec{B}_{T'}$ is critical for systems such as the Faraday disc, where the emf arises purely from kinematic interaction with a static field.

A geometric criterion for motional induction is

$$\vec{u}(\vec{r}) \nparallel \vec{B}_{T'}(\vec{r}) \quad\Rightarrow\quad \vec{u}(\vec{r})\times\vec{B}_{T'}(\vec{r}) \neq \vec{0} \quad \text{for some } \vec{r}\in\mathcal{R},$$

where $\mathcal{R}$ denotes a region occupied by the medium. This reflects the need to cross field lines to generate motional emf.

If the source distribution undergoes intrinsic rotation, the resulting velocity-dependent term modifies the static field. In such cases the total field can be written as

$$\vec{E}_{TT'} = \vec{E}_T \pm \vec{u}\times\vec{B}_{T'} = -\nabla\phi_T \pm \vec{u}\times\vec{B}_{T'}, \tag{8}$$

where ϕ_T is the potential for the given charge type. Although $\vec{E}_{TT'}$ may not satisfy Gauss's law in its original form, it can still be evaluated via a flux relation under appropriate symmetry or motion constraints:

$$\begin{aligned}\oint_{\partial V} \vec{E}_{TT'}\cdot d\vec{A} &= \oint_{\partial V} \big(-\nabla\phi_T \pm \vec{u}\times\vec{B}_{T'}\big)\cdot d\vec{A} \\ &= \frac{1}{\varepsilon_{T_0}}\int_V \rho_T\, dV,\end{aligned} \tag{9}$$

where ρ_T may include both static and motion-induced contributions.

A further extension follows from considering the flux of the force field itself. Starting from

$$\oint_{\partial V} \vec{E}_T\cdot d\vec{A} = \frac{1}{\varepsilon_{T_0}}\int_V \rho_T\, dV,$$

multiplication by q_T gives the flux of the Coulomb-like force $\vec{F}_T = q_T\vec{E}_T$:

$$\begin{aligned}\oint_{\partial V} \vec{F}_T\cdot d\vec{A} &= q_T\oint_{\partial V} \vec{E}_T\cdot d\vec{A} \\ &= \frac{q_T}{\varepsilon_{T_0}}\int_V \rho_T\, dV.\end{aligned} \tag{10}$$

If the enclosed distribution consists entirely of type-q_T charges,

$$\int_V \rho_T\, dV = q_T \quad\Rightarrow\quad \oint_{\partial V} \vec{F}_T\cdot d\vec{A} = \frac{q_T^2}{\varepsilon_{T_0}}. \tag{11}$$

Eq. (11) quantifies the total force on a type-q_T charge due to an enclosed distribution of like-sign charges. Although structurally analogous to Gauss's law, it is not a new field equation but a scaled form of the source–field relation. The resulting quantity has units of $\mathrm{N}\cdot\mathrm{m}^2$ (or $\mathrm{J}\cdot\mathrm{m}$), independent of the interaction type, underscoring the generality of the formulation.

In combination, these results extend the Maxwellian structure to incorporate motional effects and force–flux relations, providing a framework to analyze how intrinsic motion modifies field–source interactions and yields new categories of effective charges emerging from geometry.

III. MAGNETIC FIELD OF A ROTATING CHARGED SPHERE

The structural extensions introduced in the preceding section enable direct treatment of sources with intrinsic motion, where the velocity-dependent term $\vec{u}\times\vec{B}_{T'}$ modifies the conventional field–source relation. As a concrete example, we now examine a uniformly charged solid sphere of radius R and total charge q, rotating with constant angular velocity $\vec{\omega} = \omega\hat{z}$, as illustrated in Fig. 1. In the rotating frame the charge distribution is static; however, its azimuthal motion produces a steady current density, which in turn generates a magnetic field. Our goal is to determine the magnetic induction $\vec{B}(\vec{r})$ for $r \geq R$ using the Biot–Savart law in spherical coordinates, without introducing the magnetic dipole moment as an ansatz.

The uniform volume charge density is

$$\rho = \frac{q}{\frac{4}{3}\pi R^3}. \tag{12}$$

Each volume element moves with velocity $\vec{u} = \vec{\omega}\times\vec{r}'$, giving a current density

$$\vec{J}(\vec{r}') = \rho\,(\vec{\omega}\times\vec{r}') = \rho\omega r'\sin\theta'\,\hat{\phi}'. \tag{13}$$

The Biot–Savart law gives the magnetic field at an observation point $\vec{r}$:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, d^3r'. \tag{14}$$

To simplify the evaluation, we first compute the field along the z-axis, where axial symmetry ensures that only the z-component is nonzero. The source point $\vec{r}'$ is expressed in spherical coordinates as

$$\vec{r}' = r'(\sin\theta'\cos\phi',\ \sin\theta'\sin\phi',\ \cos\theta'), \tag{15}$$

with volume element

$$d^3r' = r'^2\sin\theta'\,dr'\,d\theta'\,d\phi'. \tag{16}$$

Substituting into the Biot–Savart expression and exploiting symmetry, the axial component becomes

$$B_z(r) = \frac{\mu_0\rho\omega}{2}\int_0^R\int_0^\pi \frac{r'^3\sin^3\theta'}{(r^2 - 2rr'\cos\theta' + r'^2)^{3/2}}\,d\theta'\,dr'.$$

For $r \geq R$, the field point lies outside the source, and the leading term of a multipole expansion yields the dominant contribution. Evaluating the integral and substituting for ρ gives

$$B_z(r) = \frac{\mu_0 q R^2\omega}{4\pi r^3}. \tag{17}$$

By spherical symmetry, this generalizes to arbitrary polar angle θ:

$$\vec{B}(r,\theta) = \frac{\mu_0 q R^2\omega}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right), \quad r \geq R. \tag{18}$$

This is the exact dipolar field of the rotating sphere, obtained without assuming a point dipole.

