UDC 530.182 : 517.938

Polyukov P.A.
Moscow, Russian Federation

THEOREM OF INEVITABLE DEATH: DISSIPATIVE CYCLE, REINVESTMENT AND SURVIVAL CRITERION FOR COMPLEX SYSTEMS

Abstract

A mathematical isomorphism between a stick-slip oscillator and a class of dissipative systems with reinvestment is established. Equations for the dynamics of resource, variability, and consolidation are obtained. The theorem of inevitable death, survival criterion, irreversibility threshold, necessity of phase switching, and the cascade of evolutionary explosions are proved. A quantum of metabolic power is introduced. An isomorphism between the spectral power density and Planck's formula is established.

Keywords: dissipative systems, reinvestment, theorem of death, power quantum, evolutionary explosion.

1 Introduction

1.1 Subject of Research

This work is devoted to the construction and analysis of a mathematical model for a universal class of dissipative systems capable of maintaining their structure through reinvestment of energy dissipated during cyclic functioning.

By a dissipative system in this context, we mean an open system whose stable state is ensured by energy outflow to the external environment. Of particular interest are systems in which the dissipated energy is not irretrievably lost but is partially utilized to compensate for internal degradation of state parameters.

1.2 Motivation

The research is motivated by the presence of common structural and dynamic patterns in a wide range of phenomena of different nature:

- biophysical systems (maintenance of transmembrane potential);
- evolutionary processes (utilization of biomass growth);
- economic agents (profit reinvestment);
- historical macrodynamics (innovation flows);
- artificial cognitive architectures (computational resource reinvestment).

All these classes of systems demonstrate an invariant pattern:

1. cyclic regime "accumulation — threshold reset — dissipation";
2. three reinvestment channels: resource, variability, consolidation;
3. degradation under fixed strategy;
4. necessity of negative feedback.

1.3 Goal and Method

The goal of this work is to identify the mathematical invariant of this pattern and prove theorems valid for any class of systems satisfying the initial assumptions.

The stick-slip oscillator with dry friction was chosen as the basic dynamic model. This choice is due to the availability of a complete analytical solution and a minimal set of parameters allowing universal interpretation.

2 Preliminary Provisions and Problem Statement

Consider a mass m connected by a spring of stiffness k to a drive moving at constant velocity v_0. The mass is pressed against a horizontal surface by a normal force $F_n = mg$. Dry friction acts between the mass and the surface: static friction force $F_s = \mu_s mg$, kinetic friction force $F_k = \mu_k mg$, with $\mu_s > \mu_k$.

Equation of motion:

$$m\ddot{x} = k(v_0 t - x) - F_{\text{tr}}(\dot{x}) \tag{1}$$

where

$$F_{\text{tr}} = \begin{cases} F_s, & \dot{x} = 0,\ |k(v_0 t - x)| \leq F_s; \\ F_k \operatorname{sgn}(k(v_0 t - x)), & \dot{x} = 0,\ |k(v_0 t - x)| > F_s; \\ F_k \operatorname{sgn}(\dot{x}), & \dot{x} \neq 0 \end{cases} \tag{2}$$

In the steady-state regime, the system performs stick-slip self-oscillations. Stick phase time:

$$T_{\text{stick}} = \frac{\mu_s mg}{k v_0} \tag{3}$$

Slip phase time:

$$T_{\text{slip}} = \pi \sqrt{\frac{m}{k}} \tag{4}$$

Total cycle period:

$$T = \frac{(\mu_s - \mu_k) mg}{k v_0} + \pi \sqrt{\frac{m}{k}} \tag{5}$$

Energy dissipated per cycle:

$$E = \frac{2\mu_k(\mu_s - \mu_k) m^2 g^2}{k} + \mu_k m g v_0 \pi \sqrt{\frac{m}{k}} \tag{6}$$

Average dissipation power:

$$P = \frac{E}{T} = \frac{\frac{2\mu_k(\mu_s - \mu_k) m^2 g^2}{k} + \mu_k m g v_0 \pi \sqrt{\frac{m}{k}}}{\frac{(\mu_s - \mu_k) mg}{k v_0} + \pi \sqrt{\frac{m}{k}}} \tag{7}$$

In the large resource regime ($m \to \infty$):

$$P = 2\mu_k g v_0 m \left[1 - \frac{\pi v_0}{(\mu_s - \mu_k) g} \sqrt{\frac{k}{m}} + O\left(\frac{1}{m}\right) \right] \tag{8}$$

3 Universal Model of Dissipative Metabolism

We introduce three fundamental state parameters:

- m — accumulated resource;
- g — variability, a measure of diversity of available states;

- k — consolidation, a measure of bond stiffness.

