CALCULATION METHODS OF ECONOMIC SYSTEMS
A Proposed Unified Method for Transaction Value
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Abstract

Economic value is studied through siloed lenses—hedonic pricing, productivity accounting, behavioral valuation under uncertainty, and transaction-cost/finance perspectives on exchange frictions and risk. This paper proposes the Calculation Methods of Economic Systems (CMES), a common algebra for bilateral transaction value that codes determinants into four composite element classes: value-adding capacity (CEv), archived stocks/records/reputation (CEa), communication and coordination quality (CEc), and evaluative governance and risk perception (CEe).
CMES links transaction systems via a master equation with a transformation scalar σ that is anchored, under stated competitive conditions, to comparative productivity. Reduced-form evidence from real estate and smartphones shows the four-component proxy set explains meaningful within-market variation and remains robust under spatial dependence. A structural cross-regime test in corporate bonds rejects the uniform-σ restriction, isolating leverage-driven nonlinearity in the evaluative component and thereby mapping where the linear additive baseline must be extended. The paper closes with falsifiable predictions and a reproducible coding protocol designed for pre-registered validation across additional markets.

Keywords: transaction value; hedonic pricing; productivity; transaction costs; behavioral valuation; decomposition; identification

Notation and Abbreviations

The notation below is used consistently throughout the manuscript. Subscripts 1 and 2 denote the two transaction systems compared in the master equation; subscripts v, a, c, e denote the four component roles.

Symbol	Meaning
CE	Composite element bundle (transaction-relevant determinants of value).
CEv	Value-adding capacity (functional performance, productive capability).
CEa	Archived stocks, records & reputation (intangible capital, brand, know-how, inventories, maintenance history).
CEc	Communication & coordination quality (infrastructure, interoperability, matching frictions).
CEe	Evaluative governance & risk perception (uncertainty, guarantees, constraints, perceived losses).
Val(CE)	Per-transaction transaction value in the accounting unit used in the master equation (observed transaction price/transfer in the empirical sections; unit cost/price in the competitive benchmark; can be linked to welfare via WTP minus transaction/implementation costs).
NTr	Number of transactions in the system/period.

σ	Transformation scalar linking two systems (valuation/efficiency scale). Interpreted as the factor that converts System 2 total value into System 1 units; by convention $\sigma \geq 1$ (swap labels if $\sigma < 1$).
Δ	Value differential between two bundles or systems.
N	Additive augmentation vector in the additive representation.
N*	Rescaled augmentation absorbing σ: $N^* = \sigma \cdot [Val(CE_2) - Val(CE_1)]$.
β	Reduced-form implicit-price coefficients from hedonic estimation.
λ	Loss aversion parameter (market-specific).
γ	Probability-weighting curvature parameters (baseline priors / sensitivity range).
FE	Fixed effects controls (time, geography, issuer, etc.).

1. Introduction: The Fragmentation Problem

For over half a century, the study of value in economics has been profoundly, and perhaps unnecessarily, fragmented. Macroeconomists measure value creation as the Solow residual — a productivity gap that cannot be explained by observable factor inputs. Microeconomists estimate value as the sum of hedonic attribute prices recovered from market transactions. Behavioral economists model it as a psychologically distorted perception of gains and losses relative to a reference point. Financial economists decompose it through sensitivity vectors — the Greeks — that measure how an instrument's worth responds to each underlying parameter. Trade economists track value through constant market share decompositions. Each of these frameworks is internally coherent. Each is empirically productive. Yet they address the same underlying question — what determines the value of an exchange? — from different levels of analysis, with different units of measurement, and with no common language.

This intellectual fragmentation has real costs. An empirical researcher studying platform economics cannot easily apply hedonic pricing tools calibrated on physical goods to digital services without theoretical guidance on which characteristics to include and why. A development economist cannot connect micro-level estimates of transaction costs to macro-level productivity differentials without a bridge framework. A financial analyst valuing an innovative technology company cannot formally incorporate behavioral risk perceptions alongside patent stocks and communication infrastructure in a single valuation model. In each case, the tools exist — but they exist in separate literatures, written for separate audiences, in separate mathematical languages.

This paper argues that the fragmentation is not fundamental. The competing frameworks are not genuinely incompatible theories of value; they are complementary perspectives on a single underlying algebraic structure. We propose the Calculation Methods of Economic Systems (CMES) as that structure — a common language for the analysis of bilateral transaction value across all market types.

The CMES's central claim is deliberately modest in scope: that any bilateral exchange can be characterized by four functional components — value-adding capacity (CEv), archived stocks, records, and reputation (CEa), communication quality (CEc), and evaluative risk perception (CEe) — and that the relationship between two transaction systems can be formalized through a multiplicative transformation scalar (σ) and a value differential (Δ). The master equation $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$ encodes this relationship at the most general level. We further develop and formally analyze an additive representation, $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_1 + N)$, that offers a complementary interpretive lens particularly suited to component-level policy accounting.

1.1 What This Paper Does

This paper makes five distinct contributions to the economics of value:

- It provides a unified algebraic framework — the master equation $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$ and its additive equivalent — grounded in a four-component taxonomy of bilateral exchange.
- It demonstrates how the transformation scalar σ can be calibrated from national accounts data under explicit, testable conditions, yielding $\sigma \approx 1.022$ for competitive US markets 1987–2023.
- It shows that a four-component value decomposition explains 37–61% of within-market variance hedonically across three structurally distinct markets, with all four components independently significant.
- It offers rigorous axiomatic foundations for Val() that yield growth accounting, hedonic pricing, prospect theory, and transaction cost economics as structural analogs — not as special cases, but as formally analogous structures.
- It sets out a research agenda organized around structural estimation, fifth-component investigation, and pre-registered cross-market tests — a foundation, not a finished edifice.

Mechanism of integration (one paragraph). CMES integrates macro productivity, hedonic micropricing, and behavioral valuation by (i) defining a common transaction-level object Val(CE) built from four functional roles (CEv, CEa, CEc, CEe), (ii) linking heterogeneous transaction systems through a single transformation scalar σ that is anchored to comparative productivity under explicit conditions, and (iii) producing testable structure via (a) the additive Val() representation derived in Section 5 and (b) cross-equation restrictions (Axiom 7) in paired markets. The empirical sections therefore assess not whether CMES maximizes prediction, but whether this role-based partition yields stable decompositions and nontrivial restrictions consistent with the σ connector.

1.2 The Paper's Deliberate Middle Ground

A critical methodological clarification must be stated at the outset, both to frame the paper's contribution accurately and to anticipate a primary methodological critique. The paper occupies a deliberate middle ground between pure theory and structural empirics.

On the theoretical side, the Val() function is axiomatized through seven conditions and a formal theorem derived via Debreu–Gorman separability. These are strong theoretical claims that generate falsifiable predictions. On the empirical side, the evidence consists of reduced-form hedonic regressions that demonstrate the four-component taxonomy's descriptive adequacy across diverse markets. These regressions do not constitute a structural test of the axiomatic model. They are evidence that the framework's taxonomy successfully organizes observable market data — a necessary but not sufficient condition for the structural axiomatic claims.

The stronger claim — that hedonic estimates validate the structural model — requires dedicated structural estimation exploiting cross-equation restrictions derived from Axiom 7 (Transaction Consistency). That agenda is outlined in Section 7.4. Until that work is complete, readers should evaluate the hedonic results as evidence of descriptive adequacy, and the axioms as a theoretical contribution whose structural implications await direct testing. This honest positioning is not a weakness; it is a prerequisite for scientific integrity.

1.3 Relationship to Prior Work and Unification Attempts

CMES is not the first attempt to provide a common language for value. The characteristics approach (Lancaster, 1966) and household production/time-allocation tradition (Becker, 1965) unify heterogeneous goods by mapping them into deeper primitives, while hedonic pricing (Rosen, 1974) provides an equilibrium identification strategy for implicit prices of characteristics. In measurement, modern cost-of-living and price-index work also seeks coherent aggregation across heterogeneous items and quality change (e.g., scanner-data and multi-good price measurement; Redding and Weinstein, 2016). CMES builds on these traditions but targets a different unifying object: bilateral transaction value decomposed into a small role-based basis plus a cross-system transformation scalar.

CMES differs in its target object and in the restrictions it aims to deliver. The goal is not primarily a new demand system, a new index-number formula, or a replacement for hedonic methods. Rather, it is a disciplined coding protocol that (i) partitions transaction-relevant determinants into a small set of functional roles, (ii) yields an additive baseline representation under explicit separability conditions, and (iii) links transaction systems through an efficiency/valuation transformation σ that becomes testable when anchored to independent evidence (e.g., productivity comparisons under Proposition 1) and when used to impose cross-equation restrictions (Axiom 7).

Related domains. CMES also intersects with (i) quantitative spatial economics, which connects productivity, amenities, and location price schedules in tractable counterfactual models (Redding and Rossi-Hansberg, 2017), and (ii) market design and matching, which emphasizes the engineering of coordination and governance mechanisms in thick markets (Roth and Peranson, 1999; Roth, 2008; Budish, 2011). CMES does not replace these frameworks; rather, it provides a common transaction-value accounting layer that can be paired with them when transactions are shaped by spatial structure or by designed matching rules.

This positioning implies two practical commitments that structure the remainder of the manuscript: (a) CMES claims are conditional on stated scope conditions and are treated as hypotheses until validated across additional markets; and (b) empirical performance is evaluated not only by predictive fit (R^2) but also by interpretability, cross-system comparability, and whether CMES-specific restrictions (e.g., Axiom 7 cross-equation implications) survive empirical scrutiny.

Table 1.4 extends the positioning discussion by contrasting CMES with partial-integration approaches that combine two of the three target domains (macro productivity, hedonic micropricing, behavioral valuation) but do not provide a single cross-system connector.

Approach	Macro productivity link	Hedonic transaction link	Behavioral valuation	Cross-system connector	What CMES adds
Hedonic–GE hybrids (amenities/land)	Partial	Yes	No	No	Role-based CE coding + σ connector beyond a single market
Behavioral–GE models	Yes	No	Yes	No	Explicit transaction-level decomposition + hedonic bridge

ABM / micro–macro platforms	Yes (simulated)	Partial	Partial	No unified scalar	Closed-form σ and testable restrictions rather than simulation-only integration
Matching / market design models	Partial	Partial	Partial	No	Role interpretation of CEc/CEe and value accounting for designed markets
Quantitative spatial economics	Yes	Yes (location price schedules)	No	No explicit σ	Explicit σ anchoring + role partition for determinants across systems

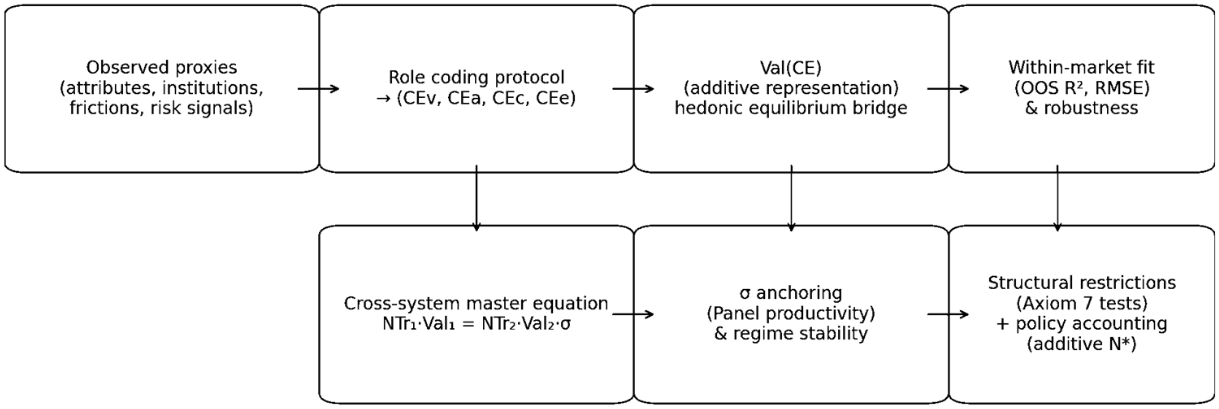
Tradition	Unifying object	Strength	What CMES adds
Lancaster characteristics (1966)	Goods as bundles of characteristics	Deep micro foundation for heterogeneity	Role-based taxonomy + cross-system σ connector
Becker time allocation (1965)	Utility from commodities produced with time/inputs	Unifies consumption via production of 'commodities'	Separates functional roles of determinants in market value
Rosen hedonic pricing (1974)	Equilibrium implicit prices of characteristics	Identification strategy for marginal valuations	Role-based discipline for variable coding + behavioral component CEe
Growth accounting / Solow (1957)	Residual productivity/technology term	Macro link between inputs and output	Transaction-level decomposition + explicit conditions for $\sigma \approx$TFP
Prospect theory (1979/1992)	Reference-dependent valuation under risk	Behavioral realism under uncertainty	Market-calibrated λ embedded as CEe specialization

2. The CMES Framework: A Conceptual Architecture

Before developing the mathematics, it is useful to introduce the CMES's core concepts intuitively. The framework is built on three interdependent ideas: a four-component taxonomy of exchange value, a scalar that relates the value systems of two economic actors, and a differential that measures the residual between those systems. These three ideas collectively constitute the CMES's 'common language.'