Applying Gauss's law for magnetism,

$$\oint_{\partial V}\vec{B}\cdot d\vec{A} = 0,$$

confirms that no net magnetic charge is present, in agreement with $\nabla\cdot\vec{B} = 0$. The total flux through a sphere of radius r is zero:

$$d\vec{A} = \hat{r}\,r^2\sin\theta\,d\theta\,d\phi, \tag{19}$$

$$\Phi_B = \int_0^{2\pi}\int_0^\pi B_r(r,\theta)\,r^2\sin\theta\,d\theta\,d\phi = 0, \tag{20}$$

since

$$\int_0^\pi \cos\theta\sin\theta\,d\theta = 0. \tag{21}$$

Although the net flux vanishes, the hemispherical contributions are nonzero:

$$\Phi_N = \frac{\mu_0 q R^2\omega}{2r}, \tag{22}$$

$$\Phi_S = -\frac{\mu_0 q R^2\omega}{2r}, \tag{23}$$

suggesting the presence of oppositely signed effective flux sources at the poles. These may be expressed as *effective magnetic charges*:

$$\tilde{q}_m^N = \frac{\Phi_N}{\mu_0} = \frac{qR^2\omega}{2r}, \tag{24}$$

$$\tilde{q}_m^S = \frac{\Phi_S}{\mu_0} = -\frac{qR^2\omega}{2r}. \tag{25}$$

Their sum,

$$q_m^{\text{eff}} = \tilde{q}_m^N + \tilde{q}_m^S = 0, \tag{26}$$

is consistent with the solenoidal constraint.

These effective quantities arise from the rotational motion of the source rather than from fundamental monopoles. For comparison, a static uniformly charged sphere can be viewed as the sum of two hemispherical charges $q/2 + q/2 = q$. In the rotating case,

$$\tilde{q}_m^m = \pm\frac{qR^2\omega}{2r}, \tag{27}$$

which leads to a convenient expression for the total magnitude of the effective magnetic charge:

$$\tilde{q}_m = \tilde{q}_m^m + \tilde{q}_m^m = \frac{qR^2\omega}{r}. \tag{28}$$

This representation compactly encodes the dipole-like structure without implying isolated monopoles. The field can be written as

$$\vec{B}(r,\theta) = \frac{\mu_0\tilde{q}_m}{4\pi r^2}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right), \quad r \geq R, \tag{29}$$

where $\tilde{q}_m$ is the emergent magnetic quantity associated with rotation. For macroscopic systems, $\tilde{q}_m$ depends on r; for an elementary particle, the same structure may be expressed in terms of a constant *emergent* magnetic charge $q_m = q\,c$. The formal derivation of q_m is presented in the next sections, providing a geometric interpretation of distributed-motion field structures consistent with Maxwell's equations.

IV. CROSS-COUPLED CONTRIBUTIONS TO THE TOTAL ELECTRIC FIELD

The rotating charged sphere analyzed in the previous section generates a magnetic field whose structure admits an effective description in terms of emergent magnetic charges. Within the extended Maxwellian framework, such rotation can also produce an additional electric field through a velocity–field cross coupling. This mechanism parallels the Faraday disc (homopolar generator), where motion through a spatially structured but temporally constant magnetic field induces an electromotive response. In a rotating continuous charge distribution, the tangential velocity interacts with the self-generated magnetic field to yield an extra electric field component, modifying the total field experienced by the system.

The total electric field $\vec{E}_{TT'}$ can be expressed as the sum of a static Coulomb field and a cross-coupled, motion-induced field:

$$\vec{E}_{ccc}(\vec{r}) \;=\; \vec{E}_c(\vec{r}) + \vec{E}_{cc}(\vec{r}) \;=\; -\nabla\phi(\vec{r}) \;\pm\; k\,\vec{u}\times\vec{B}(\vec{r}),$$

where $\vec{u} = \omega R \sin\theta\,\hat{\phi}$ is the tangential velocity of the rotating distribution on the surface. The sign reflects the sense of rotation: in macroscopic systems it can be externally chosen, whereas in elementary particles such as the electron it is fixed by internal structure and energy considerations. For generality, the cross-coupled term is written as

$$\vec{E}_{cc}(\vec{r}) \;=\; \pm\,k\,\vec{u}\times\vec{B}(\vec{r}), \tag{30}$$

where k is a context-dependent constant: $k = 1$ for macroscopic systems, and $k = \frac{3}{2}$ for elementary particles, as will be shown from flux constraints. Decomposing the field as $\vec{B}(\vec{r}) = B_r(r,\theta)\,\hat{r} + B_\theta(r,\theta)\,\hat{\theta}$, and using $\hat{\phi}\times\hat{r} = \hat{\theta}$ and $\hat{\phi}\times\hat{\theta} = -\hat{r}$, we find for $\pm k\vec{u}\times\vec{B}(\vec{r})$ that only the radial component contributes to the quantity of interest, yielding

$$E_{cc,r}(r,\theta) = \pm k\cdot\omega R\sin\theta\cdot B_\theta(r,\theta), \tag{31}$$

with

$$B_\theta(r,\theta) = \frac{\mu_0 q R^2\omega}{4\pi r^3}\sin\theta, \tag{32}$$

leading to

$$E_{cc,r}(r,\theta) = \pm k\cdot\frac{\mu_0 q R^3\omega^2}{4\pi r^3}\sin^2\theta. \tag{33}$$