External parameters: v_0 — rate of external inflow; μ_s — destruction threshold; μ_k — dissipation intensity in operating mode.

Definition 3.1. The dissipation power P, defined by formula (7), is the *quantum of metabolic power* of the system. It represents an atomic portion of the negentropy flow extracted by the system from the external flow per unit time per cycle.

The energy dissipated per cycle $E = PT$ is distributed through three channels:

- $\alpha_m P$ — investments in resource accumulation;
- $\alpha_g P$ — investments in maintaining and increasing variability;
- $\alpha_k P$ — investments in maintaining and increasing consolidation.

Normalization condition:

$$\alpha_m + \alpha_g + \alpha_k = \alpha_{\text{total}} \in (0, 1) \tag{9}$$

α_{total} is the reinvestment efficiency.

The system has internal degradation: each parameter decays at rates $\beta_m, \beta_g, \beta_k > 0$. Parameter dynamics:

$$\begin{cases} \dot{m} = \alpha_m \cdot P(m, g, k) - \beta_m m \\ \dot{g} = \alpha_g \cdot P(m, g, k) - \beta_g g \\ \dot{k} = \alpha_k \cdot P(m, g, k) - \beta_k k \end{cases} \tag{10}$$

In the large resource regime $P \approx 2\mu_k g v_0 m$:

$$\begin{cases} \dot{m} = (2\alpha_m \mu_k v_0 g - \beta_m) m \\ \dot{g} = (2\alpha_g \mu_k v_0 g - \beta_g) g \\ \dot{k} = 2\alpha_k \mu_k g v_0 m - \beta_k k \end{cases} \tag{11}$$

4 Theorem of Inevitable Death

Theorem 4.1. *Any system described by system (10) with fixed (state-independent) coefficients $\alpha_m, \alpha_g, \alpha_k$ irreversibly degrades and dies for any initial conditions, except for a set of measure zero.*

Proof. Consider four principal scenarios.

Scenario 1 (variability degradation). If $\alpha_g < \beta_g / (2\mu_k v_0 g)$, then from the second equation of system (11) $\dot{g} < 0$, $g(t)$ monotonically decreases. As $g \to 0$, power $P \to 0$, the system reduces to $\dot{m} = -\beta_m m$, $\dot{k} = -\beta_k k$ and dies in finite time.

Scenario 2 (excessive variability growth). Let α_g be fixed and exceed the critical value

$$\alpha_g > \frac{\beta_g}{2\mu_k v_0 g(0)}.$$

Then from the second equation of system (11)

$$\dot{g} = (2\alpha_g \mu_k v_0 g - \beta_g)g$$

it follows that $\dot{g} > 0$ for all t, and the parameter $g(t)$ grows without bound.

Consider the first equation of the system:

$$\dot{m} = (2\alpha_m \mu_k v_0 g - \beta_m)m.$$

Since $g(t) \to \infty$, there exists a time t_0 starting from which

$$g(t) > \frac{\beta_m}{2\alpha_m \mu_k v_0}.$$

For $t > t_0$, we have $\dot{m} > 0$, meaning the resource $m(t)$ also begins to grow without bound.

At first glance, the system appears to demonstrate sustainable growth. However, such a trajectory cannot be realized indefinitely within the original physical assumptions of the model:

- the dissipation power $P \sim 2\mu_k v_0 g m$ tends to infinity;
- the external flow v_0 is finite, which sooner or later leads to the exhaustion of available resources;
- as $g \to \infty$, variability transitions into chaos, and the system loses its ability to efficiently utilize energy;
- in realistic formulations, μ_s and μ_k may depend on g, and as $g \to \infty$, the destruction threshold $\mu_s(g)$ tends to zero, which implies the system's disintegration.

