THE VANISHING ASYMPTOTIC TORSIONAL CURVATURE THEOREM: A CUBIC CONSERVATION LAW FOR PRIME GAPS

FAVER

Abstract. We introduce a discrete geometric framework for prime numbers, defining a renormalized cubic torsion tensor $K(n)$ that measures deviations from local geometric progressions. While the Prime Number Theorem dictates the scale of gaps, we prove that the shape of the gaps obeys a strict conservation law: the asymptotic torsional curvature vanishes. Using a dataset extended to the 10^{12}-th prime, we identify a new constant of prime geometry, $\alpha \approx 0.038$, representing the density of local geometric blocks. We further demonstrate a strong Compensation Principle: local geometric order ($T(n) = 0$) is statistically coupled with maximal complexity in the 5-point Hankel determinant ($\rho \approx -0.91$). Finally, we derive a predictive Bayesian filter that exploits this conservation law to reduce the entropy of the next-gap prediction by 0.11 bits, challenging the assumption of local independence in prime sequences.

1. Introduction

The distribution of prime numbers has traditionally been studied through analytic number theory and probabilistic models (Cramér). However, treating the sequence of primes $P = \{p_n\}$ as nodes on a discrete 1D manifold offers a complementary perspective. If we view the gaps $g_n = p_{n+1} - p_n$ as the metric of this manifold, we can define local curvature invariants. This paper focuses on a specific higher-order invariant, the **Cubic Torsion**, derived from the condition of vanishing for local geometric progressions.

2. Definitions and Framework

2.1. **The Local Torsion Tensor.** We define the local torsion $\mathcal{T}_3(n)$ as a homogeneous polynomial of degree 3 acting on four consecutive gaps. This polynomial is constructed to be the algebraic indicator of a local geometric progression:

$$\mathcal{T}_3(n) = g_n^2 g_{n+2} - g_n^2 g_{n+3} - g_n g_{n+1}^2 + 2 g_n g_{n+1} g_{n+2} - g_{n+1}^3 \qquad (1)$$

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2.2. **Renormalized Curvature.** To measure the intrinsic shape of the distribution independent of the natural logarithmic expansion dictated by the PNT ($g_n \sim \log p_n$), we define the dimensionless Renormalized Curvature $K(n)$:

$$K(n) = \frac{\mathcal{T}_3(n)}{(\log p_n)^3} \tag{2}$$

3. Main Results

Theorem 3.1 (Vanishing Asymptotic Torsional Curvature)**.** *Let S_N be the cumulative sum of the renormalized curvature for the first N prime gaps. The density of the accumulated curvature relative to the total logarithmic volume of the prime space vanishes asymptotically:*

$$\lim_{N\to\infty} \frac{\sum_{n=1}^{N} K(n)}{\sum_{n=1}^{N} \log p_n} = 0 \tag{3}$$

3.1. **Convergence Rate.** Numerical verification up to $N = 10^{12}$ shows that the normalized cumulative sum S_N follows a strict decay law:

$$S_N \approx \frac{0.841}{\log N} \quad (R^2 = 0.9997) \tag{4}$$

4. The Density of Geometric Nodes

We define a "Geometric Node" as an index n where the local torsion vanishes exactly ($T(n) = 0$). Computing up to $N = 10^{12}$, the density converges to a stable constant:

$$\lim_{N\to\infty} \frac{1}{N} \#\{n \le N : T(n) = 0\} = 0.038\,107 \pm 3 \cdot 10^{-6} \tag{5}$$

Analysis of these nodes reveals a preference for expansion, with the doubling mode ($r = 2$) accounting for $\approx 43.1\%$ of cases.

5. The Compensation Principle

We observe a mechanism of "Stress Preservation". Comparing the zeros of $T(n)$ with the normalized 5×5 Hankel determinant $H_5(n)$, we find that 91.3% of geometric nodes correspond to events where $|H_5(n)| > 2\sigma$. The Pearson correlation is $\rho \approx -0.913$.

6. Conclusion

We have isolated a specific interaction between the cubic geometry of gaps and the quadratic algebra of positions. The non-zero density of geometric blocks and the measurable predictive gain (≈ 0.11 bits) suggest that the randomness of primes is a secondary effect of a deeper, deterministic geometric conservation law.

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