The contribution to the total electric flux is obtained from Gauss's law:

$$\Phi_{ccc} = \oint_{\partial V}\vec{E}_{TT'}\cdot d\vec{A} = \oint_{\partial V}\left(-\nabla\phi + \vec{E}_{cc}\right)\cdot d\vec{A}. \tag{34}$$

Over a spherical surface, only the radial components contribute:

$$\Phi_{ccc} = \int_0^{2\pi}\int_0^{\pi}\left(E_{c,r}(r,\theta) + E_{cc,r}(r,\theta)\right) r^2\sin\theta\,d\theta\,d\phi.$$

The Coulomb term gives

$$\oint_{\partial V}\vec{E}_c\cdot d\vec{A} = \frac{q}{\varepsilon_0}. \tag{35}$$

For the cross-coupled term:

$$\Phi_{cc} = \pm k\cdot\frac{\mu_0 q R^3\omega^2}{4\pi r}\int_0^{2\pi}d\phi\int_0^{\pi}\sin^3\theta\,d\theta = \pm\frac{2k\mu_0 q R^3\omega^2}{3r}.$$

Thus, the total flux is

$$\Phi_{ccc} = \frac{q}{\varepsilon_0} \pm \frac{2k\mu_0 q R^3\omega^2}{3r}. \tag{36}$$

When the induced field opposes the Coulomb field, as for the electron, the sign is negative and the effective field is reduced:

$$\vec{E}_{ccc} = -\nabla\phi - k\vec{u}\times\vec{B}(r,\theta), \tag{37}$$

$$\Phi_{ccc} = \frac{q}{\varepsilon_0} - \frac{2k\mu_0 q R^3\omega^2}{3r}. \tag{38}$$

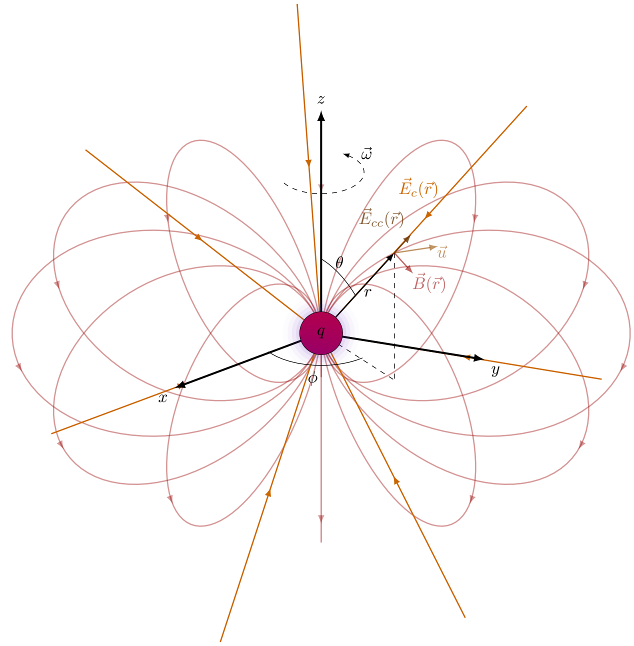

FIG. 1. **Field structure of a rotating charge distribution.** Rotation generates a dipolar magnetic field and a cross-coupled electric field $\vec{E}_{cc}$ that opposes the static Coulomb field $\vec{E}_c$, yielding a screened effective charge (original charge in red, partially obscured by the purple screening field). This effect parallels the Faraday disc, where motion through a stationary magnetic field induces an electric response.

This reduction acts as a *screening effect*, lowering the observable electric field without altering the intrinsic charge.

The physical plausibility of this configuration can be assessed from energy considerations. The total electric field energy is

$$U = \frac{1}{2}\varepsilon_0\int|\vec{E}_{\rm tot}(\vec{r},t)|^2\,d^3r, \tag{39}$$

with time derivative

$$\frac{dU}{dt} = \varepsilon_0\int\vec{E}_{\rm tot}\cdot\frac{\partial\vec{E}_{\rm tot}}{\partial t}\,d^3r. \tag{40}$$

For steady rotation, $\vec{E}_c$ is static and time variation arises solely from $\vec{E}_{cc}(t)$:

$$\frac{\partial\vec{E}_{\rm tot}}{\partial t} = \pm\frac{\partial\vec{E}_{cc}}{\partial t}. \tag{41}$$

Substituting gives

$$\frac{dU}{dt} = \pm\varepsilon_0\int(\vec{E}_c \pm \vec{E}_{cc})\cdot\frac{\partial\vec{E}_{cc}}{\partial t}\,d^3r. \tag{42}$$

In the *screening case* ($\vec{E}_{\rm tot} = \vec{E}_c - \vec{E}_{cc}$):

$$\frac{dU}{dt} = -\varepsilon_0\int(\vec{E}_c - \vec{E}_{cc})\cdot\frac{\partial\vec{E}_{cc}}{\partial t}\,d^3r. \tag{43}$$

