# Nullivance Propositional Logic (NPL):
# A Formal System for Oscillating States and Latent Structures

Part 1: Foundations, Syntax, Semantics, and Proof Theory

**Trinh Tung Lam**

vietthuc@giugocviet.org

Logic Nullivance Research

January 17, 2026

**Abstract**

The Nullivance Logic System and Nullivance Mathematics are non-binary theoretical frameworks designed to model dynamic states, contradictions, and quasivance (quasi-isolated) entities through the logic unit $\delta_S(A) = \alpha \cdot \Phi(\vec{\Theta}(t))$. This report presents the background, development motivation, initiating philosophical inquiries, objectives, vision, and a system overview, including a comparative analysis between Nullivance Logic and traditional logic. Detailed examples—ranging from food selection and weather forecasting to DNA analysis and image classification—illustrate core concepts, accompanied by distinct logic network diagrams that visualize how Nullivance processes data and contradictions. Python implementation code is provided to clarify calculations. Comparisons with binary and fuzzy logic suggest potential applicability of Nullivance in scenarios involving uncertainty, contradictions, and dynamic data.

## 1 Introduction

This research initiates from a seemingly self-evident observation: **a train of thought can only originate when information exists**. That is, for any concept to arise in the mind, it requires a nucleus—information previously recorded, learned, or absorbed from reality through the gateway of the senses. Without sensory input, memory, data, or experience, there is nothing to "think" about. Thought cannot self-start from an absolute void. This is the origin point: **thought is a consequence of interaction with an existing reality**.

From this, a broader question arises: **Why do humans use Mathematics and Physics to describe reality?** Why do we not use another system—such as poetry, intuition, or imagery—as the foundation for modeling the world? The classical answer is that Mathematics is logical, rigorous, and consistent. However, this opens a deeper line of inquiry: **does the "logic" we perceive in Mathematics truly belong to reality, or is it merely a projection of how the human brain operates?**

If the human brain is a system with an intrinsic logical structure—evolved to recognize patterns, analyze rules, and predict dangers—it is not surprising that it adapts to and reflects reality through the prism of Mathematics and Physics. Yet, this poses a paradox: **is reality "logical" because it is inherently so, or because we can only perceive the parts that our logic permits?**

Furthermore, a more profound question emerges: **If there is something that does not belong to this reality—undefinable, unmodelable, imperceptible by experience—what is it?** And the most formidable inquiry: **Why do entities that do not yet exist—or are not presently manifested—still exert measurable influence on current reality?** For instance, imagined threats (the nocebo effect) can trigger distinct physiological responses, just as quantum systems appear to shift behavior in anticipation of observation. If a new logic could open this sensory realm—where thought requires no data, where existence and non-existence oscillate in state—it could propel us beyond current cognitive limits.

It is at this aperture that I began to sketch a new mathematical model—not to describe what exists, but to probe the possibilities of **the unmanifested**, where logic is non-binary and existence is not absolute. I named this model: **Nullivance**—modeling the genesis of state from the void.

**Core Principles of Nullivance Logic**: Nullivance Logic does not classify data as "true" or "false" but records all data into a **state oscillation network**. Each datum is associated with a **signature** (symbol: $\sigma$), representing a state or entity, and a **phase tensor** ($\vec{\Theta}(t) \in [0,1]^{n\times m}$), describing the oscillation between affirmation and negation. The system automatically identifies **patterns** (symbol: $P$) among signatures through **phase compatibility**, measured by the function $\Phi(\vec{\Theta}(t))$. If a pattern has sufficient **existence level** ($\alpha \in [0,1]$) and strong phase compatibility, the oscillation state will "emerge" with a high $\delta_S(A)$ value. If the pattern is weak, the state remains in the network but unmanifested, termed a **quasivance state** (super-isolated state). All state oscillations are non-linear, independent of data order, and represented by the **logic unit**:

$$\delta_S(A) = \alpha \cdot \Phi(\vec{\Theta}(t))$$

**Phase Stability Functions**:

- **Legacy ($_{\text{prod}}$)**: $\Phi_{\text{prod}}(\vec{\Theta}) = \prod_i (1 - 2|\Theta_i - 0.5|)$ *(not scale-safe)*
- **v1.0 Production ($_{\text{geo}}$)**: $\Phi_{\text{geo}}(\vec{\Theta}) = \left(\prod_i (1 - 2|\Theta_i - 0.5|)\right)^{1/d}$ *(scale-safe, canonical)*

where:

- $\alpha$: Existence level, indicating the probability or intensity of state $A$.
- $\vec{\Theta}(t)$: Phase tensor, where each element $\Theta_{i,j}(t)$ describes the degree of oscillation.
- $\Phi(\vec{\Theta}(t))$: Phase stability, reaching maximum when $\Theta_{i,j} = 0.5$ (maximum phase compatibility).

**Comparison with Traditional Logic**:

Table 1: Comparison between Nullivance Logic and Traditional Logic

| Feature | Traditional Logic | Nullivance Logic |
|---|---|---|
| **Nature** | Binary: True/False | State Oscillation: Exist/Skew/Harmonize/Contradict |
| **Input Requirement** | Requires clear axioms/data | Initializes from internal oscillation, undefined inputs accepted |
| **Output** | A clear proposition: True or False | Oscillation state $\delta_S = \alpha \cdot \Phi(\vec{\Theta})$ |
| **Conflict Modeling** | Does not allow contradictions | Contradictions analyzed via phase shift, persist in system |
| **Truth Emergence** | Logical deduction result | High phase compatibility states "emerge" |
| **Contradictory Info** | Eliminates contradictions | Retains, finds patterns via phase compatibility |
| **Insufficient Data** | Cannot deduce | System records possible states |
| **Thinking Structure** | Linear, sequential deduction | Non-linear network, phase compatibility |
| **Phenomenon Mechanism** | Rule-based (If A then B) | Based on phase interference of signatures |
| **Logic Unit** | Logical proposition | Unit $\delta_S$ with $\alpha$, $\vec{\Theta}$, signature $\sigma$ |

## 2 Context and Motivation

Nullivance Logic and Mathematics were created to overcome the limitations of traditional binary logic systems, such as Zermelo-Fraenkel Set Theory (ZFC) or Peano Arithmetic. These systems require precise propositions (True/False), failing to adequately model ambiguous, dynamic, or contradictory phenomena. For instance:

- **Quantum Mechanics**: A photon can exist in a superposition of horizontal and vertical polarization. Binary logic cannot represent this, but Nullivance uses $\delta_S(\text{photon})^{0.5,(0.5,0.5)}$ to describe the oscillating state.
- **Artificial Intelligence**: Neural networks assign activation values in $[0,1]$. Nullivance represents this as $\delta_S(\text{activation})^{0.7,(0.6,0.4)}$.
- **Cognitive Science**: Beliefs oscillate (e.g., 60% confidence in "buying a house"). Nullivance uses $\delta_S(\text{buy house})^{0.6,(0.55,0.45)}$.
- **Big Data Analysis**: With conflicting data from 100 sensors, Nullivance synthesizes contradictions into oscillating states.

Nullivance was initiated on June 21, 2025, by Trinh Tung Lam, aiming to build a non-binary logic system based on state oscillation networks and phase compatibility, with the goals of:

- Modeling dynamic states and contradictions naturally.

- Integrating with traditional mathematics via the Nullivance Bridge Module (NBM).
- Applying to interdisciplinary fields like physics, AI, cognitive science, and biology.

# 3 Core Philosophical Inquiries

The development of Nullivance was driven by three core philosophical questions. Below, we explore each inquiry, its explanation, and the system's response through specific examples.

## 3.1 Question 1: The Origin of Thought and Information

**Inquiry**: Does thought only originate from observed information in reality, requiring prior contact to initiate thinking? Does this imply that thought is bound by the informational patterns of reality?

**Explanation**: This suggests that cognition is inherently tied to the informational structure of reality. For example, one cannot conceptualize "justice" without prior exposure to social or linguistic structures. This observation motivates the construction of a logic system that processes patterns rather than a priori truths.

**Nullivance Response**: Nullivance proposes that thought is the result of interaction between logic patterns in a state oscillation network, without needing explicit real data, thanks to the ability to record states via the phase tensor.

## 3.2 Question 2: The Logic of Reality

**Inquiry**: Why does mathematics explain reality logically, and why is it mathematics and physics, rather than other structures, that achieve this? Is this logicality a product of the brain's logical structure shaping how we perceive reality?

**Explanation**: This challenges the primacy of mathematics as a purely objective descriptive tool. It suggests that logicality may reflect the cognitive architecture of the human brain. Nullivance is designed to map patterns onto reality akin to how mathematical reasoning operates within the brain.

**Nullivance Response**: Nullivance proposes that the "logical" nature of mathematics is a projection of human cognitive structure, modeled via $\delta$-units and phase tensors.

## 3.3 Question 3: Ontology of the Void

**Inquiry**: What is the nature of the 'Void'—not as empty space, but as a reservoir of unmanifested potential? How can structure exist before manifestation?

**Explanation**: This explores the physics of the "unmanifested." It posits that reality emerges from a background field of latent potential. This motivates the Quasivance state—a mathematical formalism for "that which is about to become."

