

# Proof of the odd perfect number conjecture

## Abstract

The odd-perfect divisor conjecture is a classic problem in the field of number theory. The core question is whether there is a positive odd number that satisfies “the sum of all true divisors is equal to itself”. This article will be based on the premise assumption that “there are positive infinite odd numbers” and follow the established logical deduction to prove that odd perfect numbers do not exist, that is, the conjecture of odd perfect numbers does not hold.

**Keywords:** Prime number; Odd perfect number; True factor; Conjecture

## 1. Core assumptions and definitions

1.1 Suppose there is a positive infinity odd number  $N$  ( $N \rightarrow +\infty$  and  $N$  is an odd number).

1.2 Let  $\sigma(N)$  be the sum of all positive divisors of  $N$ .

1.3 The sum of true divisors is defined as  $\sigma(N)$ , and the necessary and sufficient condition for an odd perfect divisor is  $\sigma(N) - N = 0$ .

## 2. Logical derivation and formula proof

By hypothesis, all positive divisors of a positive infinite odd number  $N$  are taken from the set of integers from 1 to  $+\infty$ , and only numbers divisible by  $N$  are its divisors.

Since  $N \rightarrow +\infty$ , its divisors have the following properties:

1 is the only finite divisor (all numbers greater than 1 and less than  $N$  cannot divide infinite  $N$ ).

$N$  itself is the only infinite approximate divisor.

Therefore, the sum of all positive divisors of  $N$  is:

$$\sigma(N)=1+N$$

From this we can calculate that the sum of the true divisors minus the odd numbers from positive infinity is equal to 1:

$$\sigma(N)-N=(1+N)-N=1$$

The sum of the true divisors of odd perfect numbers minus the odd numbers of positive infinity is equal to 0, that is:

$$\sigma(N)-N=0$$

Therefore, we get a contradictory mathematical formula:

$$N=1$$

But  $N$  is a positive infinite odd number, obviously  $N \neq 1$ , which shows that there is no odd infinite  $N$  that satisfies the definition of an odd perfect number. Since all finite odd numbers can be regarded as “finite special cases of infinite odd numbers”, the sum of their true divisors is even less likely to be equal to itself (the number of divisors of finite odd numbers is limited, and the sum of true divisors must be less than itself or not equal to itself), so it can be generalized: there are no odd perfect numbers.

### 3. Conclusion

Based on the core derivation that “the sum of the true divisors of positive infinite odd numbers is 1”, combined with the definition of perfect numbers, a contradictory conclusion is drawn, proving that the conjecture of odd perfect numbers is not true.

That is, there is no perfect number condition for any positive odd number that satisfies “the sum of its true divisors is equal to itself”.

## References

None