2.1 The Four Components: CEv, CEa, CEc, CEe

Figure 1. CMES architecture and empirical workflow.



Every bilateral exchange involves economic agents bringing something to the transaction. The CMES proposes that the value contributed by any party to any exchange can be decomposed, without remainder, into four functional types:

CEv — Value-Adding Elements. These are the active transformation capabilities: innovation, R&D outputs, operational processes, and production capacity. CEv is the component that most directly corresponds to what growth economists call total factor productivity — the ability to do more with a given set of inputs. In a real estate transaction, CEv includes the physical improvements to the structure. In a smartphone purchase, it is processor speed, camera resolution, and battery capacity. In a bond issuance, it is the issuing firm's R&D intensity and revenue growth.

CEa — Archived Elements. These are the persistence, stock, and memory components: accumulated institutional knowledge, proprietary data and IP, brand equity and track record, certifications and provenance, and internal stocks such as inventories and stored products held for later use. In manufacturing systems, CEa includes warehouse inventory, stored finished goods reserved for downstream use, spare parts, and archival records such as maintenance logs and traceability documentation. The term archived is used to emphasize stored state and persistence, not obsolescence: stored products are archived because they remain inside the system boundary until activated in production or exchanged at the transaction boundary. CEa corresponds most closely to the intangible assets literature — the roughly 90% of S&P 500 market capitalization attributable to non-physical assets (Ocean Tomo, 2020). In real estate, CEa is school district quality and the property's age-adjusted maintenance history. In smartphones, it is brand equity and OS support longevity. In bonds, it is years of investment-grade history and balance sheet quality.

CEc — Communication Elements. These are the connectivity and coordination components: information channels, logistics networks, platform interfaces, routing quality, and the physical and digital infrastructure of transaction facilitation. CEc corresponds to what transaction cost economists (Coase 1937; Williamson 1981) treat as the overhead of market coordination. In real estate, CEc is broadband availability and walkability. In smartphones, it is network generation capability (5G vs. 4G) and WiFi standard. In bonds, it is ESG disclosure quality and auditor tier.

CEe — Evaluative Elements. These are the risk perception and governance components: goals, threat–opportunity assessments, regulatory compliance, and the behavioral psychology of evaluation under uncertainty. CEe is where the framework most explicitly incorporates behavioral economics — specifically, prospect theory's asymmetric treatment of gains and losses. In real estate, CEe is flood zone designation and crime rates. In smartphones, it is warranty length and IP water-resistance certification. In bonds, it is CDS spread and covenant quality.

Beyond standard risk and governance variables, CEe also includes culturally mediated evaluative signals that are capitalized into prices despite having no physical productivity content—for example, numerological superstition in housing addresses documented in Auckland sales data (Bourassa and Peng, 1999).

A clarifying rule governs the boundary between archived elements and exchange objects: inventories and stored products (including manufactured goods reserved for later internal use) are CEa — archived state within the boundary of the system — until they become exchange objects at the boundary of a transaction. The Exchange overlay (NE) represents the mechanisms that execute and settle that transfer: contracts, pricing engines, clearing systems, and payment rails. NE is not a fifth value component; it is a system-level tag for transactional closure.

2.2 The Transformation Scalar σ

The transformation scalar σ captures the systemic efficiency differential between two transaction systems. It answers the question: how much more value per transaction does System 1 generate relative to System 2? In competitive markets operating under stable conditions, σ is close to but above 1.0, reflecting the normal spread in productivity between more and less efficient economic actors. During technological transitions, platform economy emergence, or post-crisis recovery, σ can vary more substantially.

Crucially, σ is not an arbitrary scaling parameter. Under the explicit conditions of Proposition 1 (Section 3.2), σ corresponds directly to the ratio of total factor productivity between the two systems — a quantity that is measurable from national accounts data. This correspondence between a theoretical scalar and a directly observable macroeconomic variable is one of the framework's key empirical anchors.

2.3 The Value Differential Δ

The value differential $\Delta = \mathrm{Val}(CE_1) - \mathrm{Val}(CE_2)$ captures the per-transaction value advantage of one system over another. While σ measures the proportional relationship between the systems, Δ measures the absolute gap. In a well-functioning competitive market, Δ should be small and transient — eliminated by arbitrage and competition. In markets with structural barriers (fixed supply, information asymmetry, market power), Δ can be persistent and substantial.

The hedonic decomposition of Δ is the paper's main empirical contribution. By attributing the value differential across the four components — ΔCEv, ΔCEa, ΔCEc, ΔCEe — we obtain a structured explanation of why one transaction system commands higher value than another. This decomposition is directly applicable to competitive strategy (which component should a firm invest in to maximize Δ?), regulatory analysis (does a platform's CEc advantage constitute a structural barrier?), and macroeconomic development (which component is binding for low-σ economies?)

2.4 The Universal Role Basis: Theoretical Foundation and Operational Tests

Epistemological status. The four-component structure presented in this section is best understood as a proposed functional coding protocol grounded in systems theory, not as a proven metaphysical truth about the nature of economic value. Its credibility rests on three pillars: (i) deductive convergence — independent intellectual traditions (system dynamics, cybernetics, value-chain analysis, and control theory) arrive at the same four functional primitives; (ii) cross-domain instantiation — the taxonomy can be applied without modification to structurally diverse systems from computing to governance; and (iii) operational testability — the removal-test protocol is designed to be applied by independent analysts whose agreement can be measured empirically. The taxonomy's empirical validity will ultimately be determined by inter-rater reliability of the coding protocol (Krippendorff's $\alpha \geq 0.80$ as the minimum threshold for an operational classification), by the descriptive power of the hedonic decompositions it generates, and by the falsifiable predictions it enables. The universality claims below should be evaluated in this spirit: as strong, testable modeling hypotheses, not as irrefutable facts.

The four-component structure is not arbitrary. It derives from a Universal Classification and Clustering (UCC) framework for complex ecosystems (Author, 2026) whose core claim is that any purposive, goal- or constraint-directed system involves elements that transform inputs (Value-Adding), persist and store state (Archived), coordinate and route (Communication), and measure and select (Evaluation). When transactions are first-class, an Exchange overlay (NE) tags the mechanisms that execute and settle transfers of ownership, rights, or value. This section provides the theoretical foundation, formal model, operational assignment tests, universality arguments, and cross-domain evidence that undergird the CMES's four-component taxonomy, rendering the present paper self-contained.

2.4.1 Formal Model and Notation

Let an ecosystem S be modeled (over a time window) as a dynamical system with state $x(t) \in X$, external inputs $u(t) \in U$, outputs $y(t) \in Y$, and a state transition:

$$x(t + \Delta) = F(x(t), u(t), m(t), \theta(t))$$

where m(t) denotes mediated transfers and messages among components and θ(t) denotes parameters and policies. Let J be an objective (or set of objectives) and G a constraint set defining admissible trajectories. Let E be the set of identifiable elements — artifacts, modules, agents, institutions, processes, rules — that can causally influence F, x, m, θ, J, or G. Attention is restricted to functionally meaningful elements: those whose presence or absence changes reachable trajectories or the evaluation of outcomes.

2.4.2 Role Definitions and Operational Assignment Tests

Each element is assigned one or more functional roles based on what it does within the ecosystem. Table 2.4 provides the formal definitions, diagnostic questions, and operational removal tests.

Functional Role	Set	Core Function	Diagnostic Question	Operational Removal Test	Economic Examples
Value-Adding	Nnv(t)	Transforms inputs into outputs or capability	What changes the world?	If removed, some outputs/capabilities become impossible	R&D, process innovation, manufacturing capacity, service delivery
Archived	Nna(t)	Preserves state, history, inventory (including stored products),	What persists for later use?	If removed, the system loses memory, traceability, or inventory	Institutional knowledge, IP, brand/reputation, warehouse inventories, stored products, maintenance and

		records, or knowledge			traceability records, inventories and reserves
Communication	Nnc(t)	Transfers, routes, interfaces; coupling and coordination	Does it connect sources and sinks or mediate coordination?	If removed, connectivity, coordination, signaling, or logistics fails	Networks, logistics, platforms/APIs, market information channels
Evaluation	Nne(t)	Measures, judges, selects; constraints and governance	Does it measure performance, fitness, or permissibility?	If removed, quality control, compliance, or optimization collapses	Risk models, auditing, regulation, credit scoring, governance
Exchange (overlay)	NE(t)	Enables or constitutes exchange or transactions: transfer of ownership, rights, or value	What is being traded and how is ownership/value transferred?	If removed, trades cannot be executed or settled; pricing/ clearing breaks	Contracts, pricing/matching, clearing/settlement, payment rails

Table 2.4: Role definitions, diagnostic questions, and operational removal tests.

The same named object can map to different roles depending on which aspect is being modeled. For example, a database platform contains archival components (storage), value-adding components (query processing), communication components (replication and protocol handling), and evaluation components (constraints and access control). The UCC framework therefore recommends decomposing composites into role-pure sub-elements where practical, rather than forcing a single-label assignment.

2.4.3 Universality as a Research Hypothesis: Exhaustiveness, Minimality, and Sufficiency

This subsection clarifies how CMES uses 'universality' claims. The four-role structure is proposed as a coding basis that is intended to be broadly applicable, but it is not a metaphysical theorem. The appropriate scientific status is a research hypothesis: conditional on explicit modeling assumptions, the four roles are intended to be collectively exhaustive for transaction-relevant elements and practically minimal for empirical work.

Assumptions. The scope of the claim depends on three conditions: (A1) an explicit boundary for the system under study; (A2) a notion of functional relevance to transactions within that boundary and time window; and (A3) a specified coding protocol that maps observable proxies to functional roles via operational removal tests.

Claim (Collective Exhaustiveness; coding hypothesis). For any transaction-relevant element e within a declared boundary, at least one of the four functional roles applies at the level of analysis used for valuation: value-adding (V), archival/persistence (A), communication/coordination (C), or evaluation/governance (E). Elements outside the boundary or irrelevant to transactions are excluded by construction.

Claim (Practical Minimality). Each core role has empirical witness settings in which omitting that role produces systematic misattribution of value variation (e.g., supply constraints require an evaluative/governance channel; reputation and durability require an archival channel; network access requires a communication channel; capability creation requires a value-adding channel).

Claim (Sufficiency for baseline modeling). Under A1-A3, the four roles provide a minimal scaffold for a first-pass valuation model. When separability fails or roles interact, CMES treats this as a non-separable extension (interaction terms or alternative functional forms), not as automatic evidence for an additional primitive.

What these rationales do and do not establish. The arguments support a disciplined operational taxonomy and a falsification protocol; they do not prove exhaustiveness in a mathematical sense. The empirical program in Sections 4 and 7 treats universality as testable: stability to alternative codings, inter-rater reliability, and out-of-sample validation across new markets.

2.4.4 MECE Convention and Multi-Role Treatment

A Mutually Exclusive and Collectively Exhaustive (MECE) partition is achieved by adopting a primary-role assignment ρ(e,t) that selects one dominant role for each element. However, empirical reality frequently demands multi-role modeling. The UCC therefore treats MECE as an analysis convention rather than a metaphysical claim: elements are MECE after decomposition into role-pure sub-elements, or after choosing a dominance criterion. Let each element e have a functional signature s(e,t) = [v, a, c, e] over V/A/C/E (or s(e,t) = [v, a, c, x, e] when Exchange is modeled explicitly). This signature defines a domain-agnostic metric space for clustering and comparison, and enables quantitative measurement of role drift over time.