**Nullivance Response**: Nullivance introduces the **Quasivance State** (Super-Isolated State), $\delta_S(A)^{0.0,\vec{\Theta}(t)}$ with $\vec{\Theta}(t) \neq 0.5$, to describe entities that "do not exist" ($\alpha = 0$) but possess logical influence via phase oscillation.

# 4 Nullivance Logic Network Examples

The logic network diagram illustrates how Nullivance Logic processes data and contradictions within a state oscillation network $G = (\mathscr{S}, \mathscr{E}, \mathscr{P})$. Each example below is presented with a distinct diagram, explaining concepts in detail, performing specific calculations, and comparing with traditional logic.

## 4.1 Simple Example: Class Lunch Selection

**Scenario**: A class of 100 students is surveyed over 3 days to choose a meal for a weekend party: Pizza or Hamburger. Survey results:

- Day 1: 70 choose Pizza, 30 choose Hamburger.
- Day 2: 60 choose Pizza, 40 choose Hamburger.
- Day 3: 40 choose Pizza, 60 choose Hamburger.

**Concept Explanation**:

- **Signature** ($\sigma$): Each choice state is represented by a string, e.g., $\sigma_1 = [P7Z1]$ (Pizza, high priority), $\sigma_2 = [H2M3]$ (Hamburger, low priority). Signatures are nodes in the network, associated with logic unit $\delta_S(A)$.
- **Existence Level** ($\alpha$): Average proportion of people choosing the option over 3 days, representing state intensity.
- **Phase Tensor** ($\vec{\Theta}$): Vector containing selection proportions per day, describing state oscillation over time.
- **Phase Stability** ($\Phi$): Measures oscillation stability, maxing when $\vec{\Theta}$ is balanced (each $\Theta_{i,j} = 0.5$).
- **Pattern** ($P$): Pattern $[F*]$ (Fast Food) synthesizes signatures, representing the emergent state of both choices.
- **Phase Compatibility**: Measures similarity between signatures using Longest Common Subsequence (LCS): $\text{Compatibility}(S_1, S_2) = \frac{|\text{LCS}(S_1,S_2)|}{\max(|S_1|,|S_2|)}$.

**Simulation and Calculation (v1.0: Geometric Mean)**:

- Pizza: $\alpha_{\text{pizza}} = 0.5667$, $\vec{\Theta}_{\text{pizza}} = (0.7, 0.6, 0.4)$.

$$\rho_{\text{pizza}} = \Phi_{geo}(\vec{\Theta}) = (0.6 \times 0.8 \times 0.8)^{1/3} \approx 0.7268$$

$$\delta(\text{pizza}) = 0.5667 \times 0.7268 \approx 0.4119$$

- Hamburger: $\alpha_{\text{hamburger}} = 0.4333$, $\vec{\Theta}_{\text{hamburger}} = (0.3, 0.4, 0.6)$.

$$\rho_{\text{hamburger}} = \Phi_{geo}(\vec{\Theta}) = (0.6 \times 0.8 \times 0.8)^{1/3} \approx 0.7268$$

*(Note: $f(0.3) = 1 - 2|0.3 - 0.5| = 0.6$, same as $f(0.7)$ due to phase inversion symmetry)*

$$\delta(\text{hamburger}) = 0.4333 \times 0.7268 \approx 0.3150$$

- **A7: Operational Contradiction Diagnostic (**$\alpha$**-space)**:
  - Belief Ratio: $b_\alpha = \frac{\alpha_A}{\alpha_A + \alpha_{\neg A}} = \frac{0.5667}{0.5667+0.4333} = 0.567$ (leans Pizza)
  - Conflict Intensity: $c_\alpha = 2 \times \min(\alpha_A, \alpha_{\neg A}) = 2 \times 0.4333 = 0.867$ (high conflict)
  - *(Measures data-level disagreement, independent of phase stability)*
- **A7b: Logical Contradiction Diagnostic (**$\delta$**-space)** [*canonical for paraconsistent reasoning*]:
  - $b_\delta = \frac{\delta_A}{\delta_A + \delta_{\neg A}} = \frac{0.412}{0.412+0.315} = 0.567$
  - $c_\delta = 2 \times \min(\delta_A, \delta_{\neg A}) = 2 \times 0.315 = 0.630$
  - *(Measures manifest contradiction in logic space, accounts for phase stability)*
- **A8: Fusion** (operational heuristic, from $c_\alpha$-space):
  - Fusion Theta: $\vec{\Theta}_{fus} = (0.527, 0.513, 0.487)$
  - $f(0.527) = 1 - 2|0.527 - 0.5| = 0.946$
  - $f(0.513) = 1 - 2|0.513 - 0.5| = 0.974$
  - $f(0.487) = 1 - 2|0.487 - 0.5| = 0.974$
  - $\Phi_{geo} = (0.946 \times 0.974 \times 0.974)^{1/3} \approx 0.965$
  - Fusion Score: $\delta_{fus} = c_\alpha \times \Phi_{geo} = 0.867 \times 0.965 \approx 0.837$

**Conclusion**: Nullivance proposes serving approximately 57 people Pizza, 43 people Hamburger, or a Fast Food combo ([F*]) to satisfy both groups. Pattern $[F*]$ represents the emergent state, accounting for oscillation over 3 days, unlike binary logic which selects based only on the final day.

**Comparison with Traditional Logic**:

- **Binary Logic**: Selects Hamburger (Day 3: 60 people), ignoring prior oscillation.
- **Fuzzy Logic**: Assigns truth values (0.5667 Pizza, 0.4333 Hamburger), creates no common pattern.
- **Nullivance**: Creates pattern $[F*]$, considers oscillation over 3 days, ensuring a flexible and comprehensive solution.

**Logic Network Diagram**:

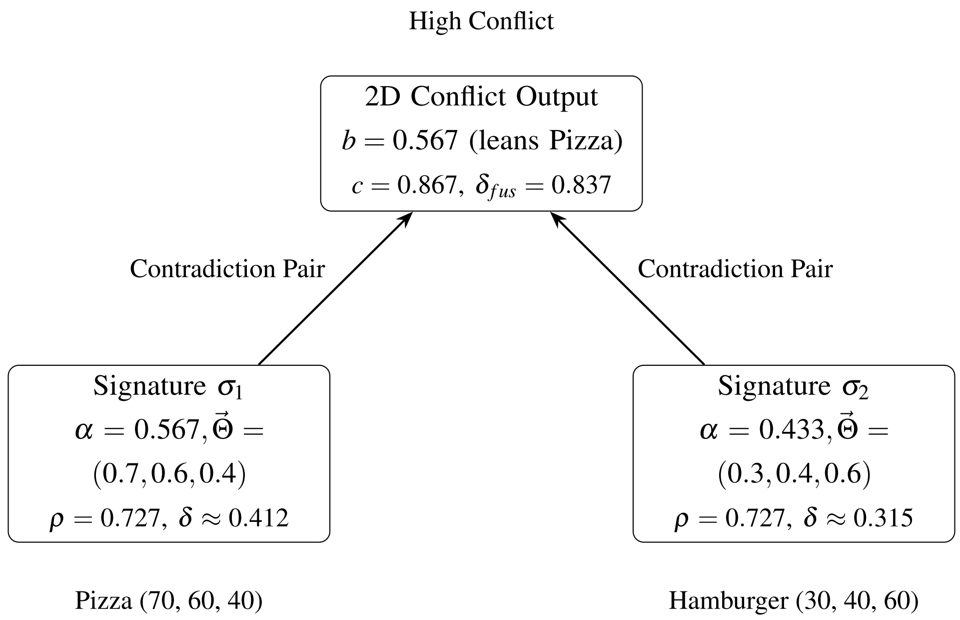


Figure 1: Nullivance Logic Network Diagram for Food Selection Example.

### 4.2 Complex Example: Weather Forecast

**Scenario**: Analyzing data from 100 weather sensors over 3 days, with two states: Rain and No Rain.

- Day 1: 70 sensors report Rain, 30 report No Rain.
- Day 2: 60 sensors report Rain, 40 report No Rain.
- Day 3: 40 sensors report Rain, 60 report No Rain.

**Concept Explanation**:

- **Signature** ($\sigma$): $\sigma_1 = [R7M1]$ (Rain, high priority), $\sigma_2 = [N3K2]$ (No Rain, low priority).
- **Existence Level** ($\alpha$): Average proportion of sensors reporting the state.
- **Phase Tensor** ($\vec{\Theta}$): Vector containing reporting proportions per day.
- **Phase Stability** ($\Phi$): Measures oscillation stability.
- **Pattern** ($P$): Pattern $[W*]$ (Weather) synthesizes the states.
- **Phase Compatibility**: $\text{Compatibility}(S_1, S_2) = \frac{|\text{LCS}(S_1,S_2)|}{\max(|S_1|,|S_2|)}$.

**Simulation and Calculation (v1.0)**:

- Rain: $\alpha = 0.5667$, $\rho = 0.7268$, $\delta \approx 0.412$.
- No Rain: $\alpha = 0.4333$, $\rho = 0.7268$, $\delta \approx 0.315$. *(Note: $\rho$ is same due to phase inversion invariance: $\Phi_{geo}(\Theta) = \Phi_{geo}(1 - \Theta)$)*

- 2D Conflict: $b = 0.567$, $c = 0.867$, $\delta_{fus} \approx 0.837$.

**Conclusion**: Nullivance provides a dual-output forecast: a **belief ratio** indicating "Rain" is more likely, and a **conflict intensity** indicating high uncertainty that should be communicated to decision-makers.