In steady state, $\partial\vec{E}_{cc}/\partial t = 0$, so

$$\frac{dU}{dt} = 0. \tag{44}$$

In the *additive case* ($\vec{E}_{\rm tot} = \vec{E}_c + \vec{E}_{cc}$):

$$\frac{dU}{dt} = \varepsilon_0 \int (\vec{E}_c + \vec{E}_{cc}) \cdot \frac{\partial \vec{E}_{cc}}{\partial t}\, d^3r. \tag{45}$$

Again, for equilibrium

$$\frac{dU}{dt} = 0, \tag{46}$$

but the total field energy becomes

$$U = U_c + U_{cc} + U_{\rm int}, \tag{47}$$

where

$$U_{\rm int} = \varepsilon_0 \int \vec{E}_c \cdot \vec{E}_{cc}\, d^3r \tag{48}$$

is generally positive, giving $U > U_c$. For an isolated elementary charge, this implies an unphysical energy surplus unless external work is supplied.

We conclude that the screening configuration is energetically consistent, whereas the additive case contradicts conservation. For elementary particles with intrinsic rotation—such as spin or internal charge circulation—the physically viable configuration is therefore

$$\vec{E}_{\rm tot} = \vec{E}_{ccc} = \vec{E}_c - \vec{E}_{cc}. \tag{49}$$

V. EXTENDED ELECTRIC FIELD OF ELEMENTARY CHARGED PARTICLES

The cross-coupled mechanism derived previously not only alters the macroscopic field of a rotating distribution but can also be applied to the intrinsic dynamics of elementary charges. For such particles, internal rotational motion generates a self-interaction between the tangential velocity and the associated magnetic field, producing a persistent screening of the electrostatic field. In this section, we adapt the formulation to the microscopic scale and determine the corresponding parameters for an individual charged particle.

For an elementary particle, the total electric field is written as

$$\vec{E}_{ccc} = -\nabla\phi - k\vec{u} \times \vec{B}(r,\theta),$$
$$\vec{u} = \omega R \sin\theta\,\hat{\phi}, \quad \tilde{q}_m = \frac{qR^2\omega}{r},$$
$$\vec{B}(r,\theta) = \frac{\mu_0 \tilde{q}_m}{4\pi r^2}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right), \quad r \geq R,$$
$$\Phi_{ccc} = \frac{q}{\varepsilon_0} - \frac{2k\mu_0 q R^3 \omega^2}{3r}.$$

Here R is replaced by the classical charge radius r_c; for the electron, $r_c = r_e$. The tangential velocity satisfies the relativistic constraint $\omega r_c = c$.

The *screening effect* reaches its maximal strength when the net electric flux at the particle surface ($r = r_c$) is zero, which fixes k through

$$\Phi_{ccc} = 0 \quad \Rightarrow \quad \frac{q}{\varepsilon_0} - \frac{2k\mu_0 q c^2 r_c}{3r} = 0 \Rightarrow k = \frac{3}{2}. \tag{50}$$

To eliminate explicit dependence on r_c, we use its known expression

$$r_c = \frac{q^2}{4\pi\varepsilon_0 mc^2}. \tag{51}$$

The quantity $\tilde{q}_m$, representing the dipole-like magnetic source term, becomes

$$\tilde{q}_m = \frac{qR^2\omega}{r} = qc\frac{r_c}{r} = qc\frac{q^2}{4\pi\varepsilon_0 mc^2 r}, \tag{52}$$

$$\tilde{q}_m = q_m \frac{q^2}{4\pi\varepsilon_0 mc^2 r}, \tag{53}$$

where q_m is the magnetic counterpart of the electric charge, co-located ($r = r_c$) with the particle's charge distribution. It is not a monopole but an *emergent* attribute arising from the particle's intrinsic rotation. The term $\frac{q^2}{4\pi\varepsilon_0 mc^2 r}$ acts as a radial scaling factor that decreases with distance, reducing the influence of q_m in $\tilde{q}_m$ as r increases.

The resulting extended electric field for an elementary charged particle is

$$\vec{E}_{ccc} = -\nabla\phi - \frac{3}{2}\vec{u} \times \vec{B}(r,\theta), \tag{54}$$

$$\vec{u} = c\sin\theta\,\hat{\phi}, \quad \tilde{q}_m = q_m \frac{q^2}{4\pi\varepsilon_0 mc^2 r}, \tag{55}$$

$$\vec{B}(r,\theta) = \frac{\mu_0 \tilde{q}_m}{4\pi r^2}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right), \quad r \geq r_c, \tag{56}$$

$$\vec{E}_{ccc} = \vec{E}_c - \vec{E}_{cc} = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}}{|\vec{r}|^3} - \frac{q^3}{16\pi^2\varepsilon_0^2 mc^2}\frac{\vec{r}}{|\vec{r}|^4}, \tag{57}$$

$$\vec{E}_c = -\nabla\phi = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}}{|\vec{r}|^3}, \tag{58}$$

$$\vec{E}_{cc} = \frac{3}{2}\vec{u} \times \vec{B}(r,\theta) = \frac{q^3}{16\pi^2\varepsilon_0^2 mc^2}\frac{\vec{r}}{|\vec{r}|^4}. \tag{59}$$

VI. EMERGENT CHARGES AND DIMENSIONAL SYMMETRY IN CLASSICAL FORCE LAWS

The extended field description for elementary charges naturally invites comparison with other interaction domains. If the structural link between field coupling constants and wave propagation speed observed in electromagnetism also applies to gravity and inertia, then analogous formulations should emerge across these systems.