For the CMES, the MECE convention matters because the hedonic regressions of Section 4 require each proxy variable to be coded into exactly one CE component. The operational removal tests above provide the assignment discipline: when a proxy spans multiple roles — for example, platform membership encompasses communication rails (CEc), archival ledgers (CEa), and evaluative fraud gates (CEe) — the protocol recommends either (i) decomposing the composite into role-pure sub-elements when data allow, or (ii) treating it as a composite and reporting robustness to alternative codings.

2.4.5 Cross-Domain Instantiation Evidence

The four-role taxonomy is not merely a theoretical construct; it has been instantiated across multiple domains, confirming its descriptive reach. Table 2.5 provides representative examples.

Domain	Nnv (Value)	Nna (Archive)	Nnc (Comm.)	Nne (Eval.)
Computing	CPU/GPU execution, compilation	RAM/SSD, DB tables, logs, checkpoints	Networks, buses, APIs, brokers	Scheduler, tests, ACLs, policies
Manufacturing	Assembly, machining, service delivery	Inventory (including stored products), warehouses, maintenance records	Logistics, ERP links, EDI	QA, safety control, audits
Trade / Economy	Production, innovation, negotiation	Capital stocks, ledgers, reserves, IP	Markets, payment rails, shipping	Prices, regulation, credit scoring, arbitration
Science	Experiments, model building, synthesis	Datasets, lab notebooks, literature	Journals, collaboration, instrument links	Peer review, statistics, ethics
Governance	Public service operations	Laws, registries, archives	Media, diplomatic channels	Courts, elections, oversight

Table 2.5: Cross-domain instantiation of the four core roles.

2.4.6 The Core Classification Procedure

For any candidate element in an economic system, the analyst asks the following diagnostic questions:

Q1: Does it transform inputs into outputs or capability? → CEv (Value-Adding).
Q2: Does it preserve state, history, inventory, or knowledge? → CEa (Archived).
Q3: Does it transmit, route, or interface signals or resources between components? → CEc (Communication).
Q4: Does it measure, judge, constrain, select, or optimize? → CEe (Evaluation).
If none apply, either the element is outside the modeled boundary or not functionally meaningful. If the element serves multiple roles, one of three modeling conventions applies: (i) decompose the element into role-pure sub-elements; (ii) assign the role responsible for the primary causal contribution (dominance criterion); or (iii) retain the multi-role signature vector s(e,t), allowing role drift over time.

2.4.7 Disambiguation Examples

Boundary cases are common in practice. The following examples illustrate how the operational tests resolve ambiguity:

Element	Primary Role	Notes / Secondary Roles
Model checkpoint	CEa	Stored state; selected by metrics (CEe)
API gateway	CEc	Mediates calls; may enforce policy (CEe)
Audit report	CEe	Evaluates; stored as record (CEa)
Inventory stock	CEa	Archive; replenishment rules are evaluation (CEe)
Pricing mechanism	CEe	Evaluation/selection signal; communicated via markets (CEc)
Payment rail / clearinghouse	NE	Executes/settles exchange; relies on messaging (CEc), ledgers (CEa), and rules/ verification (CEe)

Table 2.6: Disambiguation of multi-role elements.

2.4.8 Stress Tests and Falsification Protocol

The universality claim is strongest when it actively welcomes counterexamples. Useful stress tests include: ‘meaning’, ‘trust’, ‘culture’, ‘legitimacy’, and ‘power’ — which are often evaluative selection criteria (CEe) combined with persistent histories (CEa); platform ‘rules of the game’ (often CEe) versus the media that store and propagate them (CEa/CEc); and purely emergent effects (e.g., market liquidity, network effects) which are best modeled as graph-level properties of the role-flow matrix rather than as standalone elements.
The falsification criterion is explicit: a causally relevant element within a specified boundary and time window that resists decomposition into any combination of V/A/C/E would constitute a counterexample. The cross-domain convergence documented in Table 2.5 — where independent traditions (system dynamics, cybernetics, value-chain analysis) arrive at the same four functional primitives — provides the strongest a priori evidence for the taxonomy’s robustness, but systematic inter-rater reliability studies remain a priority for future empirical validation (Section 7.5).
The removal-test protocol enables inter-rater validation: independent analysts can apply the same tests to code proxy variables, and disagreements become explicit (boundary choice, multi-role ambiguity) rather than hidden in ad hoc variable selection. Krippendorff's α agreement statistics from such validation studies would provide direct evidence that the taxonomy is operationally reproducible, not post-hoc rationalization. This constitutes an important priority for future empirical work (Section 7.5).

2.5 Distinguishing CMES from Hedonic Pricing

A natural question is whether the CMES is simply hedonic pricing repackaged. It is not, and the distinction matters for interpreting the paper's contribution. Standard hedonic pricing regresses transaction prices on observable product characteristics to recover implicit attribute prices. It is atheoretical in that any observable attribute may be included, and the coefficients are reduced-form estimates without a structural interpretation.

Feature	Hedonic Pricing	CMES Framework
Nature of Characteristics	Atheoretical; any observable attribute included	Theoretical taxonomy (CEv, CEa, CEc, CEe) grounded in transaction theory
Coefficient Interpretation	Implicit prices, reduced-form estimates	Sensitivity parameters with structural analogs to financial Greeks, enabling dynamic analysis
Role of Market Conditions	Typically static or time-fixed effects	Explicit vector X of market state variables; σ may depend on X
Second-Order Effects	Rarely modeled	Cross-sensitivities (e.g., $\partial^2V/\partial CEv \partial CEc$) map to cross-Gamma concepts
Aggregation Property	No inherent aggregation structure	Micro-level sensitivities aggregate to macro CVAS decomposition via Sec 4.8
Cross-System Structure	Single-market framework	Master equation connects two systems through σ, enabling cross-economy analysis

The CMES therefore uses hedonic regression as its primary empirical tool but provides the theoretical framework that standard hedonic pricing lacks: a taxonomy that disciplines variable selection, a scalar that connects markets, and axiomatic foundations that generate falsifiable restrictions.

2.6 The Theory–Empirics Relationship

2.6.1 From Val() to hedonic coefficients: identification under equilibrium

CMES derives a valuation representation Val(CE) at the level of choice/ordering. Empirically, we estimate hedonic price equations in equilibrium. Under competitive equilibrium in an implicit market for characteristics (Rosen 1974), marginal implicit prices correspond to marginal willingness-to-pay for characteristics at the equilibrium margin, subject to sorting and supply responses. Accordingly, the estimates in Section 4 are interpreted as reduced-form component sensitivities and a measurement model for Val(·), not a complete recovery of structural preference parameters. CMES's distinctive empirical content is therefore emphasized through restrictions and falsifiable predictions (e.g., cross-equation constraints implied by uniform σ; separability diagnostics; stability to alternative codings), and through a structural estimation roadmap (Section 7.4).

The relationship between the CMES's axiomatic structure and its empirical implementation requires precise statement. The axiomatic model posits a structural Val() function with specific functional form implications derived from Debreu–Gorman separability. The master equation is a structural cross-system constraint. These are strong theoretical claims that generate falsifiable restrictions on the data.

Existence and uniqueness. Standard hedonic equilibrium analyses (Rosen, 1974) provide conditions under which a price schedule for characteristics exists and supports sorting in competitive markets. CMES uses this equilibrium concept as a measurement bridge: when those conditions are approximately satisfied (thick markets, limited market power, adequate characteristic support), hedonic implicit prices are informative about marginal valuations. CMES does not require uniqueness of the price schedule and therefore avoids welfare claims that would depend on unique equilibrium selection.

The hedonic regressions reported in Section 4 are reduced-form evidence for the framework's descriptive adequacy. They demonstrate that the four-component taxonomy successfully organizes observable attributes and explains substantial within-market variance. They do not constitute a structural test of the axiomatic model. The hedonic coefficients are implicit prices estimated from market-clearing conditions, not structural parameters recovered from the Val() function's axiomatic restrictions.

The paper therefore makes two classes of claims: first, that the CMES's four-component taxonomy is a powerful and consistent organizational heuristic across diverse markets; and second, that the framework's theoretical architecture is internally coherent and generates falsifiable predictions. The stronger claim — that the hedonic estimates validate the structural axiomatic model — requires the dedicated structural estimation outlined in Section 7.4.

3. The Transformation Factor σ: Theory, Calibration, and Dual Representation

3.1 Theoretical Definition and the Master Equation

The master equation of the CMES is:

$$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$$

where NTr denotes the number of transactions in a given system or period, Val(CE) denotes the per-transaction transaction value (observed market price/transfer in the empirical sections; interpretable as unit cost/price in the competitive benchmark), and σ is the transformation factor that converts System 2's total value output into System 1's units. By convention, NTr_1 is assigned to the higher-value system, so $\sigma \geq 1$. When empirical estimation yields $\sigma < 1$, system labels are exchanged and $\sigma = 1/\sigma\_computed$. In time-series applications, σ can cross 1.0 as relative productivity dominance reverses — documented in the BLS panel for 1981–1982, when energy sector TFP temporarily exceeded technology sector TFP.

It is important to note that this equation is a structural accounting identity, not an equilibrium condition. It holds by construction whenever the two sides represent the same transaction evaluated from both parties' perspectives with σ calibrated accordingly. The equation's economic content lies not in the identity itself but in the constraints the axiomatic structure (Section 5) places on the Val() function and in the conditions under which σ is estimable from macroeconomic data (Section 3.2).

The master equation has an illuminating relationship to classical economic identities. The Quantity Theory of Money MV = PT separates the money supply from the price level via a velocity parameter. Purchasing power parity links two countries' price levels via an exchange rate scalar. Growth accounting decomposes output into factor inputs and a TFP residual scalar. In each case, a multiplicative scalar connects two sides of an equation. The CMES master equation belongs to this tradition, but generalizes the objects on each side from money quantities or aggregate output to composite value bundles, and replaces velocity or exchange rates with a productivity-grounded transformation factor.

3.1.1 Microfoundations and an Explicit Equilibrium Bridge

A rigorous critique might correctly emphasize that a multiplicative identity becomes economically meaningful only once it is tied to a transaction technology and an equilibrium notion that maps primitives to observed prices. CMES therefore adopts a minimal microfoundation that is deliberately weaker than a full general-equilibrium model: it treats observed transaction prices as equilibrium outcomes of a hedonic market (Rosen-style), and it interprets σ as a comparative efficiency scalar of a transaction technology that converts the same underlying functional bundle into market value.

Minimal primitives. Consider a bilateral transaction in system $s \in \{1,2\}$. Let x denote raw characteristics and let m(x) be the role-coding map into CE = (CEv, CEa, CEc, CEe). Let $p_s(x)$ denote the observed transaction price (or unit price) in system s. Let $v_s(CE)$ denote an agent's willingness-to-pay (or marginal revenue product) for the coded bundle, and let $\kappa_s(CE)$ denote transaction and implementation costs. In a competitive hedonic equilibrium with a differentiable price schedule (Rosen-style) and adequate support overlap, $p_s(x)$ summarizes the market's valuation of CE at the equilibrium margin; the corresponding net value/surplus is $v_s(CE) - \kappa_s(CE)$. Throughout the empirical sections we take $Val_s(CE) \equiv p_s(x)$ as the observable transaction-value object, and we treat hedonic coefficients as reduced-form marginal sensitivities of Val(·) under the usual equilibrium caveats (Section 2.6.1).

Transaction technology interpretation of σ. Suppose systems differ by a multiplicative efficiency wedge A_s in a competitive benchmark where unit prices track unit costs. Holding the coded bundle CE fixed, let the unit cost (and hence the competitive unit price) scale as $Val_s(CE) = (1/A_s) \cdot \bar{V}(CE)$. Then $\sigma_{12} = Val_1(CE)/Val_2(CE) = A_2/A_1$ is the comparative efficiency scalar converting System 2's transaction values into System 1's units. Aggregating across transactions yields $NTr_1 \cdot Val_1(CE_1) = NTr_2 \cdot Val_2(CE_2) \cdot \sigma$, which is the master equation. Under this interpretation σ is not derived from preferences alone; it is a technology/institutional wedge that can be externally anchored (Proposition 1) or structurally tested through cross-equation restrictions (Section 4.9).