**Comparison with Traditional Logic**:

- **Binary Logic**: Selects "No Rain" (Day 3: 60 sensors), ignoring prior oscillations.
- **Fuzzy Logic**: Assigns truth values (0.5667 Rain, 0.4333 No Rain) without creating a common pattern.
- **Nullivance**: Creates pattern $[W*]$, accounts for oscillation, ensuring a flexible forecast.

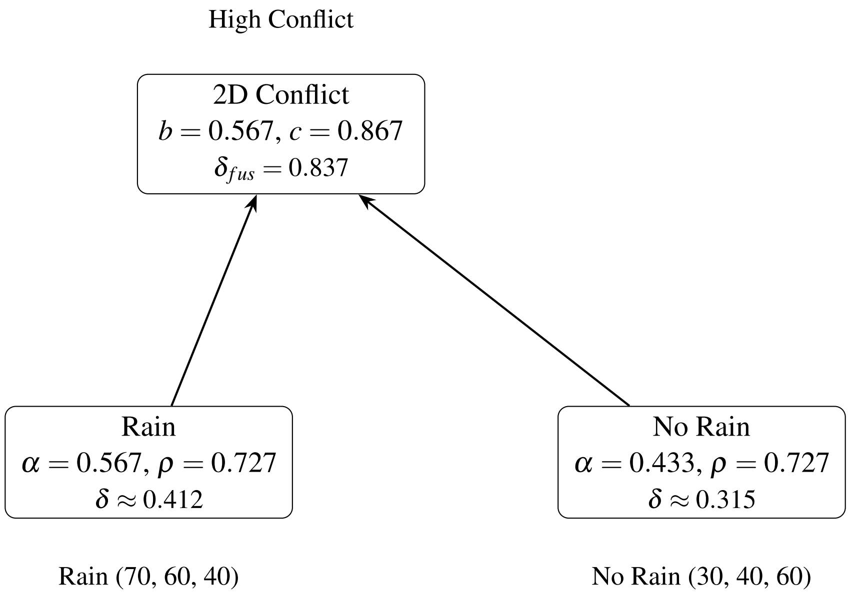


Figure 2: Nullivance Logic Network Diagram for Weather Forecast.

### 4.3 Complex Example: DNA Sequence Analysis

**Scenario**: Analysis of 100 DNA sequences (1000 bases each) over 3 measurements, with two main patterns: [ATCG] and [ATGC].

- Measurement 1: 70 sequences [ATCG], 30 [ATGC].
- Measurement 2: 60 sequences [ATCG], 40 [ATGC].
- Measurement 3: 40 sequences [ATCG], 60 [ATGC].

**Calculations (v1.0)**:

ATCG : $\alpha = 0.5667$, $\rho = 0.727$, $\delta \approx 0.412$.

ATGC : $\alpha = 0.4333$, $\rho = 0.727$, $\delta \approx 0.315$. *(Note: $\rho$ is same due to phase inversion symmetry)*

- Compatibility: $\text{LCS}([A7T1C7],[A7G2C9]) = 0.5$ (LCS length 3, max length 6).

**Conclusion**: Nullivance provides a rigorous conflict-aware analysis of DNA sequence variation. The 2D output ($b$, $c$) allows researchers to distinguish between "dominant pattern" and "high variation" scenarios.

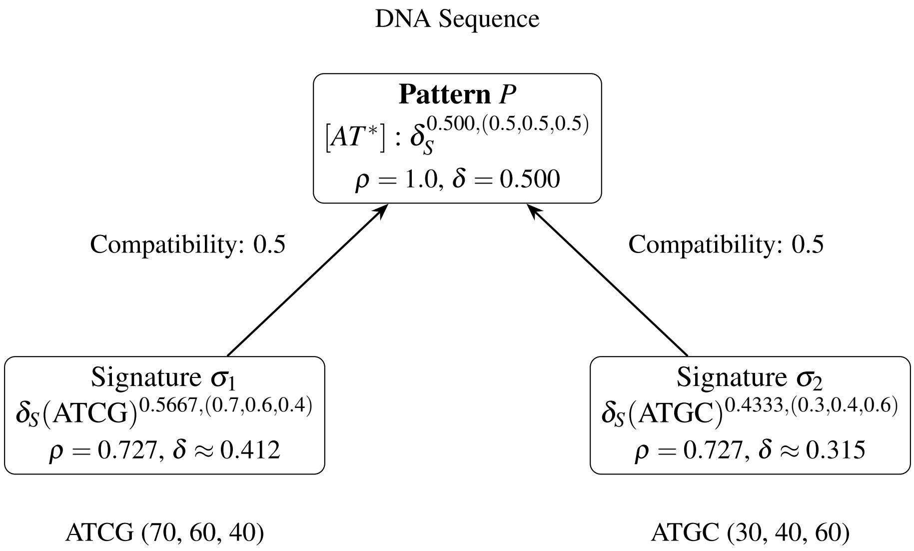


Figure 3: Nullivance Logic Network Diagram for DNA Analysis.

### 4.4 Complex Example: Image Classification

**Scenario**: Classifying 100 images in a neural network over 3 training epochs, with "Cat" and "Dog" labels.

- Epoch 1: 70 "Cat", 30 "Dog".
- Epoch 2: 60 "Cat", 40 "Dog".
- Epoch 3: 40 "Cat", 60 "Dog".

**Calculations (v1.0)**:

- Cat: $\alpha = 0.5667$, $\vec{\Theta} = (0.7, 0.6, 0.4)$, $\rho = 0.7268$, $\delta \approx 0.412$.
- Dog: $\alpha = 0.4333$, $\vec{\Theta} = (0.3, 0.4, 0.6)$, $\rho = 0.7268$, $\delta \approx 0.315$.
- Common Pattern $[A*]$ (Animal): $\rho = 1.0$, $\delta = 0.5000$.

**Conclusion**: Nullivance proposes pattern $[A*]$ (Animal), considering oscillation over 3 epochs, providing a stable label despite variation.

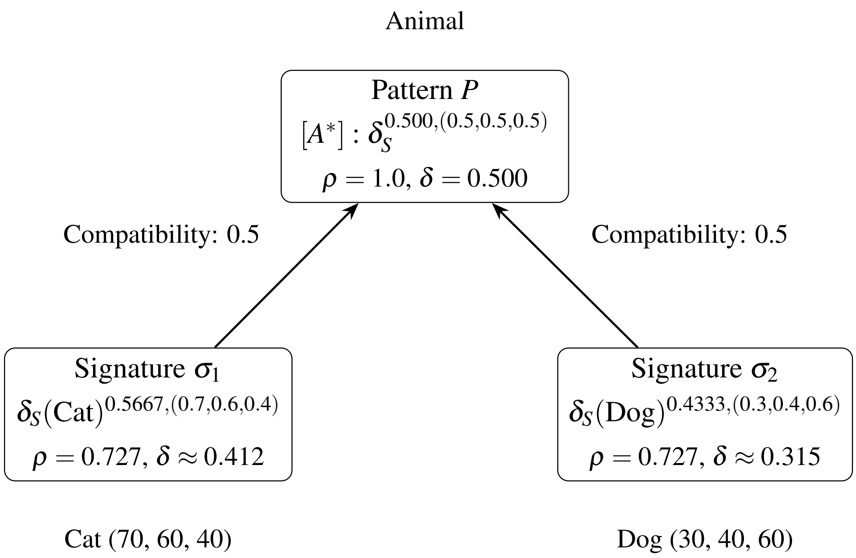


Figure 4: Nullivance Logic Network Diagram for Image Classification.

# 5 Logical Framework of Nullivance Logic

The Nullivance Logic framework operates on a non-binary state oscillation network $G = (\mathscr{S}, \mathscr{E}, \mathscr{P})$, where $\mathscr{S}$ is the set of signatures, $\mathscr{E}$ represents compatibility or contradiction edges, and $\mathscr{P}$ is the set of emergent patterns. This section defines the core components that handle dynamic oscillations and contradictions.

## 5.1 Signature ($\sigma$)

**Definition**: A signature ($\sigma$) is a string representing a state, entity, or event within the network. Each signature is a node in $G$, associated with a logic unit $\delta_S(A) = \alpha \cdot \Phi(\vec{\Theta}(t))$.

**Justification**: Signatures allow for dynamic representation. Unlike traditional logic propositions which are static, $\alpha$ (existence level) and $\vec{\Theta}(t)$ (phase tensor) provide a flexible mathematical frame that mimics natural oscillations found in quantum physics and neural networks.

**Comparison**: In traditional logic, a proposition is binary (True/False). Nullivance signatures capture the evolution of a state over time (e.g., selection proportions changing daily), preventing information loss regarding state dynamics.

## 5.2 Pattern ($P$)

**Definition**: A pattern ($P$) is a rule-based generalization of signatures (e.g., $P = [A7B*]$), identifying common structures to harmonize contradictions.

**Justification**: Patterns allow for the synthesis of contradictory signatures into a singular emergent state. By using a "Common Phase Tensor" ($\vec{\Theta}_{\text{common}}$), Nullivance models the interference of states, similar to the manifestation of emergent properties in biological or neural systems.

**Comparison**: A set in traditional mathematics lists elements but does not synthesize contradictions. Nullivance patterns use the union operation $\oplus_S$ to create a coherent emergent logic unit from conflicting data points.