In electromagnetism, the vacuum constants define the speed of light:

$$c^2 = \frac{1}{\varepsilon_0\mu_0}. \tag{60}$$

Extending this symmetry to an arbitrary interaction with charge type q_T and coupling constants ε_{T_0}, $\mu_{T_0'}$ gives

$$c^2 = \frac{1}{\varepsilon_{T_0}\mu_{T_0'}} = \frac{1}{\varepsilon_{g_0}\mu_{i_0}}, \tag{61}$$

where ε_{g_0} and μ_{i_0} represent the gravitational permittivity and inertial permeability, respectively.

For the gravitational analogy, we take

$$\varepsilon_{g_0} = 4\pi G = 8.38717 \times 10^{-10}\,\mathrm{J}/\left(\mathrm{kg}\cdot\mathrm{m}^{-1}\right)^2 \cdot \mathrm{m}, \tag{62}$$

from which

$$\mu_{i_0} = \frac{1}{c^2\varepsilon_{g_0}} = 1.32661 \times 10^{-8}\left(\mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}\right)^2/\mathrm{J}\cdot\mathrm{m}. \tag{63}$$

follows directly.

The generalized force–flux relation for a Coulomb-like field,

$$\oint_{\partial V} \vec{F}\cdot d\vec{A} = \frac{q_T^2}{\varepsilon_{T_0}},$$

retains its form for electric, magnetic, gravitational, and inertial sources. Incorporating the coupling constants and their connection through c yields

$$\frac{q_T^2}{\varepsilon_{T_0}} = \mu_{T_0'} q_{T'}^2 = \frac{q_e^2}{\varepsilon_0} = \mu_0 q_m^2 = \frac{q_g^2}{\varepsilon_{g_0}} = \mu_{i_0} q_i^2, \tag{64}$$

$$q_{T'} = \pm q_T c \quad\Rightarrow\quad q_m = \pm q_e c, \quad q_i = \pm q_g c. \tag{65}$$

Evaluating the magnetic analogue:

$$q = \pm q_e = \pm 1.60217663 \times 10^{-19}\,\mathrm{J/V}, \tag{66}$$

$$q_m = \pm q_e c = \pm 4.8032 \times 10^{-11}\,\mathrm{J}\cdot\mathrm{m/V}\cdot\mathrm{s}. \tag{67}$$

For the gravitational analogue producing the same force magnitude at a fixed separation,

$$q_g = \pm q_e\sqrt{\frac{\varepsilon_{g_0}}{\varepsilon_0}} = \pm 1.55935 \times 10^{-18}\,\mathrm{J/kg}\cdot\mathrm{m}^{-1}, \tag{68}$$

and for the inertial analogue,

$$q_i = \pm q_g c = \pm 4.67481 \times 10^{-10}\,\mathrm{J}\cdot\mathrm{m/kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}. \tag{69}$$

These results indicate that a Coulomb-type force law can be written for electromagnetic, gravitational, or inertial interactions by selecting the corresponding charge–constant pair. Although the physical interpretation of the fields differs—electric induction, gravitational acceleration, or inertial resistance—the integral structure is identical, reflecting a dimensional symmetry that extends beyond electromagnetism.

VII. FIELD EQUIVALENCE VIA GAUSS LAW FOR THE DISPLACEMENT FIELD

The dimensional symmetries established above can be expressed directly in the language of displacement fields. In electromagnetism, Gauss's law for the electric displacement field relates the flux through a closed surface to the enclosed free charge:

$$\oint_{\partial V} \vec{D}\cdot d\vec{A} = q_{\mathrm{free}}, \qquad \vec{D} = \varepsilon_0\vec{E}. \tag{70}$$

This form applies both to macroscopic systems—such as charged conductors with surface free charge—and to elementary particles, whose far-field behavior matches that of a Coulomb source.

The same structure may be applied to gravity by defining a gravitational displacement field

$$\vec{D}_G = \varepsilon_{g_0}\vec{E}_G, \tag{71}$$

with $\vec{E}_G$ as the gravitational field analogue. Using the gravitational–electric charge equivalence

$$q_g = \pm q_e\sqrt{\frac{\varepsilon_{g_0}}{\varepsilon_0}},$$

the gravitational and electric displacement fluxes are related by

$$\oint_{\partial V} \vec{D}_G\cdot d\vec{A} = \pm\sqrt{\frac{\varepsilon_{g_0}}{\varepsilon_0}}\oint_{\partial V} \vec{D}_E\cdot d\vec{A}, \tag{72}$$

where $\vec{D}_E = \varepsilon_0\vec{E}$.

Substituting $\vec{D}_G = \varepsilon_{g_0}\vec{E}_G$ and simplifying gives

$$\vec{E}_G = \pm\sqrt{\frac{\varepsilon_0}{\varepsilon_{g_0}}}\vec{E}, \tag{73}$$

showing that $\vec{D}_G$ has units of acceleration (m/s^2 or N/kg). Multiplying Eq. (73) by ε_{g_0} yields

$$\vec{D}_G = \pm\vec{E}\sqrt{\varepsilon_0\varepsilon_{g_0}}, \tag{74}$$

identifying gravitational acceleration with the gravitational displacement field rather than $\vec{E}_G$ itself. This parallels Gauss's law in electrostatics, with both interactions sharing a flux-based geometric structure.