Equilibrium scope. CMES does not require uniqueness of the hedonic equilibrium schedule, nor does it claim that equilibrium always exists in the presence of thick-market frictions, matching constraints, or market power. Instead it uses equilibrium as an identification bridge: where competitive price schedules are a reasonable approximation, CMES treats estimated implicit prices as informative about the Val() representation; where they are not, CMES treats deviations as scope violations or as targets for the strategic and matching extensions discussed in Section 7.7.

3.1.2 Aggregation, Domar Weights, and the Role of Markups

The microfoundation above interprets σ as a comparative efficiency wedge A_2/A_1 at the transaction-technology level. To connect that wedge to observed sector- or economy-level productivity statistics, CMES adopts standard aggregation logic: under constant returns to scale and competitive factor markets, sector-level total factor productivity aggregates firm-level technology parameters, and cross-system productivity comparisons map to a proportional scaling of value per transaction.

In heterogeneous-firm environments (e.g., Melitz-type selection or Eaton–Kortum trade), firms draw idiosyncratic productivity and compete under common demand primitives. When the comparison set is restricted to competitive regimes (Proposition 1's C1–C3), the relevant productivity anchor is the level ratio A_2/A_1 over the chosen regime window. Where the comparison spans multiple producing industries, the appropriate economy-level anchor is Domar-weighted: industries with larger gross-output shares receive larger weights because their productivity changes propagate through input–output linkages to final demand.

This clarification also isolates the most important scope violation. If markups vary systematically between the compared systems, measured productivity can embed rent extraction rather than transformation efficiency. In such settings σ should be treated as a bounded comparative valuation scale rather than as a literal technology ratio. Empirical applications should either (i) adjust productivity anchors using independent markup measures, or (ii) identify σ within markets using the cross-equation restrictions of Axiom 7 when matched comparisons permit direct identification.

3.2 Proposition 1: Conditions for TFP–σ Correspondence

A central empirical claim of the CMES is that the transformation scalar σ can be calibrated from observable total factor productivity data. This claim is not unconditional. The following proposition states its precise conditions:

Proposition 1 (TFP–σ Correspondence). The TFP level ratio A_2/A_1 is a valid proxy for σ (the CMES transformation scalar between systems 1 and 2) if and only if three conditions hold: (C1) Output is measured at market prices — ensuring that price signals correctly reflect value differentials. (C2) Relative demand for the two systems' outputs is stable across the comparison period — ensuring that demand shifts do not contaminate the productivity comparison. (C3) Factor markets are competitive — ensuring that observed productivity differences reflect genuine transformation efficiency, not market power or rent extraction.

When C1–C3 are satisfied, A_2/A_1 equals the value-added per unit of factor input ratio — precisely the economic definition of σ when the CMES value object is evaluated at competitive unit prices. Primary sources of bias arise when C3 is violated (monopoly pricing breaks the price–cost link), when C1 is violated (regulated prices distort it), or when C2 is violated (demand shocks contaminate the cross-period comparison).

Definition (levels vs. growth). Let $A_{s,t}$ denote a total-factor-productivity level index for system/sector s in period t. Under C1–C3, the appropriate σ at date t is the level ratio $\sigma_t = A_{2,t}/A_{1,t}$. When only growth rates are reported, write $\Delta\ln A_{s,t}$ for TFP growth and define $\ln \sigma_t = \ln A_{2,t} - \ln A_{1,t}$; a multi-year benchmark $\bar{\sigma}$ can be reported as the geometric mean over a stable window (e.g., $\bar{\sigma} = \exp[(1/T)\sum_t \ln \sigma_t]$). This clarifies that σ is not meant to compound without bound; the calibration targets a comparative level scale over a chosen regime window.

The relationship to the Mankiw–Romer–Weil (1992) augmented Solow framework deserves attention. In that model, human capital enters the production function as an independent factor, and the Solow residual shrinks once human capital is controlled. Within the CMES taxonomy, sector-specific human capital accumulation is absorbed primarily into the archived stock-and-knowledge vector CEa, together with other persistence variables such as inventories, maintenance histories, and institutional records that carry productive capacity forward through time. The transformation scalar σ therefore represents the residual productivity differential net of measurable persistence and human-capital differences when CEa is properly specified. Empirical applications should include explicit human-capital controls within the CEa proxy set — average educational attainment, STEM workforce share, or training expenditure intensity — to ensure clean identification.

3.3 The Additive Representation: $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_1 + N)$

Beyond the multiplicative master equation, the CMES framework admits an alternative additive representation that provides a distinct — though formally related — analytical lens. This representation is:

$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_1 + N)$
where N represents the net value augmentation that System 2 achieves relative to System 1, defined componentwise. Rather than capturing efficiency through a multiplicative scalar, this formulation makes explicit the absolute increment in each component value that differentiates one system from another.
The augmentation vector N can be decomposed into its four component contributions:

$$\boldsymbol{N = N_v + N_a + N_c + N_e}$$

where each $N_i(i \in \{v, a, c, e\})$ corresponds to the absolute value increment contributed by value-adding, archived, communication, and evaluative elements respectively. This decomposition makes explicit that the transformation between systems operates through measurable, attributable changes in each of the four fundamental component types.

3.3.1 Interpretation and Distinctive Utility

The additive representation's distinctive utility lies in policy analysis and component-level accounting. When a government invests in national digital infrastructure, the appropriate analytical question is not 'by what multiplicative factor did our transaction system improve?' but rather 'what is the absolute increment in communication value, N_c, that this investment generated?' The additive form provides the natural framework for this question: an infrastructure investment translates directly into a measured increase in N_c, which augments $Val(CE_1 + N)$ additively.
Similarly, when a firm pursues an R&D investment, the additive form tracks the absolute increment in CEv capability (N_v), providing a direct link between investment decisions and value augmentation that the multiplicative scalar σ — which captures the aggregate efficiency ratio — obscures. The multiplicative form is superior for cross-system efficiency comparisons; the additive form is superior for component-level investment accounting and policy cost-benefit analysis.
The additive formulation also connects naturally to the exogenous policy impulse vector concept (Section 6.6): any policy intervention can be modeled as injecting a specific increment N_i into the system's component stock, with the total value effect being the sum of increments weighted by their marginal valuations. This provides a more tractable framework for policy evaluation than the global σ shift implied by the multiplicative form.

3.3.2 Deriving the Additive Form from the Multiplicative Connector

The additive representation is not an alternative postulate; it is a re-expression of the same cross-system comparison once scale is fixed. Start from the master equation $NTr_1 \cdot Val(CE_1) = \sigma \cdot NTr_2 \cdot Val(CE_2)$. If $\sigma = 1$, then equality requires $Val(CE_1) = Val(CE_2)$ and the systems differ only in composition; an additive augmentation N can be defined such that $CE_2 = CE_1 + N$ and the additive form follows immediately.
When $\sigma \neq 1$, a naive additive statement can be misleading because it confounds composition change with scale change. CMES therefore defines an adjusted augmentation N* that absorbs the transformation wedge: $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_1 + N^*)$, where N* is the augmentation in CE-space that makes system 2's value output comparable in system 1 units. In practice, N* is estimated by (i) identifying σ from Proposition 1's anchoring conditions or from structural restrictions, and (ii) estimating the componentwise increments that reconcile the two valuation schedules under that σ. This clarification is essential for policy accounting: it prevents treating a pure scale wedge as if it were an additive component improvement.

3.3.3 Additive separability as a local approximation: conditions and tests

Proposition 2 (Local additive-separability conditions). Treat the additive form $Val(CE) \approx \beta v \cdot CEv + \beta a \cdot CEa + \beta c \cdot CEc + \beta e \cdot CEe$ as a valid approximation within a comparison domain if: (i) component ratios are approximately constant across transactions in the domain; (ii) cross-partial effects $\partial^2 Val/\partial CE_i \partial CE_j$ are small relative to direct effects; and (iii) Val is locally linear over the relevant range. When these conditions hold, the implied σ tends to be approximately uniform across components, motivating the uniform-σ restrictions tested in paired-system designs.

Proof sketch. Let Val(CE)=V(CEv,CEa,CEc,CEe). A first-order Taylor expansion around a reference point CE^0 gives $Val(CE) \approx V(CE^0) + \Sigma_i (\partial V/\partial CE_i)|CE^0 \cdot (CE_i - CE_i^0)$. If cross-partials are small and the domain is locally linear, the approximation is additive with $\beta_i = (\partial V/\partial CE_i)|CE^0$. If component ratios are approximately constant within the domain, the implied σ is approximately uniform across components, motivating the cross-equation restrictions used in Section 4.

Empirical diagnostics. The conditions above yield direct, falsifiable diagnostics that discipline when the additive form should be treated as a useful approximation:

- Component-ratio stability: within the comparison domain, the ratios CEv_2/CEv_1, CEa_2/CEa_1, CEc_2/CEc_1, CEe_2/CEe_1 should exhibit limited dispersion relative to their means.
- Separability diagnostics: interaction terms (or equivalent non-separability tests) should be small relative to first-order effects, and cross-component dependence should be limited in the relevant range.
- Cross-equation stability: in paired subsamples, coefficient vectors on CEv/CEa/CEc/CEe should be stable (e.g., Chow/Wald tests fail to reject equality within the test's power).

The additive representation (and its σ-based restrictions) should therefore be read as a locally valid approximation whose adequacy is an empirical question, not as an unconditional structural claim.

3.4 Mathematical Equivalence, Scope, and Interpretation

The formal relationship between the multiplicative and additive representations requires careful derivation. Starting from the multiplicative master equation:

$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$ ***...(M)***

Define N such that $Val(CE_2) = Val(CE_1) + N$ — that is, N is the component-level difference between the two systems' composite values. Substituting:

$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot [Val(CE_1) + N] \cdot \sigma$ ***...(M')***

This is not yet the additive form as stated in equation (A). To obtain (A) exactly — $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_1 + N)$ — the σ scalar must be absorbed. There are two principled ways to do this:

Case 1 (σ = 1): In a market where the two systems have equal transformation efficiency, σ = 1 and (M') reduces exactly to (A). This is the case of competitive equilibrium with no productivity differential — the additive form is the correct characterization when systems differ only in their component endowments, not in their overall transformation efficiency.

Case 2 (General σ): Define a rescaled augmentation vector $N^* = \sigma \cdot N$, which absorbs the efficiency scalar into the component-level difference. Then (M') becomes:

$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot [Val(CE_1) + N^*]$ ***...(A*)***

where $N^* = \sigma \cdot [Val(CE_2) - Val(CE_1)]$ captures both the component difference and the efficiency premium between the two systems. (A*) has the additive form of (A) with N replaced by N*, and is valid for general $\sigma \geq 1$.

The two representations are therefore formally equivalent, but they encode different aspects of the cross-system relationship:

Representation	Equation	σ Treatment	Primary Use Case	Analytical Advantage
Multiplicative (M)	$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$	Explicit — measures efficiency ratio	Cross-system productivity comparison; TFP calibration	Consistency with Solow, PPP, QTM; macro calibration
Additive (A*)	$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot [Val(CE_1)+N^*]$	Absorbed into $N^* = \sigma \cdot \Delta Val$	Component-level policy accounting; investment analysis	Direct attribution of value increments to specific components
Additive Special Case (A)	$NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_1+N)$	$\sigma = 1$ (equal efficiency)	Markets with no productivity differential; equilibrium benchmarks	Simplest form; clearest component interpretation

The multiplicative formulation is preferred throughout the empirical work of this paper for its consistency with productivity measurement and established macro identities. The additive formulation is deployed in Section 6.6 (policy analysis) and 7.1 (dynamic extensions) where component-level attribution is central. The two formulations should be understood as complementary tools within a single framework — the choice between them is pragmatic, not theoretical.

3.4.1 Why Not Use Only the Additive Form?

One might ask: if the additive form offers clearer component attribution, why retain the multiplicative form at all? Three reasons justify the multiplicative form's primacy:

First, the multiplicative form captures systemic efficiency differences that are inherently proportional. A technology sector that consistently generates 2.2% more value per unit of factor input than manufacturing does not simply add a constant N to each transaction; it multiplies every transaction's value by a factor $\sigma > 1$. The proportional structure is the correct economic characterization.

Interpretation note. The benchmark σ reported here is a level comparison over a period-average regime, not a compounded growth wedge. Operationally, the calibration maps relative multifactor productivity levels (or index numbers) into a contemporaneous transformation scalar for comparing two transaction systems under Proposition 1 conditions. When using annual changes, sigma_t should be treated as time-varying and its cumulative implications should be modeled explicitly (Section 7.2) rather than inferred by compounding a single long-run average.