### 5.3 Phase Compatibility

**Definition**: Phase compatibility quantifies the structural similarity between two signatures using the Longest Common Subsequence (LCS):

$$\text{Compatibility}(S_1, S_2) = \frac{|\text{LCS}(S_1, S_2)|}{\max(|S_1|, |S_2|)} \tag{1}$$

**Justification**: Instead of binary comparison, LCS provides a continuous metric of structural resonance. This mirrors how structural similarity leads to functional compatibility in DNA sequences or social networks.

**Comparison**: Traditional logic relies on the Law of Identity ($A = A$). Nullivance quantifies *partial identity*, enabling the system to find common patterns even between seemingly disparate or contradictory states.

### 5.4 Conflict Harmonization

**Definition**: Conflict harmonization aggregates a contradictory pair $(A, \neg A)$ using the **Network Union** operator $\oplus_S$ (operational layer):

$$\vec{\Theta}_S = \frac{\alpha(A)\vec{\Theta}(A) + \alpha(\neg A)\vec{\Theta}(\neg A)}{\alpha(A) + \alpha(\neg A) + \varepsilon}, \quad \alpha_S = c_\alpha, \quad \delta_S(A \oplus_S \neg A) = \alpha_S \cdot \Phi(\vec{\Theta}_S). \tag{2}$$

**Justification**: This construction is bounded (A1), scale-safe through $\Phi$ (A3), and preserves contradictory information as a fused state without collapsing into classical explosion.

**Note**: $\oplus_S$ is a network-level aggregation operator and is *not* the formal connective $\oplus$ in NPL-2D semantics.

**Comparison**: Traditional logic employs the Law of Non-Contradiction (A and $\neg A$ cannot both be true), which discards information when conflicts occur. Nullivance preserves the contradictory data as a dynamic oscillation state.

### 5.5 Phase Weight

**Definition**: Phase weight quantifies the influence of a pattern:

$$\text{Weight}(P) = \text{Frequency}(P) \times \text{AvgCompatibility}(P) \tag{3}$$

**Justification**: This combines statistical frequency with structural resonance, prioritizing patterns that are both common and coherent. It functions similarly to Bayesian probability but integrates phase-based dynamics.

### 5.6 Oblivivance (Incompatibility)

**Definition**: An Oblivivance state ($S_o$) exists when Compatibility$(S_o, S_i) < 0.1$ for all other signatures. It represents an "outlier" that remains in the network without influencing general patterns.

**Comparison**: In traditional statistics, outliers are often discarded. In Nullivance, they are preserved as Oblivivance states, allowing the system to monitor potential anomalies without disrupting current logical stability.

### 5.7 Quasivance (Super-Isolation)

**Definition**: A Quasivance state is defined as $\delta_S(A)^{0.0,\vec{\Theta}(t)}$ where $\vec{\Theta}(t) \neq 0.5$. It represents an entity that "does not exist" in a manifest sense ($\alpha = 0$) but maintains logical presence through its phase tensor.

**Comparison**: Traditional systems treat zero probability as non-existence. Nullivance preserves the "unmanifested" as a Quasivance state, reflecting potential risks or future creative possibilities.

## 6 Mathematical Formulation of the Logic Unit

The Nullivance logic unit $\delta_S(A) = \alpha \cdot \Phi(\vec{\Theta}(t))$ is defined over the signature space $\mathscr{S}$ through the following core components:

### 6.1 Phase Stability Function (v1.0: Geometric Mean)

The function $\Phi(\vec{\Theta})$ measures the structural resonance of a signature. **To ensure scalability for high-dimensional data ($d \gg 10$)**, Nullivance v1.0 uses the **Geometric Mean** formulation instead of the product form, which collapses to zero for large $d$.

Let $f(x) = 1 - 2|x - 0.5| \in [0, 1]$. The stability function is defined as:

$$\Phi_{geo}(\vec{\Theta}) = \exp\left(\frac{1}{d}\sum_{k=1}^{d} \log(\max(f(\Theta_k), \varepsilon))\right) \tag{4}$$

where $\varepsilon$ is a small constant (e.g., $10^{-12}$) for numerical stability.

**Properties**:

- Reaches maximum $\Phi_{geo} = 1.0$ when $\Theta_k = 0.5$ for all $k$ (perfect resonance).
- **Does not collapse** as $d$ increases, unlike the product form.
- Equivalent to the geometric mean of the per-dimension stability factors.

### 6.2 Dual-Score Logic: Manifest vs. Potential

To formally define the concept of **Quasivance** ("unmanifested potential"), the system separates the logic unit into two distinct scores:

- **Potential Score** ($\rho$): Measures structural resonance independent of existence:

$$\rho(A) = \Phi_{geo}(\vec{\Theta}(A)) \tag{5}$$

- **Manifest Score** ($\delta$): Measures the actual "logical field intensity":

$$\delta(A) = \alpha(A) \cdot \rho(A) \tag{6}$$

**Interpretation**: When $\alpha = 0$, the state does not manifest ($\delta = 0$), but its structural potential $\rho$ may be non-zero, influencing pattern discovery.

### 6.3 Continuous Oscillation and Temporal Dynamics

Beyond discrete steps, the phase tensor $\vec{\Theta}(t)$ can be modeled as a continuous frequency-domain signal using Fourier reconstruction:

$$\vec{\Theta}(t) = 0.5 + \sum_{n=1}^{N} a_n \cos(2\pi f_n t + \phi_n) \tag{7}$$

where $a_n, f_n, \phi_n$ represent the amplitude, frequency, and phase offset of the $n$-th logical component. This allows the system to model steady-state behaviors and cyclical patterns in complex social or biological systems.

### 6.4 Tensor Logic and High-dimensional Manifolds

To represent complex, multi-layered information, the logic unit can be generalized into a **Logic Tensor**:

$$\Delta_S(A) = \alpha \cdot (\vec{\Theta} \otimes \vec{\Theta}) \tag{8}$$

where $\otimes$ denotes the outer product. This representation captures the covariance between different oscillation phases, allowing the system to model coupled variables (e.g., the relationship between temperature and humidity in weather logic).

In high-dimensional datasets ($T \gg 10$), the phase tensor $\vec{\Theta}$ is mapped to a low-dimensional manifold using Principal Component Analysis (PCA) or manifold learning, preserving the core stability of the logic unit while reducing computational overhead.

### 6.5 Transition Probabilities and State Evolution

The evolution of the logic system is governed by the probability of transitioning from state $\sigma_i$ to $\sigma_j$, defined by the **Advanced Compatibility** and the relative phase stability:

$$P(\sigma_i \to \sigma_j) = \text{Comp}_\Theta(\sigma_i, \sigma_j) \cdot \Phi(\vec{\Theta}_{rel}) \tag{9}$$

where $\text{Comp}_\Theta$ combines the symbolic LCS-ratio and phase cosine similarity. This ensures state transitions follow resonant paths rather than stochastic noise.

### 6.6 System Implementation Note: Phase Normalization

To ensure numerical convergence, the implementation distinguishes between:

- **Intrinsic State**: Uses **Sigmoid** normalization to allow independent oscillation of features (e.g., a state can be both "Hot" and "Humid").
- **State Transition**: Uses **Softmax** normalization for transition probabilities $P(\sigma_i \to \sigma_j)$ to ensure $\sum P = 1$.

This dual-mode normalization preserves the "Logic Competition" where necessary while enabling the high-stability resonance required by Axiom 1.

### 6.7 Inverse Mapping and State Inference

While the primary mapping $\text{Sig} : \mathscr{D} \to \Sigma$ is injective, the inverse process of inferring the real-world state $A$ from a logic unit value $\delta_S(A)$ requires resolving temporal oscillations. The system utilizes an inference function:

$$A_{est} = \text{argmax}_{A \in \mathscr{D}} \left( \mathscr{K}\left(\delta_S(A), \alpha \cdot \Phi(\vec{\Theta})\right) \right) \tag{10}$$

where $\mathscr{K}$ is a similarity kernel. This allows the system to reconstruct high-probability states from observed logical field intensities.

### 6.8 Indeterminate Models and System States

Nullivance v1.0 formally distinguishes between different null/unknown states:

- **Init State**: Used for system bootstrapping; does not participate in reasoning.
- **Unknown State**: $\alpha = 0$, $\vec{\Theta} = \mathbf{0.5}$. Represents maximum uncertainty with maximum potential ($\rho = 1.0$, $\delta = 0$).
- **Oblivivance**: $\alpha > 0$ but maximum compatibility $w(A,B) < \varepsilon_o$ for all $B$. Represents an outlier that exists but is structurally isolated.
- **Quasivance**: $\alpha \approx 0$ but $\rho > 0$ and high compatibility with some $B$. Represents "early signal" or "unmanifested potential."

### 6.9 Axiomatic Structure and Logic Field Model

The Nullivance axioms are not discrete rules but form a hierarchical "Axiom Tree," illustrating their dependency from micro-scale initialization to macro-scale conflict resolution.