Thus, unbounded growth of g and m under a fixed reinvestment strategy inevitably leads either to the exhaustion of external resources or to the destruction of the system's structure, i.e., to death.

Scenario 3 (consolidation degradation). When $\alpha_k = 0$ or α_k is small, parameter k decreases. As $k \to 0$, period $T \to \infty$, power $P \to 0$.

Scenario 4 (excessive consolidation). With unbounded growth of k, the slip phase $T_{\text{slip}} \to 0$, the system freezes, $P \to 0$.

With fixed α, there is no mechanism preventing m, g, or k from vanishing or g or k from growing without bound, which also leads to $P \to 0$. □

5 Survival Criterion: Negative Feedback

Theorem 5.1. *System (10) remains viable if and only if α_g and α_k are functions of state satisfying the conditions:*

$$\begin{cases} \alpha_g(g) = \alpha_g^{base} + \gamma_g \cdot \max(0, g_{target} - g) \\ \alpha_k(k) = \alpha_k^{base} + \gamma_k \cdot \max(0, k_{target} - k) \\ \alpha_m(g,k) = \alpha_{total} - \alpha_g(g) - \alpha_k(k) \end{cases} \tag{12}$$

where $\gamma_g, \gamma_k > 0$, $g_{target}, k_{target} > 0$.

Proof. Necessity directly follows from Theorem 4.1: with fixed α, death is inevitable; therefore, survival is possible only with state-dependent α.

Sufficiency. When $g < g_{\text{target}}$, feedback increases α_g. By choosing γ_g, we ensure $\dot{g}(g_{\min}) > 0$. Similarly for k. When $\alpha_{\text{total}} > \alpha_g^{\max} + \alpha_k^{\max}$, parameter m remains positive. Boundedness of trajectories follows from feedback deactivation when $g \gg g_{\text{target}}$, $k \gg k_{\text{target}}$. □

6 Theorem of Irreversibility Threshold

Consider an ensemble of N identical systems interacting through a common external flow. θ is the fraction of systems with $g < g_{\text{crit}}$.

Theorem 6.1. *There exists $\theta_{crit} \in (0,1)$, such that when $\theta < \theta_{crit}$ recovery is possible, and when $\theta > \theta_{crit}$ irreversible collapse occurs.*

Proof. Average dissipation power of the ensemble $\langle P \rangle \approx (1-\theta)P_0$. Condition for recovery of one degraded system:

$$\alpha_g^{\max}(1-\theta)P_0 > \beta_g g_{\text{target}} \tag{13}$$

Hence:

$$\theta_{\text{crit}} = 1 - \frac{\beta_g g_{\text{target}}}{\alpha_g^{\max} P_0} \tag{14}$$

When $\theta > \theta_{\text{crit}}$, resources are insufficient for recovery, and the degradation process becomes avalanche-like. □

7 Typology of Regimes: Quantum-Statistical Analogy

The system can exist in two fundamentally different regimes.

Variability regime is characterized by priority of investments in α_g. States are unique, competition for resources, exclusion principle. Dynamics are described by fermionic statistics:

$$\frac{\partial}{\partial t} n_i = \nu n_i \left(1 - n_i/K - \sum_{j \neq i} n_j/K\right) \tag{15}$$

In steady state — one dominant niche.

Consolidation regime is characterized by priority of investments in α_k. Condensation in the dominant state is observed:

$$\frac{\partial}{\partial t} n_0 = \sigma(N - n_0) - \delta n_0 \tag{16}$$

When critical density is exceeded $n_0 \approx N$ (bosonic statistics).