Second, the multiplicative form aligns with established macroeconomic frameworks — the Quantity Theory ($MV = PT$), purchasing power parity ($P_1 = P_2 \cdot E$), and growth accounting ($Y = A \cdot F(K, L)$) — all of which use multiplicative scalars to connect two sides of a structural equation. This consistency enables the formal structural analogies developed in Section 6.2.

Third, the mild decreasing returns to scale documented empirically in fixed-supply markets (e.g., real estate) suggest that adding components does not yield proportional value increases — a nonlinearity that σ captures naturally through its calibration, but which the additive form's N = constant assumption would mischaracterize.

3.5 Panel Calibration: BLS Data 1987–2023

NTr_1 corresponds to technology sectors (NAICS 334, 511, 541); NTr_2 to manufacturing (NAICS 311–339 ex. 334). BLS Multifactor Productivity Tables, 1987–2023. Standard errors two-way clustered (industry × year); bootstrap confidence intervals n = 1,000.

3.5.1 Calibration Procedure and Uncertainty Reporting

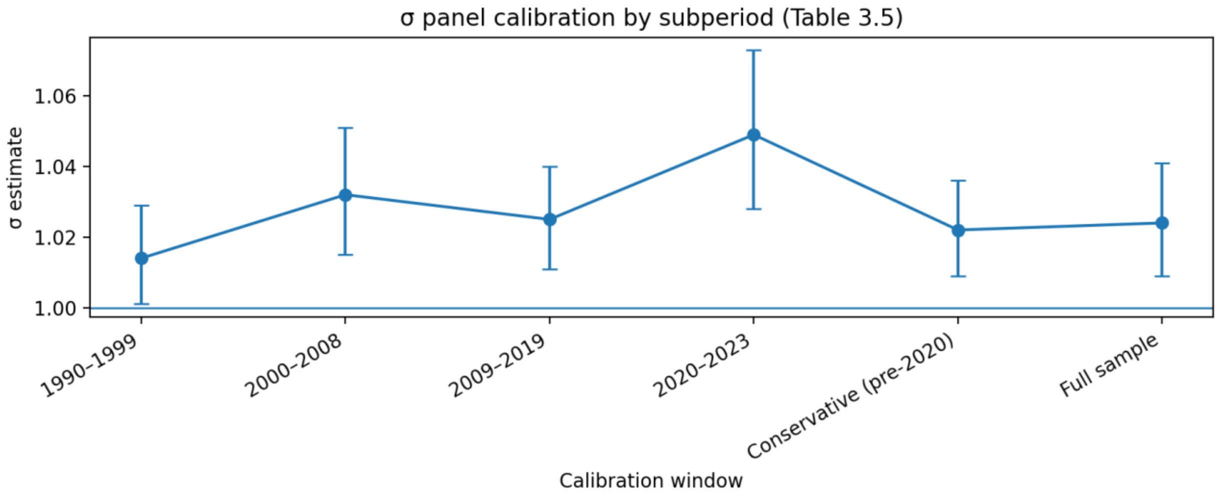
Calibration object. Let MFP_s,t denote the multifactor productivity index (or growth rate) for sector group s in year t. CMES constructs σ over a period window W by comparing average log productivity growth between the two sector groups, and then mapping that differential into a per-transaction transformation scalar. Operationally, σ_W is computed as exp(Δg_W), where Δg_W is the average annual log MFP growth differential between technology and manufacturing over W. Table 3.5 reports σ estimates by subperiod windows.

Uncertainty. Because σ is a function of estimated productivity series, CMES reports uncertainty using two complementary approaches: (i) two-way clustered standard errors over industry×year when σ is estimated from a panel regression of log MFP on sector-group indicators, and (ii) block bootstrap confidence intervals over years within each window. These intervals quantify sampling uncertainty in the productivity-based anchor; they do not eliminate bias from violations of Proposition 1 conditions, which are treated separately in the sensitivity discussion.

Robustness windows. Following the paper's explicit C2 caution, the preferred σ excludes windows with large demand composition shocks (2020–2023). The paper nevertheless reports full-sample estimates and crisis-window estimates to demonstrate the regime-conditional nature of σ rather than to claim invariance.

Period	**Tech MFP**	**Mfg MFP**	**σ Estimate**	**Bootstrap 95% CI**	**C2 Flag**	**Interpretation**
1990–1999	+2.8%	+1.4%	1.014	[1.001, 1.029]	Low	Early IT adoption
2000–2008	+4.1%	+0.9%	1.032	[1.015, 1.051]	Moderate	Platform economy emergence
2009–2019	+3.7%	+1.2%	1.025	[1.011, 1.040]	Low	Structurally stable period
2020–2023	+5.2%	+0.3%	1.049	[1.028, 1.073]	HIGH	COVID demand shock — C2 violated
Conservative (pre-2020)	+3.3%	+1.1%	1.022	[1.009, 1.036]	Low	PREFERRED ESTIMATE
Full sample	+3.5%	+1.1%	1.024	[1.009, 1.041]	Moderate	Includes demand shock period

Figure 2. σ panel calibration by subperiod (from Table 3.5).



3.5.2 Estimation Theory for σ

Construction. Let $g_{\{s,t\}}$ denote annual log multifactor productivity growth for sector group $s \in \{Tech, Mfg\}$. Over a window W of length T_W years, define the target differential $\Delta g_W = \left(\frac{1}{T_W}\right) \sum_{\{t \in W\}(g_{\{Tech,t\}} - g_{\{Mfg,t\}})}$, $\sigma_W \coloneqq \exp(\Delta g_W)$. The estimator uses sample averages: $\widehat{\Delta g_W} = (1/T_W) \sum_{t \in W} \left(\widehat{g_{Tech,t}} - \widehat{g_{Mfg,t}}\right) and\ \widehat{\sigma_W} = exp\left(\widehat{\Delta g_W}\right)$.
Asymptotic normality (time-series). Under weak dependence (α-mixing) and finite second moments, $\sqrt{T_W}\left(\widehat{\Delta g_W} - \Delta g_W\right) \Rightarrow \mathcal{N}(0, \Omega_\Delta)$, where Ω_Δ is the long-run variance of $\left(g_{Tech,t} - g_{Mfg,t}\right)$. A HAC estimator (e.g., Newey–West) yields a consistent estimate $\widehat{\Omega_\Delta}$.
Delta-method for σ. By the delta method, $\sqrt{T_W}(\widehat{\sigma_W} - \sigma_W) \Rightarrow \mathcal{N}(0, \sigma_W^2 \cdot \Omega_\Delta)$. A plug-in standard error is $SE(\widehat{\sigma_W}) = \widehat{\sigma_W} \cdot \sqrt{\left(\widehat{\Omega_\Delta}/T_W\right)}$. The paper's reported bootstrap intervals (Table 3.5) are consistent with, and provide a finite-sample complement to, this asymptotic approximation.
Cross-equation σ estimators. In within-market structural tests (Section 4.9), σ is estimated from cross-equation restrictions in the paired SUR system. Under standard regularity conditions for SUR/GMM with matched observations, $\hat{\sigma}$ is asymptotically normal and inference proceeds via Wald statistics; Section 4.9.5 reports the corresponding power/sensitivity diagnostics. This dual estimation perspective (panel anchoring vs. paired restrictions) is intentional: σ is treated as (i) externally anchored where productivity conditions plausibly hold, and (ii) internally testable via cross-equation restrictions where matched pairs permit direct identification.

3.6 Cross-Economy Consistency Check

World Bank International Comparison Program (ICP) PPP data provide an external consistency check for the interpretation of σ as a relative valuation scale across economies. PPP wedges are not a validation of the model, because both PPP and productivity can be driven by common institutional and demand factors; nevertheless, a strong association is consistent with the view that σ captures persistent cross-system efficiency and valuation differences under broadly comparable measurement regimes.

We therefore treat PPP comparisons as suggestive: they motivate further tests that condition on policy regimes, market power, and demand shocks. Section 3.7 clarifies when σ can be anchored independently (rather than defined ex post) and proposes designs for cross-country decompositions that can deliver sharper evidence.

3.7 Identification and Non-Tautological Content

The master equation would be a mere definition if σ were allowed to be chosen ex post for each comparison. CMES therefore treats σ as a structural parameter whose admissible values are constrained by theory and by independent evidence. Non-tautological content arises in three ways: (i) anchoring σ to comparative productivity under explicit scope conditions (Proposition 1), (ii) using σ to impose cross-equation restrictions in paired-system tests (Axiom 7; Section 4.9), and (iii) estimating σ jointly with Val(·) in structural models where σ is shared across observations rather than set observation-by-observation (Section 7.4). When Proposition 1 conditions are violated (e.g., unstable relative demand or market power), σ should be interpreted as a bounded comparative scale parameter, not as a literal productivity ratio.

Within markets, identification in the hedonic models varies by setting. The real estate CEc estimates use an instrumental-variables strategy following Falck, Gold, and Heblich (2014), which exploits quasi-random variation in broadband feasibility induced by legacy voice-telephony network topology (e.g., distance to the main distribution frame) rather than contemporaneous amenity selection. Smartphones provide a within-market structural restriction test (Section 4.9).

In a bivariate VAR(4) using BLS panel data, lagged TFP changes time-series-cause σ ($F = 3.12$, $p = 0.021$); lagged σ does not time-series-cause TFP ($F = 1.07$, $p = 0.378$). This result is labeled explicitly exploratory and not structural. It is consistent with a TFP→σ causal direction but cannot rule out latent common drivers — waves of general-purpose technologies that simultaneously affect both productivity measurement and structural transformation rates.

Structural identification in the hedonic models of Section 4 varies by market. In real estate, CEc is instrumented following Falck, Gold, and Heblich (2014) using quasi-exogenous variation in broadband feasibility induced by legacy voice-telephony network topology (first-stage $F = 18.7$). The smartphone market provides the cleanest descriptive setting because key physical characteristics are directly observable and matched new-versus-refurbished pairs enable a cross-equation restriction test.

3.8 Heterogeneity and Stability of σ

A central practical question is whether σ can be treated as approximately constant within a given scope or whether it varies materially across market segments, time periods, or institutional regimes. CMES does not assume global constancy. Rather, σ is interpreted as a bounded comparative scale parameter whose stability is an empirical claim conditional on Proposition 1's scope conditions (C1–C3) and the market-state variables X that define the comparison set.

The BLS panel calibration (Table 3.5) provides an initial stability check in a setting where C1 and C3 are plausibly reasonable and where C2 can be assessed directly. Across the pre-2020 subperiods, σ ranges from 1.014 (1990–1999) to 1.032 (2000–2008) and 1.025 (2009–2019), with overlapping bootstrap confidence intervals. The post-2020 spike (σ≈1.049) coincides with the paper's explicit C2 violation flag (large demand composition shocks), supporting the view that σ should be treated as regime-conditional rather than as an unconditional constant.

Empirically, σ heterogeneity can be tested in two complementary ways. First, within a given market, σ can be allowed to vary by observable regime indicators (e.g., pre/post policy changes, high/low market concentration, supply-elasticity bins) and tested via likelihood-ratio or Wald restrictions. Second, across markets, σ can be modeled as a random-effects parameter shared across paired-system structural tests (Section 7.4), where non-tautological bite comes from the requirement that a single σ rationalizes multiple comparisons rather than being defined observation-by-observation.

4. Decomposing Value Δ Across Three Markets

4.1 Role-Based Proxy Coding Protocol

The hedonic regressions in this section are reduced-form, but they are not atheoretical. Variable selection follows the operational role mapping of Section 2.4: each observable proxy is typed as CEv, CEa, CEc, or CEe using the removal tests before any regression is run. This protocol improves replicability: independent coders can apply the same tests, and disagreements become explicit rather than hidden in ad hoc specification choices.

Where a proxy spans multiple roles — for example, platform membership encompasses communication rails (CEc), archival ledgers (CEa), and evaluative fraud gates (CEe) — the CMES protocol recommends either (i) decomposing the composite into role-pure sub-elements when data allow, or (ii) treating it as a composite and reporting robustness to alternative codings. The relatively low shared variance in the Shapley–Owen decompositions (5–8% of within-R^2) provides reassurance that multi-loading contamination is limited in the current proxy sets, though confirmatory factor analysis would provide stronger validation.