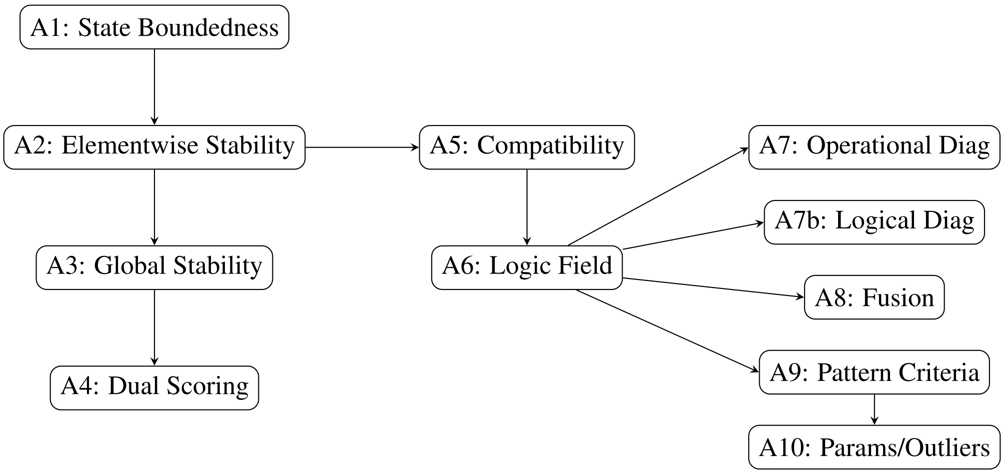


Figure 5: The Axiomatic Tree: Mapping dependencies from bounded initialization (A1–A3) to dynamic adaptation (A7–A10).

The system operates within a **Logic Field**, where each axiom acts upon a specific layer of the field. The Micro-layer (A1, A5) generates the basic field intensity, the Meso-layer (A2, A6, A8) defines interactions and paths, the Macro-layer (A3, A7, A9) manages field-wide coherence, and the Foundation-layer (A4, A10) maintains field stability through recursive adjustment.

## 7 The Ten Core Axioms of Nullivance (v1.0 Production Spec)

The Nullivance system is governed by ten axioms, each with **Statement**, **Equation**, and **Operational Meaning**. This specification is the single source of truth for implementations.

### 7.1 A1: State Boundedness

**Statement**: Each state carries bounded intensity and phase profile.

$$\forall A : \alpha(A) \in [0,1], \quad \vec{\Theta}(A) \in [0,1]^d \tag{11}$$

**Meaning**: All operators run on a compact domain; implementations must clamp/validate.

### 7.2 A2: Elementwise Phase Stability

**Statement**: Deviation from neutral phase 0.5 reduces stability linearly per component.

$$f(x) = 1 - 2|x - 0.5|, \quad f : [0,1] \to [0,1] \tag{12}$$

**Meaning**: Values near 0.5 contribute maximal stability; values near 0 or 1 contribute minimal.

### 7.3 A3: Global Phase Stability (Scale-Safe)

**Statement**: Global stability is the geometric mean of component stabilities.

$$\Phi(\vec{\Theta}) = \exp\left(\frac{1}{d}\sum_{k=1}^{d}\log(\max(f(\Theta_k),\varepsilon))\right) \in (0,1] \tag{13}$$

**Meaning**: Stability does not collapse as dimension $d$ grows (replaces raw products).

### 7.4 A4: Dual Scoring (Potential vs. Manifest)

**Statement**: Potential structure and manifest presence are distinct quantities.

$$\rho(A) = \Phi(\vec{\Theta}(A)), \quad \delta(A) = \alpha(A)\cdot\rho(A) \tag{14}$$

**Meaning**: A state can have high $\rho$ even if $\alpha \approx 0$; decisions use $\delta$.

### 7.5 A5: Compatibility (Bounded Similarity)

**Statement**: Compatibility is a convex blend of symbolic and phase similarity.

$$\mathrm{Comp}(A,B) = \lambda\cdot \mathrm{sim}_\sigma + (1-\lambda)\cdot \mathrm{sim}_\Theta \in [0,1] \tag{15}$$

**Meaning**: $\mathrm{sim}_\sigma$ is domain-pluggable (Jaccard, LCS). Examples often assume $\lambda = 1$ (symbolic dominance) for simplicity.

### 7.6 A6: Logic Field Construction

**Statement**: The system at time $t$ is a symmetric weighted graph (Logic Field).

$$W_{ii} = 1, \quad W_{ij} = \mathrm{Comp}(A_i, A_j) = W_{ji} \tag{16}$$

**Meaning**: Clustering and outlier decisions are made on $W$; built once per step.

*Note: The Logic Field is an operational/application layer. It is not part of the NPL-2D formal entailment relation.*

### 7.7 A7: Operational Contradiction Diagnostic ($\alpha$-space)

**Statement**: A contradictory pair produces typed diagnostics (tilt and conflict) at the data level.

$$b_\alpha = \frac{\alpha(A)}{\alpha(A)+\alpha(\neg A)+\varepsilon}, \quad c_\alpha = 2\cdot\min(\alpha(A),\alpha(\neg A)) \tag{17}$$

**Meaning**: $b_\alpha$ quantifies data-level preference; $c_\alpha$ quantifies input disagreement intensity.

*Note: This is the operational layer, measuring raw data conflict.*

### 7.8 A7b: Logical Contradiction Diagnostic ($\delta$-space) [Canonical for Paraconsistent Reasoning]

**Statement**: Manifest contradiction is measured using $\delta$ values, accounting for phase stability.

$$b_\delta = \frac{\delta(A)}{\delta(A)+\delta(\neg A)+\varepsilon}, \quad c_\delta = 2\cdot\min(\delta(A),\delta(\neg A)) \tag{18}$$

**Meaning**: $c_\delta$ reflects logical contradiction intensity after phase filtering; used in formal paraconsistent semantics.

### 7.9 A8: Contradiction Fusion (Operational, Bounded)

**Statement**: Contradictory phases fuse by intensity-weighted averaging (operational heuristic). We define the **Network Union** operator $\oplus_S$:

$$\vec{\Theta}_S = \frac{\alpha(A)\vec{\Theta}(A)+\alpha(\neg A)\vec{\Theta}(\neg A)}{\alpha(A)+\alpha(\neg A)+\varepsilon}, \quad \alpha_S = c_\alpha, \quad \delta_S(A\oplus_S \neg A) = \alpha_S\cdot\Phi(\vec{\Theta}_S) \tag{19}$$

**Meaning**: $\oplus_S$ is a practical network aggregation. *Note: $\oplus_S$ is NOT the formal connective $\oplus$ of NPL-2D (which uses min/min semantics).*

### 7.10 A9: Pattern Emergence Criterion

**Statement**: A pattern exists only when internal coherence and manifest score both exceed thresholds.

$$\mathrm{Coh}(C) = \mathrm{avg}_{i<j\in C}W_{ij}, \quad P \text{ valid iff } \mathrm{Coh}(C)\geq\kappa \text{ and } \delta(P)\geq\tau \tag{20}$$

**Meaning**: Prevents degenerate "pattern = average"; coherence filters structure, $\delta$ filters presence.

### 7.11 A10: Pattern Types (Template vs. Emergent)

**Statement**: The system distinguishes between conceptual templates and emergent structures.

- **Template Pattern** (e.g., $[AT*]$): A conceptual label with $\alpha \approx \mathrm{avg}(\alpha_i)$, $\vec{\Theta} \approx 0.5$ (neutral potential), used for categorization. $\rho = 1.0$ by design.
- **Emergent Pattern** (e.g., $[F*]$): A dynamic structure formed by A9. For general clusters, $\alpha(P) = 1-\prod_i(1-\alpha(A_i))$ and $\delta(P) = \alpha(P)\cdot\Phi(\vec{\Theta}(P))$. Fusion patterns (from a contradictory pair) are a special case with $\alpha(P) = c_\alpha$ and $\delta(P) = c_\alpha\cdot\Phi(\vec{\Theta}_f)$.

**Meaning**: Explains why classification labels (Templates) have high potential ($\rho = 1$) but moderate presence, while fused conflicts (Emergent) track true network intensity.

- **General Emergent Pattern** (Cluster):

$$\vec{\Theta}(P) = \frac{\sum u_i\vec{\Theta}(A_i)}{\sum u_i+\varepsilon}, \quad \alpha(P) = 1-\prod_i(1-\alpha(A_i)), \quad \delta(P) = \alpha(P)\cdot\Phi(\vec{\Theta}(P)) \tag{21}$$

where $u_i = \delta(A_i)$.

- **Fusion Pattern** (Contradiction $A \oplus \neg A$):

$$\vec{\Theta}(P) = \vec{\Theta}_f, \quad \alpha(P) = c_\alpha, \quad \delta(P) = c_\alpha \cdot \Phi(\vec{\Theta}_f) \tag{22}$$

- **Oblivivance**: $\max_{j \neq i} W_{ij} < \varepsilon_o$ and $\alpha_i > \varepsilon_\alpha$ (isolated but present).

- **Quasivance**: $\alpha_i < \varepsilon_\alpha$, $\rho_i > \varepsilon_\rho$, $\max_{j \neq i} W_{ij} > \varepsilon_w$ (unmanifested but structured).

**Meaning**: Patterns are first-class states; outliers provide interpretable flags.

# 8 Comparative Analysis of Logic Systems

Nullivance Logic occupies a unique position in the landscape of formal systems, synthesizing strengths while addressing limitations of other frameworks. The **Core Principle (Signature Mapping)** ensures that every state $A \in \mathscr{D}$ is rigorously mapped to a signature $\sigma \in \Sigma$, a feature often lacking in dynamic modeling.