To illustrate the scale of the coupling, consider the electric field in vacuum required for $D_G = 9.81\,\mathrm{N/kg}$:

$$E = \frac{D_G}{\sqrt{\varepsilon_0\varepsilon_{g_0}}}, \quad D_E = \varepsilon_0 E, \tag{75}$$

$$E = 1.13838 \times 10^{11}\,\mathrm{V/m}, \quad D_E = 1.0079\,\mathrm{J/V}\cdot\mathrm{m}^2. \tag{76}$$

This enormous value highlights the relative weakness of gravity despite the formal analogy.

An analogous correspondence follows from the magnetic–inertial charge relation in Eq. (64):

$$\mu_{i_0} q_i^2 = \mu_0 q_m^2 \quad\Rightarrow\quad q_i = \pm q_m\sqrt{\frac{\mu_0}{\mu_{i_0}}}. \tag{77}$$

The resulting flux equivalence between inertial and magnetic displacement fields is

$$\oint_{\partial V} \vec{D}_I\cdot d\vec{A} = \pm\sqrt{\frac{\mu_0}{\mu_{i_0}}}\oint_{\partial V} \vec{D}_B\cdot d\vec{A}, \tag{78}$$

with $\vec{D}_B = \vec{B}/(\mu_0 c) = \vec{H}/c$ and $\vec{D}_I = \vec{B}_I/(\mu_{i_0} c) = \vec{H}_I/c$, where $\vec{H}_I$ is an inertial analogue to the magnetic field.

From Eq. (78), the magnetic field needed to induce $D_I = 9.81\, m/s^2$ is

$$H = cD_I\sqrt{\frac{\mu_{i_0}}{\mu_0}}, \quad B = cD_I\sqrt{\mu_0\mu_{i_0}}, \tag{79}$$

$$H = 3.02\times 10^8\,\text{A/m (or J/V}\cdot\text{s}\cdot\text{m)}, \quad B = 379.72\,\text{T}. \tag{80}$$

As in the electric–gravitational case, the required field is exceptionally large, emphasizing that while the displacement-field analogy preserves formal structure, the coupling strengths differ greatly.

Because the framework is flux-based, these analogies extend from point particles to macroscopic systems—such as the rotating charged spheres considered earlier—by substituting alternative coupling constants without altering the underlying Gauss-law form. This provides a unified geometric and dimensional description for electric, gravitational, magnetic, and inertial interactions.

VIII. UNIFIED FIELD EQUATION

The flux-based analogies established in the displacement-field framework can be extended to a single formulation that applies uniformly across electric, magnetic, gravitational, and inertial domains. Building on the structural extensions to Maxwell's equations and the emergent charge relations derived from dimensional symmetry, we write the generalized static-field equation for an arbitrary charge type (Fig. 2) in a form parallel to the extended electric field:

$$\vec{E}_{TT'} = -\nabla\phi_T - \frac{3}{2}\vec{u}\times\vec{B}_{T'}(r,\theta), \tag{81}$$

$$\vec{u} = c\sin\theta\,\hat{\phi}, \quad \tilde{q}_{T'} = q_{T'}\frac{q_T^2}{4\pi\varepsilon_{T_0}mc^2 r}, \tag{82}$$

$$\vec{B}(r,\theta) = \frac{\mu_{T'_0}\tilde{q}_{T'}}{4\pi r^2}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right), \quad r \geq r_c, \tag{83}$$

$$\vec{E}_{TT'} = \vec{E}_T - \vec{E}_{T'} = \frac{q_T}{4\pi\varepsilon_{T_0}}\frac{\vec{r}}{|\vec{r}|^3} - \frac{q_T^3}{16\pi^2\varepsilon_{T_0}^2 mc^2}\frac{\vec{r}}{|\vec{r}|^4}, \tag{84}$$

$$\vec{E}_T = -\nabla\phi_T = \frac{q_T}{4\pi\varepsilon_{T_0}}\frac{\vec{r}}{|\vec{r}|^3}, \tag{85}$$

$$\vec{E}_{T'} = \frac{3}{2}\vec{u}\times\vec{B}_{T'}(r,\theta) = \frac{q_T^3}{4\pi\varepsilon_{T_0}^2 mc^2}\frac{\vec{r}}{|\vec{r}|^4}, \tag{86}$$

$$\vec{D}_{TT'} = \frac{q_T}{4\pi}\frac{\vec{r}}{|\vec{r}|^3} - \frac{q_T^3}{16\pi^2\varepsilon_{T_0}mc^2}\frac{\vec{r}}{|\vec{r}|^4}. \tag{87}$$

The formalism applies to specific interaction pairs by assigning the appropriate charges and constants:

$$TT' = EM \begin{cases} q_T = \pm q_e, & q_{T'} = \pm q_m, \\ \varepsilon_{T_0} = \varepsilon_0, & \mu_{T'_0} = \mu_0, \\ \phi_T = \phi, & B_{T'} = B \end{cases}, \tag{88}$$

$$TT' = GI \begin{cases} q_T = \pm q_g, & q_{T'} = \pm q_i, \\ \varepsilon_{T_0} = \varepsilon_{g_0}, & \mu_{T'_0} = \mu_{i_0}, \\ \phi_T = \phi_G, & B_{T'} = B_I \end{cases}. \tag{89}$$