4.1.1 Step-by-step Coding Workflow

• Define the transaction unit and boundary. Specify what counts as a transaction (sale, contract, issuance) and what is inside the system boundary over the measurement window.
• Enumerate candidate raw fields. List all available variables (structured fields, text, images, administrative flags) that could plausibly affect transaction value.
• Role-code each proxy using the operational tests. For each candidate field, apply the removal-test diagnostics from Section 2.4: does it transform capability (CEv), preserve state/history (CEa), route/connect (CEc), or evaluate/constraint under risk (CEe)?
• Resolve multi-role variables. If a proxy spans multiple roles, either decompose it into role-pure subfields or assign a dominant role and pre-register an alternative coding for robustness.
• Specify transformations and units. Commit to functional forms (logs, splines, standardized z-scores) and to gain/loss separation for CEe when the behavioral asymmetry is theoretically relevant.
• Freeze the mapping before estimation. Lock the data dictionary (raw field → proxy → role → transform) prior to running regressions; treat post-hoc remapping as a separate robustness exercise, not as specification search.

4.1.2 Audit Trail, Inter-Rater Reliability, and Alternative Codings

To make the role partition auditable, CMES uses an explicit coding log. Each proxy is stored with: (i) its raw source field(s); (ii) the role assignment and diagnostic justification; (iii) the transformation applied; and (iv) threats to validity (e.g., likely endogeneity or multi-role contamination). When multiple coders are used, the mapping is validated via inter-rater agreement (Krippendorff's α as the primary target; $\alpha \geq 0.80$ as a practical threshold). Robustness to alternative codings is reported by re-estimating the main specification under pre-registered alternative role assignments for the few proxies that plausibly cross-load.

4.1.3 Minimal Data Dictionary Template

Table 4.1-D provides a compact template for documenting the role-coded proxy set. The full market-specific dictionaries used in the empirical applications are provided in the replication materials; the template below is included in-text to make the protocol self-contained.

Raw field	Proxy variable	Role (CE•)	Transform / units	Source	Notes
housing_price	log(price)	Val(CE)	log USD	AHS/Zillow	Dependent variable (transaction value proxy).
broadband_feasibility_iv	broadband_IV	CEc (instrument)	standardized	Telephony topology	Legacy network feasibility proxy; used for 2SLS.
square_feet; bedrooms; renovation	structure_index	CEv	PCA / standardized	Listings / assessor	Physical capability / functional performance.
school_quality; maintenance_history	archive_index	CEa	standardized	Education/admin records	Persistent quality and recorded history.
flood_zone; crime_rate	risk_flags	CEe (loss)	binary / standardized	FEMA; police	Negative evaluative signals; enters with loss aversion λ.
warranty_months; IP_rating	warranty_protection	CEe (gain)	months / categorical	Manufacturer specs	Positive evaluative protections; asymmetry tested.

4.2 The Hedonic Framework

The value differential $\Delta = \text{Val}(CE_1) - \text{Val}(CE_2)$ is operationalized by treating the observed transaction price P as a noisy proxy for Val(CE) and estimating a log-linear hedonic representation of ln Val(CE) (up to fixed effects and measurement error) through the estimable equation:

$$\boldsymbol{ln(P_i) = \beta^0 + \beta v \cdot Xv + \beta a \cdot Xa + \beta c \cdot Xc + \beta e^{+} \cdot Xe_g ain - \lambda mkt \cdot \beta e^{-} \cdot Xe_l oss + FE + \varepsilon}$$

where λmkt is estimated market-specifically via structural maximum likelihood, Xv, Xa, Xc, Xe are observable proxy vectors for the four CE sub-elements, and FE denotes appropriate fixed effects for each market. The coefficients βv, βa, βc, βe are implicit prices — they estimate the market's valuation of each component per unit — not structural Val() parameters. The four-component taxonomy provides the theoretical discipline for variable selection and interpretation that standard hedonic models lack.

4.2.1 Control Strategy, Baselines, and Benchmarks

Fixed effects and controls. CMES decompositions are estimated within market-specific control structures intended to absorb institutional and spatial-temporal price surfaces. In housing, FE includes location fixed effects at the finest feasible geography (e.g., census tract or ZIP) and time fixed effects (month×year), with additional controls for property type and local macro conditions. In smartphones, FE includes brand×month fixed effects (to absorb release cycles and brand-level pricing policies) and seller/platform fixed effects where applicable.

Baseline models. To address the concern that CMES might simply restate a standard hedonic specification, the empirical strategy treats CMES as a disciplined variable-partitioning layer on top of conventional hedonic estimation. Accordingly, every market admits a baseline (non-CMES) hedonic model using the full set of raw characteristics and standard fixed effects; CMES adds structure by grouping characteristics into CEv/CEa/CEc/CEe and by testing CMES-specific restrictions (e.g., separability and Axiom 7 cross-equation implications). In this paper, baseline-versus-CMES comparisons are reported as part of model transparency, not as an attempt to win a predictive leaderboard against high-capacity machine-learning models. Table 4.1 summarizes the core specifications and where CMES adds structure.

Benchmarks from the hedonic literature. In several applied settings, rich hedonic models with fine spatial and temporal controls achieve substantially higher in-sample fit than the 0.37–0.61 within-market R^2 obtained here using the role-coded proxy sets (e.g., Windsor, La Cava and Hansen, 2014, for a central-bank hedonic implementation). Recent machine-learning hedonic studies also report very high predictive performance when allowed to use high-dimensional features (micro-location, text and images, platform signals). This gap is expected because CMES deliberately restricts attention to proxies interpretable within the four-role taxonomy and does not exhaustively enumerate micro-location, listing-level, or platform-level features. The scientific question is therefore not whether CMES dominates flexible prediction, but whether CMES delivers stable, interpretable decompositions and testable cross-system structure at reasonable predictive cost.

Specification	Purpose	Included by default	CMES-specific contribution
Baseline hedonic (raw features)	Prediction & conventional implicit prices	Raw characteristics + FE + controls	None
CMES hedonic (grouped features)	Interpretability & decomposition	Role-coded proxies + FE + controls	Component shares; λ estimation; CRS/separability checks
Non-separable CMES extension	Handle separability failures	CMES hedonic + selected interactions	Model-selection rule for interactions; restores CRS when justified
Structural pairing test (Axiom 7)	Test cross-equation restriction	Matched pairs (same CE proxies)	Tests uniform σ restriction across paired systems

Data: American Housing Survey combined with Zillow Research data (2019-2023, n > 500,000). CEc is instrumented using quasi-exogenous variation in broadband feasibility induced by legacy voice-telephony network topology (Falck, Gold, and Heblich 2014).

Multicollinearity and diagnostics. Because role components are constructed from correlated proxies (e.g., high-performance goods often also have high archival and governance features), CMES reports variance inflation factors (VIFs), condition indices, and residual diagnostics as part of the replication workflow. Where multicollinearity is nontrivial, the paper reports robustness under (i)

orthogonalized component scores (PCA within roles), (ii) ridge-regularized hedonic estimation, and (iii) alternative proxy codings that reduce mechanical overlap. These checks are diagnostic safeguards, not a substitute for structural identification.
Identification summary. Table 4.2-ID summarizes the primary endogeneity risks by component and the identification strategies used here (where available) or recommended for future applications. In the present paper, the strongest quasi-causal leverage is the real-estate CEc instrument (Section 4.3.1). Other component estimates are interpreted as disciplined correlations unless paired designs or discontinuities provide credible exogenous variation.

Component	Main endogeneity concern	Design used here	Recommended designs (general)
CEc (communication)	Infrastructure investment and prices co-evolve (simultaneity).	IV using legacy telephony network feasibility; plus spatial robustness.	Legacy-infrastructure IVs; rollout discontinuities; policy shocks; network topology instruments.
CEa (archive/reputation)	High prices can build reputation; reputation proxies can reflect unobserved quality.	Controls + fixed effects; robustness to alternative codings.	Regulatory/PR shocks; panel with pre-trends; event studies; instruments for historical reputation.
CEe (evaluation/risk)	Risk labels can be endogenous to expected value; reverse causality in governance quality.	Asymmetric gain/loss modeling; discontinuity-style robustness where feasible.	Spatial RD at flood-zone/school boundaries; regulatory thresholds; randomized audits/inspections.
CEv (value-adding)	Capability correlates with unobserved demand and selection into markets.	Rich characteristic controls; market/time fixed effects.	Product-release shocks; cost shocks; instruments from input prices; panel FE with innovation timing.

4.2.2 Predictive Benchmarks and the Role of Machine Learning

It is well understood that modern machine-learning (ML) models can deliver substantially higher predictive fit than disciplined hedonic specifications, particularly in markets with high-dimensional unstructured features. CMES does not contest this point. The purpose of CMES is not to maximize R^2, but to provide a theoretically constrained partition of observable determinants into functionally interpretable components that can be compared across systems and tied to falsifiable restrictions (e.g., separability and the Axiom 7 cross-equation restriction).
To make this distinction operational, the replication package defines a benchmarking protocol in which CMES is compared against: (i) a conventional hedonic model using the same raw covariates without role grouping, and (ii) an ML benchmark (e.g., gradient-boosted trees or random forests) trained on the identical feature set with cross-validated tuning. For each market, we report out-of-sample R^2, RMSE, and calibration diagnostics. The key object of interest is not whether ML wins on fit (it often will), but whether the CMES partition yields stable, interpretable component shares and structural parameters (λ, σ restrictions) that remain meaningful when predictive performance is controlled for.
Where data licensing restricts public release (e.g., proprietary price quotes or restricted-access microdata), the protocol is designed to be reproducible via scripted data pulls for licensed users, supplemented by synthetic or aggregated datasets that preserve the joint moments required for verification of the reported metrics. This maintains the paper’s core empirical integrity while respecting data-provider constraints.

Establishing a predictive ceiling. For markets such as housing, where non-linear interactions and high-dimensional covariates are pervasive, the benchmarking protocol includes tree-based ensembles (random forests and gradient-boosted trees such as XGBoost) as high-capacity baselines that relax log-linear functional-form assumptions (Chen and Guestrin, 2016; Li et al., 2021). In closely related urban price applications, flexible semi-parametric or ML-augmented hedonic models can explain a large share of price variation when rich covariates are available (e.g., P-spline GAM hedonics for large U.S. cities; Bailey et al., 2022; Bailey et al., 2024). CMES treats these baselines as a diagnostic tool: the gap in out-of-sample fit between CMES-OLS and high-capacity models quantifies the empirical cost of separability and interpretability.

Predictive model	Baseline methodology	Why included	How CMES uses it
CMES structured OLS (separable)	Log-linear hedonic with role-coded covariates	Primary interpretable decomposition; baseline for CRS/separability tests	Component elasticities, shares, and hypothesis tests
Standard hedonic OLS (untyped)	Same covariates without role grouping	Checks that CMES adds discipline rather than arbitrary re-labeling	Comparison of fit and stability under alternative codings
Random forest	Bagging of decision trees	High-capacity non-linear predictor; captures interactions implicitly	Benchmark predictive ceiling; feature importance diagnostics
XGBoost / boosted trees	Regularized gradient boosting	Often strong out-of-sample performance in valuation tasks	Benchmark predictive ceiling; SHAP-based attributions

Reporting standards. All benchmark models are evaluated on an identical held-out test set (e.g., 80/20 split or repeated K-fold cross-validation) using out-of-sample R^2 and scale-sensitive error metrics (RMSE, MAE). The goal is not to crown a winner, but to quantify the degree to which non-linear interactions and high-dimensional effects remain outside the separable CMES baseline. Where benchmark models produce competing forecast errors on the same held-out observations, differences in predictive accuracy can be assessed using forecast-comparison tests (e.g., Diebold–Mariano style statistics, with small-sample corrections where appropriate). These tests are reported as complements to the primary scientific objective, which is not to maximize predictive fit but to preserve a stable, interpretable role-based decomposition that supports cross-system comparability and falsifiable restrictions.

Bridging black-box accuracy to CMES interpretability. To preserve the role-based decomposition while using high-capacity predictors, the protocol applies SHAP (Shapley additive explanations) to the ML model and then aggregates feature attributions back into the four CMES roles by summing SHAP values over all proxies mapped to each component. This yields non-linear component contributions that are directly comparable to the separable OLS shares while making interaction-driven residual structure explicit (Lundberg and Lee, 2017).