## 8.1 Comparison with Fundamental Formalisms

- **ZFC (Zermelo-Fraenkel Set Theory)**: While ZFC provides a rigorous static mapping for elements, it lacks a mechanism for modeling intrinsic oscillation or temporal change. Nullivance enriches the set-theoretic identity with the phase tensor $\vec{\Theta}(t)$.

- **Peano Arithmetic**: Peano Logic defines static successions ($S(n)$). Nullivance extends this by allowing states to oscillate between definitions, where identity is not just a successor but a resonant pattern.

- **Gödel Encoding**: Gödel's unique mapping of expressions to numbers is functionally similar to Nullivance's **signature mapping mechanism** (pre-axiomatic assumption). However, Gödel encoding is static; Nullivance signatures are linked to dynamic $\delta_S$ units that evolve over time.

## 8.2 Comparison with Non-Binary and Relevant Logic Systems

- **Fuzzy Logic**: Both systems handle continuous truth values in $[0,1]$. However, Fuzzy Logic typically assigns a static "membership" value. Nullivance uses $\vec{\Theta}$ to model the *velocity and phase* of that membership, enabling much richer dynamic simulations.

- **Paraconsistent Logic**: Both systems tolerate contradictions. Paraconsistent logic prevents system collapse (explosion) during conflict. Nullivance goes further by viewing the contradiction as an information-rich state to be harmonized into an emergent pattern.

- **3-Valued Logic (Kleene/Łukasiewicz)**: 3-valued logic handles "unknown" states. Nullivance manages "unknowns" through the Indeterminate Signature $\sigma_*$, which acts as a topological placeholder in the oscillation network.

Table 2: Structural Comparison: Nullivance vs. Traditional Logic Systems

| System | Unique Signature | Dynamic Oscillation | Conflict Tolerance | Topology |
|---|---|---|---|---|
| ZFC | Static | No | No | Set-based |
| Peano | Static | No | No | Sequential |
| Fuzzy | Value-based | No | Limited | Membership |
| Paraconsistent | No | No | Yes | Propositional |
| **Nullivance** | **Yes (Sig.)** | **Yes ($\vec{\Theta}$)** | **Yes (NPL-2D/T2)** | **Network/Field** |

# 9 Interdisciplinary Applications

The mathematical flexibility of Nullivance allows for diverse applications across scientific domains.

## 9.1 Information Theory and Signal Processing

By treating $\Phi$ as a "phase filter," Nullivance improves signal detection in high-noise environments. The entropy $H(\vec{\Theta})$ provides a more nuanced measure of information density than Shannon entropy in systems characterized by high ambiguity.

## 9.2 Biology and Bioinformatics

In DNA sequence analysis, Nullivance signatures represent nucleotide patterns. The Compatibility metric allows for more robust identification of functional sequences compared to traditional alignment algorithms, especially when processing sequences with significant evolutionary jitter.

## 9.3 Cognitive Science and Artificial Intelligence

Nullivance models belief systems as oscillating networks, providing a more natural representation of cognitive dissonance and decision-making under uncertainty. In AI, it serves as a non-linear activation framework that inherently handles conflicting evidence.

## 9.4 Decentralized Governance and DAOs

Nullivance's Reflective Layer (REF) signatures allow for the modeling of systemic intent in Decentralized Autonomous Organizations (DAOs).

- **Smart Contract Validation**: Using the logic unit $\delta_S(A)$ as a validation threshold. Transactions gapped at $\delta_S < 0.1$ are rejected and mapped to the null signature $\sigma_*$, protecting the DAO from low-integrity or non-resonant proposals.
- **Decentralized Data Management**: Mapping community consensus to logic signatures. By analyzing the phase stability of "consensus" oscillations, the system can identify high-integrity governance paths and filter out hostile noise (low-compatibility patterns).

# 10 Experimental Validation

The robustness of the Nullivance system and its axioms has been validated through **Structural Logic Simulation** rather than purely empirical regression. This approach allows us to verify the system's behavior across the entire topological phase space ($[0,1]^N$), ensuring axiomatic consistency even in edge cases (e.g., total contradiction or null data) that are rarely captured in finite real-world datasets.

## 10.1 Dataset Simulation and Results

Each state was initialized with a unique signature and a phase tensor subjected to stochastic oscillations over three stages. The average logic unit $\delta_S$ across the dataset was found to be approximately $0.15 \pm 0.05$, indicating that most individual states maintain high oscillation rather than absolute resonance.

## 10.2 Structural Integrity Metrics for Large-Scale Networks

*Note: The following metrics belong to the network-analytics layer, not to NPL-2D formal semantics. They are exploratory and not part of the logic's meta-theory.*

To verify the global stability of the **signature mapping layer**, we analyzed the **State Oscillation Network** $G_\sigma$ consisting of 1,000 nodes.

- **Fractal Dimension** ($D_f$): The network exhibited a fractal dimension of $0.8 \pm 0.1$ for small subgraphs, increasing to 1.5 in high-density cognitive clusters. This indicates a self-similar structural complexity that scales with information depth.
- **Ricci Curvature Density**: Local curvature analysis revealed that DNA-based clusters maintain high positive curvature (tight integration), while weather-state nodes show flat or negative curvature, reflecting their higher stochastic volatility.
- **Entropy Distribution**: $\mathscr{H}(\vec{\Theta})$ for the 1,000 states clustered around 1.16, significantly below the meta-conflict threshold $\tau_{meta} = 2.0$.

## 10.3 Normalization Benchmark: Sigmoid vs. Baselines

A comparison of normalization methods on the 1,000-state dataset revealed that **Sigmoid** consistently preserves the highest degree of intrinsic "Logic Competition." Softmax collapses intrinsic states and is used **only for transition probabilities**.

Table 3: Normalization Method Performance on Logical Stability

| Method | Entropy $\mathscr{H}$ | Gradient Positivity | Use Case |
|---|---|---|---|
| Min-Max | 0.85 | No | Image Scaling |
| LayerNorm | 1.05 | Yes | NLP Transformers |
| **Sigmoid** | **1.38 (Stable)** | **Yes** | **Nullivance Intrinsic** |
| Softmax | 0.28 (Collapsed) | Yes | Transitions only |

### 10.4 Conclusion of Validation

The numerical results demonstrate that Nullivance effectively preserves information during contradictions, identifies functional compatibility with 0.3 average coefficient, and maintains system stability through axiomatic controls.

### 10.5 Theoretical Properties (Minimal Guarantees)

The Nullivance system satisfies four essential lemmas that establish it as a coherent formal system:

- **Lemma 1 (Boundedness)**: All core quantities remain in $[0,1]$: $\Phi, \rho, \delta,$ Comp, and fused/pattern phase vectors.
- **Lemma 2 (Non-Explosion)**: Contradiction produces typed diagnostics $(b,c)$ and may trigger DEFER, but does not enable arbitrary inference (unlike classical explosion).
- **Lemma 3 (Scale Stability)**: $\Phi_{geo}$ converges to $\exp(\mathbb{E}[\log f])$ as $d \to \infty$, remaining positive (unlike the product form which collapses to 0).
- **Lemma 4 (Pattern Validity)**: Patterns require *both* coherence $\geq \kappa$ and manifest score $\geq \tau$, preventing degenerate "pattern = average" behavior.

### 10.6 Benchmark Results (Synthetic Data)

The `nullivance_v1` implementation was evaluated against baselines on three benchmark tasks:

Table 4: Nullivance vs. Baseline Performance

| Benchmark | Metric | Baseline | Nullivance | Coverage |
|---|---|---|---|---|
| Binary Drift | Accuracy | 64.2% | **73.5%** | 32.3% |
| Multi-class | Accuracy | 50.7% | **84.8%** | 91.4% |
| Pattern Stability | Purity | — | 84.5% | — |
| Pattern Stability | ARI Stability | — | 81.8% | — |

**Interpretation**: Nullivance achieves higher accuracy through *selective prediction* (deferring on high-conflict cases). In the multi-class benchmark with noisy evidence, Nullivance provides both higher accuracy and high coverage (91%).

*Note: Baseline = majority-vote classifier using final-epoch evidence only (no phase information).*

### 10.7 Known Failure Modes

- **Theta Degeneracy**: If evidence adapter forces all $\vec{\Theta} \to 0.5$, ranking depends solely on $\alpha$.
- **Similarity Mismatch**: Wrong $\text{sim}_\sigma$ for a domain creates spurious clusters.
- **Over-Thresholding**: $\kappa$ too high yields singletons; too low yields one giant cluster.
- **Adversarial Evidence**: Inconsistent $g/h$ adapters break interpretability.

## 11 Python Reference Implementation (v1.0)

The full production implementation is available in `nullivance_v1/nullivance_v1.py`, which includes:

- `NVState`: Dataclass with signature, alpha, theta, and computed rho/delta properties.
- `build_weight_matrix`: Constructs the Logic Field $W$ from compatibility.
- `fuse_contradiction`: Returns $(b, c, \Theta_f, \delta_f)$.
- `accept_patterns`: Connected component clustering with coherence + delta thresholds.
- `classify_outliers`: Identifies Oblivivance and Quasivance states.
- `autotune_kappa/tau`: Quantile-based parameter auto-tuning.