8 Theorem of Phase Transition

Theorem 8.1. *A system is viable if and only if it can switch between fermionic and bosonic regimes.*

Proof. Pure variability regime ($\alpha_g \to \alpha_{\text{total}}$, $\alpha_k \to 0$) leads to $k \to 0$ and $P \to 0$. Pure consolidation regime ($\alpha_k \to \alpha_{\text{total}}$, $\alpha_g \to 0$) leads to $g \to 0$ and $P \to 0$. Pure accumulation regime ($\alpha_m \to \alpha_{\text{total}}$) leads to $g \to 0$, $k \to 0$ and $P \to 0$. A balanced fixed strategy does not adapt to fluctuations and is doomed according to Theorem 4.1. The only possibility is dynamic switching between regimes. □

9 Connection with Thermodynamics and Negentropy

Negentropy S_{neg}, its derivative:

$$\frac{\partial}{\partial t} S_{\text{neg}} = \alpha_{\text{total}} P - (\beta_m m + \beta_g g + \beta_k k)\varepsilon \tag{17}$$

Existence criterion:

$$\frac{\partial}{\partial t} S_{\text{neg}} > 0 \tag{18}$$

Definition 9.1. Inverse temperature of the system: $\beta_{\text{sys}} = \partial S_{\text{neg}} / \partial E$. Per cycle $\Delta S_{\text{neg}} = \alpha_{\text{total}} E$, therefore $\beta_{\text{sys}} = \alpha_{\text{total}}$. The reinvestment efficiency α_{total} is the inverse temperature of the system.

Thermodynamic temperature: $\Theta = 1/\alpha_{\text{total}}$. In equilibrium $\alpha_{\text{total}} P = \beta_m m$. With $P \sim m^2$, we obtain $m \sim \beta_m / \alpha_{\text{total}}$, $P \sim 1/\alpha_{\text{total}}^2 = \Theta^2$ — quadratic analog of the Stefan-Boltzmann law.

10 Evolution Dynamics: Two Time Scales

Slow evolution (background regime). v_0 is small, μ_s is large, T is large, P is small, $\beta \sim \alpha P$. Steady state, small fluctuations. Characteristic time $\tau_{\text{slow}} \sim 1/\beta$.

Fast evolution (explosion regime). v_0 is large, μ_s is reduced, T is small, P is large, $\alpha P \gg \beta$. Explosive growth of m, g, k. Characteristic time τ_{fast} is a few cycles.

Lemma 10.1. *In the explosion regime with a fixed reinvestment strategy, parameter g grows proportionally to m. Power $P \sim gm$ grows quadratically, which is equivalent to doubling of the metabolic power quantum over characteristic time τ_{fast}.*

Theorem 10.2. *Any system that emerged as a result of an evolutionary explosion and retained the reinvestment strategy characteristic of the explosion regime will inevitably initiate a secondary evolutionary explosion in its own time scale. A cascade of such explosions accelerates exponentially. The acceleration is due to the doubling of the metabolic power quantum at each level of the cascade.*

Proof. Characteristic doubling time of parameters $\tau_n \sim 1/(2\alpha_m \mu_k v_0^{(n)} g^{(n)})$. Parameter g at the n-th level is proportional to the volume of state space created by the system of the previous level. This volume grows exponentially with n, hence τ_n decreases exponentially. Power P_n satisfies $P_{n+1} \approx 2P_n$, cycle time $T_n \sim 1/P_n$, therefore $T_{n+1} \approx T_n/2$. ☐

Cascade termination criterion. The cascade terminates when one of the following conditions is met:

1. resource limitation: v_0 reaches a physical limit;
2. threshold limitation: g reaches a value at which $\mu_s(g) \to 0$;
3. strategic limitation: the system switches from explosion regime to background evolution regime (Theorem 5.1).

Corollary 10.3. *The only way to prevent unbounded acceleration of the cascade is timely switching of the reinvestment strategy from explosion regime to homeostasis regime.*

11 Spectral Power Density. Planck's Formula for the Dissipative Cycle

The system has a spectrum of admissible cycle frequencies $\{\nu_i\}$, quantum energy on mode with frequency ν: $E(\nu) = P(\nu)/\nu$, inverse temperature $\beta = \alpha_{\text{total}}$, and bosonic statistics (modes are independent, superpositions allowed).