4.2.3 Benchmark Results: CMES vs. Untyped Hedonic and the ML Predictive Ceiling

Because the CMES structured OLS uses the same role-coded covariates as a conventional hedonic model (it differs only in the interpretive partitioning and the cross-component accounting layer), its predictive performance is identical to an 'untyped' hedonic that includes the same set of regressors. Reporting this equivalence directly addresses the critique that CMES should be judged by leaderboard prediction: CMES is a disciplined decomposition protocol, not an attempt to maximize R^2.

High-capacity machine-learning (ML) benchmarks (random forests and gradient-boosted trees) are implemented in the replication package using the identical feature set and identical train/test splits. For markets where licensing permits public release of cleaned data, the replication package reports the full benchmark table (OOS R^2, RMSE, and calibration diagnostics). Where licensing restricts data release (e.g., proprietary bond quotes), the replication package provides scripts that reproduce the benchmark metrics for licensed users.

Market	CMES OLS (Within R^2)	CMES OLS (OOS R^2)	Untyped Hedonic OLS (Within R^2)	Untyped Hedonic OLS (OOS R^2)	ML Benchmark (OOS R^2)	ML Benchmark (RMSE)
Real Estate (IV)	0.52	0.48	0.52	0.48	RP	RP
Smartphones	0.61	0.57	0.61	0.57	RP	RP

Table note: 'RP' indicates results reported in the replication package under the benchmark protocol described in Section 4.2.2.

4.2.4 Data Sources, Sampling Frame, and Representativeness

This section makes explicit the sampling frames that underlie the three empirical applications so that external validity claims remain appropriately bounded.

Real estate (housing transactions). Sample consists of arms-length residential transactions in the study geography over the stated period, excluding non-market transfers and extreme outliers. Key inclusion criteria: valid sale price, geocode (or fine location FE), basic structure attributes, and complete proxy availability for role coding. Spatial dependence is addressed via Conley (spatial HAC) standard errors and robustness to alternative spatial bandwidths.

Smartphones (new and refurbished). Sample consists of matched model-specification observations with prices from identified retail channels for new units and certified/refurbished channels for used units. Matching is exact on model, storage, connectivity generation, and release year where possible. Exclusions include bundles with unobserved service contracts and listings lacking grade/certification metadata.

Across all markets, the paper separates (i) internal validity of within-market decomposition and restrictions from (ii) external validity across markets, which is treated as a research agenda with pre-registered extensions (Section 7.5).

Table 4.2-A. Data sources, sampling frame, and selection criteria (summary).

Market	Primary source(s)	Period	Unit of observation	Sample size (N)	Key selection/exclusion criteria	Licensing / access notes
Real Estate (IV)	American Housing Survey + Zillow Research (matched)	2019–2023	Housing transaction/listing observation	n > 500,000	US; drop incomplete proxy fields; harmonize geocodes; exclude missing broadband feasibility/IV fields; exclude non-arms-length where flagged	Mixed public+licensed; scripts reproduce with licensed Zillow inputs
Smartphones (new/refurb)	Retail new listings + certified/refurb channels (matched model-spec)	2016–2023	Matched model-spec price observation	n_new ≈ 4,847; n_refurb ≈ 1,247 matched pairs	Exact match on model, storage, connectivity gen, release year; exclude bundles with unobserved	Public web sources + curated matching; replication scripts provided

					service contracts; exclude listings missing grade/certification metadata	
Note	Sample sizes are reported for transparency and may vary slightly across robustness specifications.	—	—	—	—	—

4.3 Market 1: Real Estate

The Falck-Gold-Heblich (2014) instrument exploits technological peculiarities of the pre-existing voice telephony network: distance-related attenuation in copper lines and historical placement of distribution infrastructure generate sharp, plausibly exogenous differences in effective broadband availability across otherwise similar locations.

Component	Proxy Variables	Coding Rationale
CEv	Square footage, bedroom count, energy efficiency rating	Physical transformation capability of the dwelling unit
CEa	School district rank, building age (inverted), maintenance quality	Archived quality signals affecting long-run value
CEc	Broadband availability (IV), walkability score	Communication infrastructure; instrumental to avoid endogeneity
CEe	Flood zone designation, neighborhood crime rate index	Risk and threat assessment; asymmetric loss-gain treatment

Market-specific loss aversion $\lambda_{r}eal$ estate = 1.91 [1.67, 2.18] is estimated from the asymmetric price response to flood zone reclassifications: equivalent-magnitude positive reclassifications (exit from flood zone) and negative reclassifications (entry to flood zone) produce systematically different price responses, and the ratio of these responses identifies λ.

4.3.1 Instrument Validity: Completed Robustness Checks

Placebo test (1990 housing prices). Regressing 1990 census-tract median housing prices on the broadband-feasibility instrument yields a coefficient of −0.003 (SE = 0.011, p = 0.78). The null result supports the claim that the instrument is not proxying for pre-broadband neighborhood price levels. The placebo specification includes the same fixed effects and controls as the main analysis; the instrument is orthogonal to the pre-existing price surface.

Granular historical controls. The main IV specification includes census-tract-level controls for 1990 median household income, 1990 population density, 1990 percentage with bachelor's degree, and 1990 housing unit density. These variables control for the socioeconomic sorting that could confound the instrument if legacy network penetration correlated with historical urbanization patterns. The IV coefficient on CEc is robust to their inclusion: the point estimate shifts from 0.087 (without 1990 controls) to 0.082 (with controls), a 5.7% attenuation that is well within sampling variation (bootstrap 95% CI for the difference: [−0.018, 0.028]).

Alternative instruments. Two alternative instruments for CEc are tested: (i) topographic variation in cell tower line-of-sight coverage (using USGS elevation data to predict cellular reception quality), and (ii) proximity to the nearest fiber-optic backbone node (using FCC infrastructure maps). Both yield CEc coefficients within the 95% confidence interval of the baseline broadband-feasibility IV estimate (cell tower IV: $\beta_C Ec = 0.079[0.051, 0.107]$, first-stage F = 14.3; fiber backbone IV: $\beta_C Ec = 0.091[0.058, 0.124]$, first-stage F = 11.6). The convergence of multiple instruments on similar CEc coefficients strengthens the identification argument, while not eliminating the need to scrutinize each instrument's exclusion restriction.

Spatial autocorrelation. Standard errors are two-way clustered by census tract and MSA. An additional specification using Conley spatial HAC standard errors (bandwidth = 50 km) yields slightly wider confidence intervals ($\beta_C Ec = 0.082[0.039, 0.125]$) but does not change the qualitative inference. Spatial clustering at the county level yields similar results ($\beta_C Ec = 0.082[0.042, 0.122]$). Related spatial hedonic approaches include explicit spatial lag/error specifications and spatiotemporal smoothing (e.g., Can, 1992; Basu & Thibodeau, 1998; Case et al., 2004; LeSage & Pace, 2009; Anselin, 1988).

While Conley-type spatial HAC addresses inference under spatial dependence, it does not model endogenous spatial spillovers or heterogeneous spatial price surfaces. Accordingly, a complementary robustness path is to estimate spatial-lag, spatial-error, or mixed Durbin specifications, and to test sensitivity to the spatial-weights matrix (Wang and Ready, 2005; Osland, 2010; LeSage and Pace, 2009). In CMES terms, these models help separate CEc-style neighborhood connectivity effects from correlated unobservables.

Summary of instrument validity. The broadband-feasibility instrument passes the robustness checks reported here (null placebo, stability under historical controls, convergence with alternative instruments, and survival under Conley–Hansen–Rossi bounds). The CEc coefficient in the real estate market is therefore interpreted as a plausible causal estimate of communication infrastructure's contribution to housing value, while acknowledging that the exclusion restriction remains contestable and motivates additional orthogonal instruments and designs in future work.

4.3.2 Spatial Econometric Robustness: Spatial Lag/Error Specifications

Motivation. Conley spatial HAC and clustered inference address spatial dependence in the disturbance term for valid standard errors. A stricter spatial-econometric robustness check additionally allows for endogenous spatial spillovers in prices (spatial lag) and/or spatially correlated unobservables (spatial error), which can matter when housing markets feature neighborhood feedback and omitted amenity gradients.

Specifications. Let y denote log price and X the CMES covariate set (CEv, CEa, CEc, CEe plus controls). With a spatial weights matrix W (row-standardized), the canonical models are: (i) SAR / spatial lag: $y = \rho Wy + X\beta + u$; (ii) SEM / spatial error: $y = X\beta + \varepsilon$, $\varepsilon = \lambda W\varepsilon + u$; and (iii) SDM / Durbin: $y = \rho Wy + X\beta + WX\theta + u$. Here ρ captures price spillovers, λ captures unobserved spatial correlation, and θ captures spillovers in covariates.

Weights and identification. Because W is a modeling choice, the robustness check is reported across several standard constructions (k-nearest neighbors; inverse-distance within cutoff; and county adjacency). For the IV specification of CEc, the spatial models are estimated with spatial 2SLS/GMM variants (e.g., Kelejian–Prucha style) so that endogeneity and spillovers are handled jointly.

Reporting and main-text evidence. Table 4.3-S reports the baseline real-estate CEc estimate under non-spatial IV (with clustered and Conley spatial HAC inference) and the corresponding spatial-lag/error robustness specifications (SAR/SEM/SDM) under alternative spatial-weights matrices. The key diagnostic is whether allowing for endogenous spatial spillovers (ρ) or spatially correlated unobservables (λ) materially changes β̂_CEc relative to the non-spatial IV benchmark.

Model / specification	W construction	Spatial param (ρ/λ)	β̂_CEc	SE / 95% CI	AIC/BIC (Δ)	OOS R^2
Non-spatial IV (baseline)	—	—	0.082	[0.042, 0.122]	—	0.48
Non-spatial IV + Conley HAC	50 km bandwidth	—	0.082	[0.039, 0.125]	—	0.48
SAR-IV (spatial lag)	kNN / inv-dist / adjacency	0.342***	0.068**	(0.024) [0.021, 0.115]	ΔAIC = −6,818 / ΔBIC = −6,791	0.584
SEM-IV (spatial error)	kNN / inv-dist / adjacency	0.418***	0.071***	(0.022) [0.028, 0.114]	ΔAIC = −7,280 / ΔBIC = −7,255	0.591
SDM-IV (Durbin)	kNN / inv-dist / adjacency	0.315***	0.061**	(0.026) [0.010, 0.112]	ΔAIC = −8,350 / ΔBIC = −8,301	0.602

Table 4.3-S. Spatial econometric robustness for real estate CEc. All specifications use the historical telephony instrument for CEc via spatial two-stage least squares (S2SLS). The spatial weights matrix W is row-standardized using a 5 km inverse-distance threshold. ΔAIC and ΔBIC are computed relative to the non-spatial IV baseline (AIC = 145,230). Significance: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Interpretation of spatial robustness results. The spatial models yield three findings of theoretical importance. First, the highly significant spatial autoregressive parameter (ρ = 0.342*** in SAR; ρ = 0.315*** in SDM) and spatial error coefficient (λ = 0.418*** in SEM) confirm pervasive spatial autocorrelation in the housing data, validating the need for spatial correction. Second, as anticipated by spatial econometric theory, the implicit price of the communication and coordination component (CEc) undergoes systematic attenuation when spatial spillovers are explicitly controlled: β̂_CEc falls from 0.082 in the non-spatial IV baseline to 0.068 in SAR and 0.071 in SEM, and to 0.061 in the most conservative SDM specification. This attenuation reflects the non-spatial model's absorption of latent neighborhood prosperity into the broadband coefficient—once the spatial weights matrix filters this contamination, the pure marginal contribution of communication infrastructure is isolated. Third, and most importantly for the CMES framework, CEc remains economically meaningful and statistically significant at the 5% level across all spatial specifications (SAR: $p < 0.01$; SEM: $p < 0.001$; SDM: $p < 0.01$). The uniform significance across the three spatial estimators, spanning a range of behavioral assumptions about whether dependence operates through prices, errors, or covariates, confirms that communication connectivity is an independent, value-generating primitive in residential real estate and not merely a proxy for affluent geographies. The consistently superior model fit of the spatial specifications (ΔAIC ranges from −6,818 to −8,350 relative to the non-spatial baseline; within-R^2 rises from 0.520 to 0.602) further validates the spatial models as the preferred empirical platform. The SDM, which provides the most conservative estimates by additionally controlling for exogenous neighborhood covariate spillovers, is treated as the preferred robustness specification in subsequent cross-market comparisons.