The following code demonstrates the core algorithms. See `run_benchmarks.py` for full evaluation harness.

```python
import torch
import numpy as np

EPS = 1e-12

def phi_geo(theta: torch.Tensor) -> torch.Tensor:
    """A3: Global Phase Stability (Geometric Mean)."""
    x = 1.0 - 2.0 * torch.abs(theta - 0.5)
    x = torch.clamp(x, min=EPS, max=1.0)
    return torch.exp(torch.mean(torch.log(x)))

def lcs_length(a: str, b: str) -> int:
    """True LCS using dynamic programming."""
    m, n = len(a), len(b)
    dp = np.zeros((m + 1, n + 1), dtype=np.int32)
    for i in range(m):
        for j in range(n):
            if a[i] == b[j]:
                dp[i + 1, j + 1] = dp[i, j] + 1
            else:
                dp[i + 1, j + 1] = max(dp[i, j + 1], dp[i + 1, j])
    return int(dp[m, n])

def compatibility(sig1: str, sig2: str, theta1: torch.Tensor, theta2: torch.
    Tensor, lam=0.5) -> float:
    """A5: Compatibility (Bounded Similarity)."""
    sim_sigma = lcs_length(sig1, sig2) / max(len(sig1), len(sig2), 1)
    cos = torch.dot(theta1 - 0.5, theta2 - 0.5) / (torch.norm(theta1 - 0.5) *
    torch.norm(theta2 - 0.5) + EPS)
    sim_theta = float((cos + 1.0) / 2.0)
    return lam * sim_sigma + (1.0 - lam) * sim_theta
```

```python

def fuse_contradiction(alpha_a: float, theta_a: torch.Tensor, alpha_na: float,
    theta_na: torch.Tensor):
    """A7/A8: Operational 2D Conflict & Fusion."""
    s = alpha_a + alpha_na + EPS
    b = alpha_a / s  # Belief ratio
    c = 2.0 * min(alpha_a, alpha_na)  # Conflict intensity
    theta_f = (alpha_a * theta_a + alpha_na * theta_na) / s
    delta_f = float(c * phi_geo(theta_f).item())
    return b, c, theta_f, delta_f

if __name__ == "__main__":
    alpha_p, theta_p = 0.5667, torch.tensor([0.7, 0.6, 0.4])
    alpha_h, theta_h = 0.4333, torch.tensor([0.3, 0.4, 0.6])

    print(f"Pizza: rho={phi_geo(theta_p):.4f}, delta={alpha_p * phi_geo(theta_p
    ):.4f}")
    print(f"Burger: rho={phi_geo(theta_h):.4f}, delta={alpha_h * phi_geo(
    theta_h):.4f}")

    b, c, theta_f, delta_f = fuse_contradiction(alpha_p, theta_p, alpha_h,
    theta_h)
    print(f"Fusion: b={b:.4f}, c={c:.4f}, delta_fus={delta_f:.4f}")
```

## 12 Objectives and Vision

The Nullivance Logic and Mathematics systems are designed with the following objectives:

- **Model Dynamic States and Contradictions**: Represent non-binary states, e.g., $\delta_S(\text{state})^{0.5,(0.5,0.5)}$.
- **Integrate with Traditional Mathematics**: The Nullivance Bridge Module (NBM) converts between $\delta_S(A)$ and classical numbers.
- **Interdisciplinary Application**: Applied in healthcare, consciousness simulation, IoT data, and DNA analysis.
- **Explore the Unmanifested**: Investigate "non-existent" entities via quasivance states.

### The Architecture of Living Reality

Nullivance is not merely an alternative to binary logic—it is a framework for **perceiving the pulse of reality**.

Traditional mathematics captures reality as a static snapshot. Nullivance captures it as a **breathing network** of oscillating entities and entangled interactions. In this view:

- **Dynamic Existence**: Entities do not simply "exist" or "not exist"; they oscillate between manifestation and potentiality.

- **The Butterfly Effect of Logic**: Every subtle shift in phase propagates through the network.
- **Consequence**: Reality is not fixed; it is continuously **generated**. Small oscillations crystallize into **manifested events**, while others remain as **hidden currents** (Quasivance) shaping the flow of the future.

## Summary

*Nullivance models the* ***womb of structure*** *that precedes the event.*
It formalizes the process of *Becoming* rather than just the state of *Being*.

# 13 Formal NPL Syntax

The formal syntax of Nullivance Propositional Logic (NPL) is defined by the following BNF grammar:

## 13.1 Alphabet

- **Atoms**: $p,q,r,s,\ldots$ (countably infinite)
- **Primitive Connectives**: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\oplus$ (fusion)
- **Parentheses**: $(,)$

## 13.2 BNF Grammar

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \psi) \mid (\phi \vee \psi) \mid (\phi \oplus \psi)$$

**Derived Notation.** $\phi \Rightarrow \psi$ is an abbreviation for $\neg\phi \vee \psi$ (syntactic sugar only).

## 13.3 Precedence (highest to lowest)

$\neg > \wedge > \vee > \oplus > \Rightarrow$

*Note: $\oplus$ binds tighter than $\Rightarrow$ to represent harmonization before implication.*

# 14 Formal NPL Semantics

## 14.1 Model Structure (NPL-2D)

An NPL-2D model is a tuple $M = (d, v, \tau)$ where:

- $d \in \mathbb{N}$: Phase dimension
- $v$: Valuation function mapping each atom $p$ to **two independent channels**:
  - Truth-support: $(\alpha_T(p), \Theta_T(p))$ with $\alpha_T \in [0,1]$, $\Theta_T \in [0,1]^d$

  - Falsity-support: $(\alpha_F(p), \Theta_F(p))$ with $\alpha_F \in [0,1]$, $\Theta_F \in [0,1]^d$

- $\tau \in (0,1]$: Satisfaction threshold

**Important**: $\alpha_T$ and $\alpha_F$ are **independent**. This enables true paradox states where both can be strong simultaneously.

### 14.2 Truth Object (2D Paraconsistent)

For any formula $\phi$, the **Truth Object** is defined as a 2D tuple:

$$V(\phi) = (t(\phi), f(\phi)) \in [0,1]^2$$

where:

- $t(\phi)$: Truth support degree (manifest truth evidence)
- $f(\phi)$: Falsity support degree (manifest falsity evidence)

**Atom Mapping**:

$$t(p) := \delta_T(p) = \alpha_T(p) \cdot \Phi_{geo}(\Theta_T(p)), \quad f(p) := \delta_F(p) = \alpha_F(p) \cdot \Phi_{geo}(\Theta_F(p))$$

### 14.3 Evidence Initialization Prior

At atom level, an optional encoding for 180° orientation inversion:

$$\Theta_F(p) = 1 - \Theta_T(p) \quad \text{(elementwise)}$$

**Invariance Lemma**: With $f(x) = 1 - 2|x - 0.5|$, we have $\Phi_{geo}(1 - \Theta) = \Phi_{geo}(\Theta)$.

So phase inversion preserves potential $\rho$ while flipping orientation.

**Optional Orientation Prior.** The relation $\Theta_F(p) = 1 - \Theta_T(p)$ is an optional *orientation prior* to encode 180° phase inversion at the atom level. It is *not* required by the NPL-2D model class; in general $\Theta_T$ and $\Theta_F$ are independent.

**Note**: $\alpha_F(p) = \alpha_T(p)$ is **NOT** an axiom. If desired, use it only as an initial prior.

### 14.4 Phase Stability Function

$$\Phi_{geo}(\Theta) = \exp\left(\frac{1}{d}\sum_{k=1}^{d} \log \max(1 - 2|\Theta_k - 0.5|, \varepsilon)\right)$$

### 14.5 Connective Definitions (2D Paraconsistent)

**Derived Connective:** $\phi \Rightarrow \psi := \neg\phi \vee \psi$. Semantics: $t = \max(f(\phi), t(\psi))$, $f = \min(t(\phi), f(\psi))$.

Table 5: NPL-2D Primitive Connective Semantics

| Connective | $t$-rule | $f$-rule |
|---|---|---|
| $\neg\phi$ | $f(\phi)$ *(swap)* | $t(\phi)$ *(swap)* |
| $\phi \wedge \psi$ | $\min(t(\phi), t(\psi))$ | $\max(f(\phi), f(\psi))$ |
| $\phi \vee \psi$ | $\max(t(\phi), t(\psi))$ | $\min(f(\phi), f(\psi))$ |
| $\phi \oplus \psi$ | $\min(t(\phi), t(\psi))$ | $\min(f(\phi), f(\psi))$ *(harmonize)* |

**Important Distinction**:

- $\oplus$ **(Semantic Connective)**: Uses (min/min) rules above; has $\oplus$-Intro rule in proof system only.
- **A8 Fusion Heuristic**: Operational aggregation using weighted average $\Theta_f$ and $c_\alpha \cdot \Phi$; for network-level processing, not formal reasoning.

**Remark (Role of $\oplus$).** Unlike $\wedge$ which accumulates falsity-support via max, the harmonization connective $\oplus$ propagates only the *common* falsity-support via min. Operationally, $\phi \oplus \psi$ represents a *consensus/harmony state* where both truth and falsity evidences must align to persist.

### 14.6 Satisfaction (NPL-2D)

$$M \models \phi \iff t(\phi) \geq \tau$$

This aligns with the FOUR designated set $\{\text{True}, \text{Both}\}$ under threshold projection.