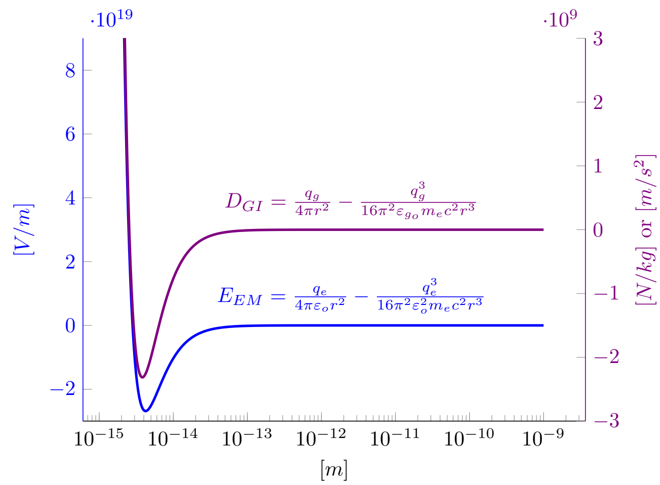

FIG. 2. **Unified static-field representation.** Comparison of the electron's total electromagnetic field E_{EM} and its gravitational displacement field D_{GI} within the unified formulation. At $r = 1.12500\cdot 10^{-10}$ m, the fields have magnitudes $E_{EM} = -1.13775\cdot 10^{11}$ V/m and $D_{GI} = -9.80475$ N/kg. Both arise from the same structural equation with domain-specific coupling constants, illustrating the geometric and dimensional symmetry between electromagnetic and gravitoinertial interactions.

Choosing TT′ = EM in Eq. (88) recovers the extended electric field of an elementary charged particle—such as the electron—or, more generally, its composite electromagnetic field combining electric and emergent magnetic charges. Likewise, TT′ = GI in Eq. (89) yields the extended gravitational field, or composite gravitoinertial field, uniting gravitational and emergent inertial charges.

IX. FIELD QUANTITIES AND UNITS

The following is a summary of the field quantities, accompanied by their respective units:

TABLE I. Field Quantities and Units (Part I)

Symbol	Unit	Symbol	Unit
E	[V/m]	H	[J/V · s · m] = [A/m]
D_E	[J/V · m^2]	B	[V · s/m^2]
ε_0	[J/V^2 · m]	μ_0	[(V · s)^2 /J · m]
q_e	[J/V]	q_m	[J · m/V · s]

TABLE II. Field Quantities and Units (Part II)

Symbol	Unit	Symbol	Unit
E_G	[kg · m^{-1}/m]	H_I	[J/kg · m^{-1} · s · m]
D_G	[J/kg · m^{-1} · m^2]	B_I	[kg · m^{-1} · s/m^2]
ε_{g0}	$[\text{J}/\left(\text{kg}\cdot\text{m}^{-1}\right)^2\cdot\text{m}]$	μ_{i_0}	$[\left(\text{kg}\cdot\text{m}^{-1}\cdot\text{s}\right)^2/\text{J}\cdot\text{m}]$
q_g	[J/kg · m^{-1}]	q_i	[J · m/kg · m^{-1} · s]

X. FIELD-INDUCED MASS AND FORCE EFFECTS

The expressions linking electromagnetism to gravity and inertia—assumed here by symmetry—can be generalized to apply across different media. In this broader formulation, the electromagnetic and gravito-inertial constants are complemented by the medium's relative permittivity ε_r and permeability μ_r. Furthermore, instead of using the vacuum speed of light c, one may substitute the group velocity u_g, which in conductive media is approximated by a diffusive-like expression. Thus, the effective expressions for gravitational-like and inertia-like acceleration (a) become:

$$a = D_G = \pm E\, \varepsilon_r \sqrt{\varepsilon_0 \varepsilon_{g0}}, \tag{90}$$

$$a = D_I = \pm \frac{B}{u_g\, \mu_r \sqrt{\mu_0 \mu_{i_0}}}. \tag{91}$$

Let us consider confined electromagnetic energy, such as a standing wave, which can be modulated through an angular velocity shift $\Delta\omega$. This applied shift gives rise to a force, initially described by the radiation pressure relation $F = \frac{1}{c}\frac{dU}{dt}$. An equivalent formulation expresses this force in terms of the electromagnetic energy density w_{EM} and the cross-sectional area A through which the energy undergoes accelerated translation:

$$F = A \cdot w_{\mathrm{EM}} \cdot \frac{\Delta\omega}{\omega_g}, \tag{92}$$

where ω_g denotes the base angular velocity.

The equivalent electromagnetic mass m_{EM} can now be expressed by dividing the force by the corresponding acceleration expressions introduced above:

$$w_{\mathrm{EM}} \approx \varepsilon_0 \varepsilon_r E^2 \approx \frac{B^2}{\mu_0 \mu_r}, \tag{93}$$

$$m_{\mathrm{EM}} = \pm A \cdot E \sqrt{\frac{\varepsilon_0}{\varepsilon_{g0}}} \cdot \frac{\Delta\omega}{\omega_g} = \pm A \cdot u_g B \sqrt{\frac{\mu_{i_0}}{\mu_0}} \cdot \frac{\Delta\omega}{\omega_g}. \tag{94}$$

Eq. (94) suggests that confined electromagnetic energy may give rise to an effective mass that exhibits both inertial and gravitational-like behavior—despite originating from a massless configuration.