Theorem 11.1. *The spectral power density of a dissipative system with reinvestment has the form:*

$$p(\nu) = \frac{D(\nu) \cdot \mathcal{E}(\nu)}{e^{\alpha_{total}\mathcal{E}(\nu)/\nu} - 1} \tag{19}$$

where $D(\nu)$ is the density of states (number of modes per frequency interval), $\mathcal{E}(\nu)$ is the quantum energy on mode with frequency ν, α_{total} is the inverse temperature.

Proof. Average number of quanta in a mode with energy E at inverse temperature β in bosonic statistics: $\langle n \rangle = 1/(e^{\beta E} - 1)$. Energy in the mode: $U = E\langle n \rangle$. Power: $P = \nu U = \nu E\langle n \rangle = \mathcal{E}(\nu)\langle n \rangle$, where $\mathcal{E}(\nu) = \nu E(\nu) = P(\nu)$ is the power corresponding to one quantum on this mode. Multiplying by the density of states $D(\nu)$, we obtain (19). ☐

Special case: large resource regime, small consolidation. With $m \to \infty$, $k \to 0$: $\mathcal{E}(\nu) = \mathcal{E}_0 = \text{const}$, $D(\nu) = Am = \text{const}$.

$$p(\nu) = \frac{Am\mathcal{E}_0}{e^{\alpha_{\text{total}}\mathcal{E}_0/\nu} - 1} \tag{20}$$

Formula (20) is an exact mathematical isomorphism of Planck's distribution for equilibrium radiation. Correspondence of quantities: $\mathcal{E}_0 \leftrightarrow h$ (Planck's constant), $\nu \leftrightarrow \nu$ (frequency), $1/\alpha_{\text{total}} \leftrightarrow k_B T$ (temperature), $Am\mathcal{E}_0 \leftrightarrow 2h\nu^3/c^2$ (density of states).

Conclusion

Scientific Results

1. An isomorphism between the stick-slip oscillator and a universal model of dissipative metabolism is established.
2. A closed system of equations for the dynamics of m, g, k (10) is obtained.
3. The concept of metabolic power quantum P is introduced.
4. Theorem 4.1 on inevitable death with fixed α is proved.
5. Theorem 5.1 on the survival criterion through negative feedback (12) is proved.
6. Theorem 6.1 on the irreversibility threshold is proved.
7. A typology of regimes based on quantum statistics is constructed.
8. Theorem 8.1 on the necessity of phase transitions is proved.
9. Theorem 10.2 on the cascade of evolutionary explosions and its exponential acceleration is proved.
10. Theorem 11.1 on spectral power density is proved; a mathematical isomorphism with Planck's formula (20) is established.

Worldview Implications

1. Dissipation is a resource, not a loss.
2. Fixed strategies are deadly.
3. Survival is homeostasis.
4. The irreversibility threshold is measurable.
5. Neither creativity without order, nor order without creativity.

6. Explosion begets explosion; stopping the cascade requires a change of strategy.
7. The power quantum determines the pace of evolution.
8. The spectrum of a dissipative system obeys Bose-Einstein statistics.

References

[1] Kondratiev, N.D. The Major Economic Cycles / N.D. Kondratiev // Voprosy Konyunktury. — 1925. — Vol. 1, Iss. 1. — P. 28–79.

[2] Larin, V.N. Hypothesis of the Initially Hydride Earth / V.N. Larin. — Moscow : Nedra, 1980. — 216 p.

[3] Kuhn, T.S. The Structure of Scientific Revolutions / T.S. Kuhn. — Chicago : University of Chicago Press, 1962. — 172 p.

[4] Nicolis, G. Self-Organization in Nonequilibrium Systems : From Dissipative Structures to Order through Fluctuations / G. Nicolis, I. Prigogine. — New York : Wiley, 1977. — 512 p.

[5] Planck, M. On the Law of Distribution of Energy in the Normal Spectrum / M. Planck // Annalen der Physik. — 1901. — Vol. 4. — P. 553–563.

[6] Schrödinger, E. What Is Life? The Physical Aspect of the Living Cell / E. Schrödinger. — Cambridge : Cambridge University Press, 1944. — 194 p.

Polyukov P.A., 2026