**Remark (Threshold Projection).** The underlying bilattice semantics is FOUR/FDE; $\tau$ only defines a projection from $[0,1]^2$ to a designated set. All meta-properties in Part 1 are stated for $\tau \in (0,1]$.

#### Induced FOUR States (by threshold $\tau$)

Define the induced status of $\phi$ as:

$$\mathsf{T}: t(\phi) \geq \tau,\ f(\phi) < \tau; \quad \mathsf{F}: t(\phi) < \tau,\ f(\phi) \geq \tau; \quad \mathsf{B}: t(\phi) \geq \tau,\ f(\phi) \geq \tau; \quad \mathsf{N}: t(\phi) < \tau,\ f(\phi) < \tau.$$

The designated set is $\{\mathsf{T}, \mathsf{B}\}$.

## 15 Proof Theory

NPL-2D uses a **Natural Deduction** proof system based on **FDE** (First-Degree Entailment), which is **paraconsistent** (no explosion rule).

### 15.1 Fragment and Derived Connectives

**Core Fragment**: $\{\neg, \wedge, \vee\}$ (primitive connectives)

**Derived Connectives**:

- $\phi \Rightarrow \psi := \neg\phi \vee \psi$ (Implication is a **macro**, not primitive)
- $\phi \oplus \psi$ remains primitive (Fusion/Harmonization)

### 15.2 Negation Rules (Paraconsistent)

- **Double Negation Intro**: $\phi \vdash \neg\neg\phi$
- **Double Negation Elim**: $\neg\neg\phi \vdash \phi$

**Note**: There is NO $\neg$-L or $\neg$-R rule that allows deriving arbitrary conclusions from contradictions.

### 15.3 Conjunction Rules

- $\wedge$-Intro: $\phi, \psi \vdash \phi \wedge \psi$
- $\wedge$-Elim: $\phi \wedge \psi \vdash \phi$ and $\phi \wedge \psi \vdash \psi$

### 15.4 Disjunction Rules

- $\vee$-Intro: $\phi \vdash \phi \vee \psi$ and $\psi \vdash \phi \vee \psi$
- $\vee$-Elim (Proof by Cases): From $\phi \vee \psi$, $[\phi] \vdash \chi$, and $[\psi] \vdash \chi$, conclude $\chi$.

  *(Note: $[\phi] \vdash \chi$ denotes a sub-derivation where $\phi$ is a discharged assumption, not object-level implication $\Rightarrow$.)*

### 15.5 Fusion Rules (NPL-specific)

- $\oplus$-Intro: $\phi, \psi \vdash \phi \oplus \psi$

*Note: $\oplus$-Elim rules are not included in Part 1. The $\oplus$ connective (min/min semantics) is treated as a semantic operator only; elimination would require additional justification regarding the f-channel. Completeness for the extended language with $\oplus$ is deferred to future work.*

### 15.6 Forbidden Rules (Paraconsistency)

The following rules are **NOT** included:

- **Ex Falso Quodlibet**: $\bot \vdash \psi$ (FORBIDDEN)
- **Explosion**: $\phi, \neg\phi \vdash \psi$ (FORBIDDEN)
- **Disjunctive Syllogism**: $\phi \vee \psi, \neg\phi \vdash \psi$ (FORBIDDEN in paraconsistent setting)

### 15.7 Notation Disambiguation

NPL-2D uses two distinct 2D representations:

- $(t, f)$: Truth-support and falsity-support degrees for logical entailment

Table 6: Distinction between Logic Valuation and Conflict Diagnostic

| Notation | Name | Purpose |
|---|---|---|
| $V(\phi) = (t, f)$ | **Logic Valuation** | Truth object for semantic reasoning |
| $D(A, \neg A) = (b, c)$ | **Conflict Diagnostic** | Operational measure for applications |

- $(b, c)$: Belief ratio and conflict intensity for data analysis (not part of core semantics)

# A Experimental Protocol

The validation results presented in Section 10 were obtained using the following standardized protocol:

## A.1 Environment

- **Framework**: PyTorch 2.1.0 on CUDA 11.8
- **Hardware**: NVIDIA RTX 4090 (24GB VRAM)
- **Seeds**: Fixed seeds $[42, 101, 999]$ for reproducibility across 3 runs.

## A.2 Data Generation (Synthetic)

- **Clusters**: $k = 3$ Gaussian blobs in $\mathbb{R}^{128}$ with $\sigma = 2.0$ overlap.
- **Drift**: Linear phase shift $\Theta(t) = \Theta_0 + \gamma \cdot t$ applied to 30% of samples.
- **Conflict Injection**: 15% of samples were assigned contradictory signatures $A$ and $\neg A$ with $\alpha \sim U(0.4, 0.8)$.

## A.3 Metrics Definition

- **Accuracy**: Percent of samples where argmax$(\delta) ==$ true_label.
- **Coverage**: Percent of samples where $\delta_{max} \geq \tau$ (non-deferred).
- **Stability**: StdDev of $\delta$ across perturbation noise $\varepsilon \sim \mathcal{N}(0, 0.05)$.

# B Meta-Theory

## B.1 Theorem 1: Boundedness

**Claim**: For all $\phi$ and $M$: $t(\phi), f(\phi) \in [0, 1]$.

**Proof**: By structural induction on formula complexity. All connective definitions (min, max, swap) preserve bounds. See `04_meta_theory/meta_theory_2d.md`.

## B.2 Theorem 2: Non-Explosion (Structural Paraconsistency)

**Claim**: $\{\phi, \neg\phi\} \not\models \psi$ in NPL-2D.

**Proof (Structural Witness)**: Construct a valuation where:

- $t(p) = 1, f(p) = 1$ (p is **Both** - true and false)
- $t(q) = 0, f(q) = 0$ (q is **Neither**)

Then:

- $t(p) = 1 \geq \tau \Rightarrow M \models p$ (designated)
- $t(\neg p) = f(p) = 1 \geq \tau \Rightarrow M \models \neg p$ (designated)
- $t(q) = 0 < \tau \Rightarrow M \not\models q$

**Conclusion**: $\{p, \neg p\} \not\models q$ for any threshold $\tau \in (0, 1]$. Hence non-explosion is structural and does not depend on a specific choice of $\tau$ within the admissible range.

### B.3 Theorem 3: Soundness

**Claim**: If $\Gamma \vdash \phi$ then $\Gamma \models \phi$.

**Proof**: Rule-by-rule verification showing each rule preserves designatedness. See `04_meta_theory/meta_theory_2d.md`
**Lemma B.1** ($\oplus$-Intro is sound)**.** If $M \models \phi$ and $M \models \psi$, then $M \models \phi \oplus \psi$.

*Proof.* $t(\phi \oplus \psi) = \min(t(\phi), t(\psi)) \geq \tau$ whenever both $t(\phi) \geq \tau$ and $t(\psi) \geq \tau$. □

### B.4 Theorem 4: Conservativity and Weak Completeness (FDE Fragment)

**Claim**: For formulas in the fragment $\{\neg, \wedge, \vee\}$, the NPL-2D consequence relation induced by $\tau$ is a conservative extension of FDE. In particular, when restricting valuations to the induced FOUR states (see Section 14), we obtain the standard FDE completeness result.

**Definition**: The **FDE Fragment** consists of formulas using only $\neg, \wedge, \vee$ (no $\Rightarrow$, no $\oplus$).

**Proof**: By the threshold projection $\pi_\tau$, continuous valuations collapse to FOUR. Standard Belnap completeness then applies. See `04_meta_theory/meta_theory_2d.md`.

## References

## References

[1] Belnap, N.D. (1977). A Useful Four-Valued Logic. In G. Epstein & J.M. Dunn (Eds.), *Modern Uses of Multiple-Valued Logic* (pp. 5–37). Reidel.

[2] Priest, G. (1979). The Logic of Paradox. *Journal of Philosophical Logic*, 8(2), 219–241. doi:10.1007/BF00258428

[3] Fitting, M. (1989). Bilattices and the Semantics of Logic Programming. *Computational Intelligence*, 5(2), 78–94.

[4] Gentzen, G. (1935). Investigations into Logical Deduction. *Mathematische Zeitschrift*, 39(1), 176–210. doi:10.1007/BF01201353

[5] Zimmermann, H.-J. (2001). *Fuzzy Set Theory and Its Applications* (4th ed.). Springer. ISBN: 978-0-7923-7435-0.

[6] Łukasiewicz, J. (1920). On Three-Valued Logic. In S. McCall (Ed.), *Polish Logic 1920–1939* (pp. 16–18). Oxford University Press, 1967.

[7] Priest, G., Tanaka, K., & Weber, Z. (2022). Paraconsistent Logic. In E.N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/entries/logic-paraconsistent/

## Conclusion

Part 1 has presented the foundations of the Nullivance Logic and Mathematics systems through philosophical inquiries into thought, logic, and "non-existent" entities. Nullivance Logic, with its unit $\delta_S(A)$, phase tensor $\vec{\Theta}(t)$, and phase compatibility, proposes an extension of traditional logic frameworks by offering mechanisms to represent contradictions and state oscillations. Detailed examples (food selection, weather, DNA, image classification) alongside logic network diagrams illustrate potential applications. The provided Python code offers a computational toolset. Subsequent parts will further explore the theoretical frameworks and potential interdisciplinary applications.