An insightful expression emerges for the electron or positron when we set $A = 4\pi r_e^2$, $E = \mp \frac{q_e}{4\pi\varepsilon_0 r_e^2}$, $u_g = c$, and $\varepsilon_r = \mu_r = 1$. Under these conditions, the electromagnetic mass becomes:

$$m_{\mathrm{EM}} = \mp \frac{q_e}{\sqrt{\varepsilon_0 \varepsilon_{g0}}} \cdot \frac{\Delta\omega}{\omega_g}, \tag{95}$$

$$m_g = \mp \frac{q_e}{\sqrt{\varepsilon_0 \varepsilon_{g0}}} = \mp 1.85921 \cdot 10^{-9}\,\mathrm{kg}. \tag{96}$$

By dividing m_g by the Planck mass m_p and squaring the result, we obtain the fine-structure constant:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0 \hbar c}, \tag{97}$$

$$\left(\frac{m_g}{m_p}\right)^2 = \alpha. \tag{98}$$

XI. DISCUSSION

Embedding the Lorentz force into Maxwell's flux form exposes motional and rotational terms that are normally hidden when force laws are treated separately from field equations. In this view, a spinning charged sphere acquires an emergent dipole-like magnetic charge and a rotation-induced electric screening, both following directly from geometry. Such effects, while consistent with Maxwell's equations, have been overlooked because the flux form is rarely used as a foundation for including source motion intrinsically. Beyond electromagnetism, we construct a *Gauss Force–Flux* law by multiplying Gauss's field–flux relation by the corresponding charge type q_T. The resulting force flux has fixed units of $\mathrm{N \cdot m^2}$, and its closed-surface integral evaluates to q_T^2/ε_{T_0}. The reference to q_T-independence pertains to the dimensional form, which is the same for any interaction type. This fixed dimensionality enables direct substitution of electric parameters with magnetic, gravitational, or inertial ones while preserving the inverse-square force magnitude. In this way, the hidden motion–field coupling found in electromagnetism extends naturally to a broader class of static-field interactions, all within a purely classical, geometric framework.

XII. CONCLUSION

The present formulation reveals latent geometric structure in Maxwell's theory and extends it through a charge-independent (in units) force–flux relation to encompass magnetic, gravitational, and inertial domains. While the integral of the force flux depends on q_T^2/ε_{T_0}, the fixed units of $\mathrm{N \cdot m^2}$ allow different charge types to be substituted directly without altering the underlying inverse-square structure. This preserves the compact elegance of the original equations while offering a unified, nonrelativistic description of static interactions. This perspective suggests that Maxwell's framework is not merely the grammar of electromagnetism, but a universal geometric language for all inverse-square forces.

[1] M. Faraday, “Experimental Researches in Electricity”, Cambridge Library Collection, 3 vols. (1839–1855, reprinted 2013).
[2] J. J. Thomson, “On the Electric and Magnetic Effects Produced by the Motion of Electrified Bodies”, Philosophical Magazine 11, 229–249 (1881).
[3] J. C. Maxwell, “A Treatise on Electricity and Magnetism”, Dover reprint of the 3rd ed. revised by J. J. Thomson, Cambridge University Press (2005).
[4] O. Heaviside, “A Gravitational and Electromagnetic Analogy”, The Electrician (1893), reprinted in *Electromagnetic Theory*, Vol. 1, Dover Publications (1950).
[5] P. A. M. Dirac, “Quantised Singularities in the Electromagnetic Field”, Proceedings of the Royal Society A 133, 60–72 (1931).
[6] E. T. Newman and T. R. Newman, “Unification of the Generalized Gravitational and Electromagnetic Fields by Complex Mass and Charge”, Journal of Mathematical Physics 6, 89–92 (1965).
[7] G. ’t Hooft, “Magnetic Monopoles in Unified Gauge Theories”, Nuclear Physics B 79, 276–284 (1974).
[8] A. M. Polyakov, “Particle Spectrum in Quantum Field Theory”, JETP Letters 20, 194–195 (1974).
[9] B. Mashhoon, F. W. Hehl, and D. S. Theiss, “On the Gravitational Analogy of Electromagnetism”, General Relativity and Gravitation 16, 711–750 (1984).
[10] J. Polchinski, “Monopoles, Duality, and String Theory”, arXiv:hep-th/0304042 (2003).
[11] B.V. Ivanov, “Strong Gravitational Force Induced by Static Electromagnetic Fields”, arXiv:gr-qc/0407048 (2004)
[12] L. F. O. Costa and J. Natário, “Gravito-electromagnetic Analogies”, General Relativity and Gravitation 46, 1792 (2014).
[13] M. W. Ray, E. Ruokokoski, S. Kandel, M. Möttönen, and D. S. Hall, “Observation of Dirac Monopoles in a Synthetic Magnetic Field”, Nature 505, 657–660 (2014).
[14] P. Ghose, “Unification of Gravity and Electromagnetism and Cosmology”, arXiv:1605.08263 (2016).
[15] R. J. De Souza, “On the History of Classical Unified Field Theories”, Foundations of Physics 47, 138–174 (2017).
[16] V. A. Mitsou, “Searches for Magnetic Monopoles: A Review”, Proceedings 13, 10 (2019).
[17] N. E. Mavromatos and V. A. Mitsou, “Magnetic Monopoles Revisited: Models and Searches at Colliders and in the Cosmos”, International Journal of Modern Physics A 35, 2030012 (2020).
[18] M. L. Ruggiero, “A Note on the Gravitoelectromagnetic Analogy”, Universe 7, 451 (2021).
[19] D. Milstead and E. J. Weinberg, “Magnetic Monopoles”, in Review of Particle Physics (Particle Data Group), Physical Review D 110, 030001 (2024).