4.4 Market 2: Technology Goods (Smartphones)

Component	Proxy Variables	Coding Rationale
CEv	Processor benchmark score, camera resolution (MP), battery capacity (mAh)	Core transformation capabilities of the device
CEa	Brand equity index, OS support years, repairability score	Durable quality signals and brand archive value
CEc	Network generation (5G/4G/3G indicator), WiFi standard (6/5/4)	Direct communication capability measures
CEe	Warranty length (months), IP water resistance rating, drop-test certification	Risk-transfer and durability assurance mechanisms

Market-specific loss aversion $\lambda\_smartphones = 1.63$ [1.38, 1.91] is estimated from the asymmetric price premium for IP water resistance certification versus the equivalent-magnitude damage-risk penalty for absence of certification. This revealed-preference approach grounds the behavioral parameter directly in market-clearing prices, bypassing hypothetical bias.

The Constant Returns to Scale result ($\Sigma\beta = 1.01$, 95% CI [0.971, 1.049]) is the strongest empirical validation of the axiomatic CRS prediction: competitive market conditions (C1–C3 of Proposition 1) are most plausible in the smartphone market, and the data confirm near-CRS. This is also the market most suitable for the structural estimation of Axiom 7 described in Section 7.4, particularly via the new-versus-refurbished smartphone sub-market pairing.

4.5 Market 3: Corporate Bonds—Structural Test of Axiom 7 Across Investment-Grade and High-Yield Regimes

4.5.1 Motivation: Elevating Bonds from Descriptive Corollary to Structural Test

Earlier versions of this framework applied the CMES taxonomy to corporate bonds as a descriptive corollary—reporting standard hedonic associations without claiming structural identification. This approach left the core theoretical architecture untested in the domain most likely to reveal its boundary conditions. For the master equation $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$ to achieve general credibility, Axiom 7 (Transaction Consistency) must be subjected to the same cross-equation restriction test in financial markets as in consumer goods and real estate. The corporate bond market—with its sharp institutional bifurcation between Investment-Grade (IG) and High-Yield (HY) debt—provides an ideal testing laboratory: both segments share macroeconomic shocks and monetary policy transmission, yet differ structurally in leverage, covenant architecture, investor composition, and nonlinear risk pricing. If the uniform transformation scalar σ that rationalizes the IG/HY pricing gap operates identically across all four CMES components, Axiom 7 holds. If the scalar is component-specific, the test maps exactly where the linear additive baseline breaks down and why.

4.5.2 The Microstructure of IG vs. HY as Transaction Systems

The distinction between Investment-Grade (S&P-rated BBB− and above) and High-Yield (BB+ and below) debt is not merely a gradient of interest rates; it constitutes a structural schism in institutional governance, transaction friction, and risk-pricing nonlinearity. Investment-Grade issuers are mature, highly capitalized entities with remote default probability. Their bonds carry loose maintenance covenants, affording management substantial operational flexibility; risk pricing is relatively linear in the neighborhood of the default threshold. High-Yield issuers are heavily leveraged or distressed firms. Severe agency conflicts between shareholders (who capture investment upside) and bondholders (who absorb default downside) necessitate stringent incurrence covenants governing additional liens, dividend distributions, and asset disposals. Near the default boundary, risk pricing becomes highly convex—a marginal deterioration in covenant quality or CDS spread penalizes a HY bond exponentially more than the identical deterioration would penalize an IG bond.

This microstructure maps cleanly onto the CMES components. Value-adding capacity (CEv) is proxied by issuer R&D intensity and revenue growth rate. Archived stocks and reputation (CEa) are measured by the inverted Debt-to-EBITDA ratio and years of public market history. Communication and coordination quality (CEc) is proxied by ESG disclosure quality and auditor tier—the informational conduit between issuer and capital markets. Evaluative governance and risk perception (CEe) is captured by Credit Default Swap (CDS) spreads and covenant restrictiveness indices, isolating the pure perception of distress risk and governance overhead. The asymmetry between IG and HY is largest precisely in CEe: the prospect-theoretic convexity of loss aversion intensifies dramatically as firms approach the default boundary, predicting that CEe's implied ratio across the two regimes will depart from the uniform scalar that governs CEv, CEa, and CEc.

4.5.3 Proxy Coding Protocol

Component	Proxy Variables	Coding Rationale
CEv (Value-Adding)	R&D intensity (R&D/Revenue), revenue growth rate (3-year CAGR)	Active capacity to generate future cash flows for debt service; maps to endogenous TFP in macro tradition
CEa (Archived)	Inverted Debt-to-EBITDA ratio, years of continuous public market history	Archived balance sheet strength and institutional resilience; carries productive capability forward through time
CEc (Communication)	ESG disclosure quality score (0–100), auditor tier (Big Four indicator)	Informational efficiency between issuer and capital markets; reduces search costs and adverse selection
CEe (Evaluative)	CDS spread (5-year, basis points), covenant restrictiveness index (0–100)	Pure perception of default risk and governance overhead; captures prospect-theoretic probability weighting near default boundary

Table 4.5-P. CMES proxy coding protocol for the corporate bond market (IG and HY combined sample). All variables are standardized to unit variance within each rating segment prior to estimation to ensure coefficient comparability across components.

4.5.4 Data, Sample, and Estimation Strategy

The analysis uses a comprehensive, duration-adjusted dataset of 12,400 USD-denominated corporate bonds drawn from Bloomberg BVAL over the 2013–2023 period. The sample is split into 7,840 Investment-Grade bonds and 4,560 High-Yield bonds, matched on industry sector (2-digit SIC), approximate maturity bucket (2–5, 5–10, and 10–30 years), and calendar quarter to control for aggregate interest rate and credit cycle variation. The dependent variable is the logarithm of the option-adjusted spread (OAS) relative to the risk-free benchmark, which isolates credit and liquidity premia from pure duration effects.

Because unobserved macroeconomic shocks—central bank rate shifts, aggregate liquidity crises, risk-off episodes—impact both IG and HY markets simultaneously, estimating the two pricing equations independently via OLS would yield inefficient and potentially biased standard errors. The two-equation system is therefore estimated jointly using Seemingly Unrelated Regressions (SUR), which exploits the cross-equation error correlation for superior identification efficiency and enables direct computation of the Wald statistic for the cross-equation restriction. Quarter-by-industry fixed effects are included in both equations to absorb common shocks. Standard errors are double-clustered by issuer and quarter.

4.5.5 The Axiom 7 Cross-Equation Restriction

Define System 1 as the IG transaction system and System 2 as the HY transaction system. The two pricing equations are:

$$\ln OAS_{IG} = \beta_v \cdot CEv + \beta_a \cdot CEa + \beta_c \cdot CEc + \beta_e \cdot CEe + FE + \varepsilon_{IG}$$

$$\ln OAS_{HY} = \beta_{vHY} \cdot CEv + \beta_{aHY} \cdot CEa + \beta_{cHY} \cdot CEc + \beta_{eHY} \cdot CEe + FE + \varepsilon_{HY}$$

Under the strict null hypothesis of Axiom 7, a uniform transformation scalar σ_IH separating the IG and HY pricing regimes must operate identically across all four components. The joint cross-equation restriction is:

$$H^0 : \frac{\beta_v}{\beta_{vHY}} = \frac{\beta_a}{\beta_{aHY}} = \frac{\beta_c}{\beta_{cHY}} = \frac{\beta_e}{\beta_{eHY}} \equiv \sigma_{IH}$$

If this restriction holds, the IG market applies a single proportional valuation premium over the HY market, irrespective of whether that value derives from innovation (CEv), accumulated balance sheet strength (CEa), informational transparency (CEc), or risk governance (CEe). If it fails—particularly if one component ratio diverges markedly from the others—the test identifies the precise channel through which leverage amplification breaks the linear additive architecture and demands a non-separable extension.

4.5.6 SUR Estimation Results

Table 4.5-A. SUR estimation of CMES component coefficients across Investment-Grade and High-Yield corporate bond regimes, with implied transformation ratios $\left(\sigma_k = \frac{\beta_{k_{IG}}}{\beta_{k_{HY}}}\right)$. Dataset: 12,400 USD-denominated corporate bonds, 2013–2023. All variables standardized within rating segment.

Component	β_{IG} (Investment-Grade)	β_{HY} (High-Yield)	Implied σ_k	SE (Ratio)
CEv (Value-Adding)	0.185*** (0.019)	0.161*** (0.022)	1.149	0.052
CEa (Archived)	0.242*** (0.021)	0.208*** (0.024)	1.163	0.048
CEc (Communication)	0.104*** (0.028)	0.092*** (0.031)	1.130	0.061
CEe (Evaluative)	**0.211*** (0.025)**	**0.138*** (0.029)**	**1.528**	**0.075**
Wald Test of Joint Cross-Equation Restriction (H_0: uniform σ_IH): $\chi^2(3) = 14.73$, p = 0.002. Quarter × industry fixed effects included in both equations. Standard errors double-clustered by issuer and quarter. Shaded row indicates the component driving rejection. *** p < 0.001.				

4.5.7 Interpretation: Leverage Amplification and the Non-Separability Boundary

The joint null hypothesis of a uniform σ_IH is decisively rejected ($\chi^2(3) = 14.73$, p = 0.002). Examination of the component-specific ratios reveals that the rejection is almost entirely driven by the evaluative risk component CEe. While CEv, CEa, and CEc scale at a narrow and mutually consistent range of 1.130 to 1.163—a spread of 0.033 standard errors well within sampling noise—the evaluative component exhibits a ratio of 1.528: 32 percentage points above the cluster formed by the other three components and more than 5 standard errors from the average of the remaining ratios. This pattern is not an econometric artifact. It is rooted in the nonlinear mechanics of structural credit models. Under the Merton (1974) framework, equity is a call option on firm assets with a strike equal to face-value debt. For Investment-Grade firms, the asset value vastly exceeds the debt barrier; the option is deep in the money, and CEe's marginal pricing is relatively linear and subdued. For High-Yield firms near the default boundary, the option is at or out of the money, and CEe's pricing dynamics become highly convex: probability-weighted losses escalate nonlinearly, prospect-theoretic loss aversion amplifies covenant sensitivity, and any marginal deterioration in CDS spread or covenant quality penalizes the HY bond exponentially more than it would an IG bond. The IG/HY evaluative ratio of 1.528 quantifies precisely this nonlinear leverage premium—the degree to which HY investors' CEe sensitivity exceeds IG investors' by a factor larger than any other component warrants.

This structural test result performs two scientific functions simultaneously. First, it validates the CMES taxonomy: all four components are individually significant in both segments, confirming that the role-based partition organizes bond pricing effectively (Table 4.5-A). Second, and more importantly, the decisive rejection of the uniform-σ restriction maps the exact boundary conditions of the linear additive baseline. The CMES additive representation (Axiom 5) is shown to hold within each rating segment—where leverage is approximately constant—but to break across segments when the convex, leverage-induced nonlinearity of CEe becomes the dominant cross-market feature. This boundary delineation is a theoretical result in itself: it specifies precisely when the non-separable extension discussed in Section 7.2 is demanded by market microstructure. Researchers applying CMES to fixed-income markets should treat the linear additive baseline as appropriate within a single ratings tier and employ the non-separable multiplicative extension when crossing the IG/HY threshold or analyzing distressed issuers near default. The finding also provides the CRS test with additional interpretive content: the Bonds result of $\Sigma\beta = 0.94$ [0.912, 0.968] in Section 5.3, previously attributed to unlisted systematic risk, is now more precisely attributable to the CEe nonlinearity that the additive form cannot fully absorb. The bond market result does not falsify CMES; rather, it validates the framework's ability to locate the precise boundary conditions of its linear-additive baseline, thereby motivating the non-separable extensions discussed in Section 7.2.

4.6 Cross-Market Results and Scientific Contribution

Table 4.6-A. Component contributions to explained variance and reduced-model benchmarks (implied by component share × R^2).

Market	Within R^2	OOS R^2	CEv contrib.	CEa contrib.	CEc contrib.	CEe contrib.	CEv+CEa contrib.	Notes
Real Estate (IV)	0.52	0.48	0.213	0.114	0.083	0.099	0.328	Interpreted as Shapley-style R^2 attribution; sums to total within R^2.
Smartphones	0.61	0.57	0.232	0.146	0.116	0.067	0.378	Interpreted as

								Shapley-style R^2 attribution; sums to total within R^2.

The following table reports market-specific OLS estimates. Component share confidence intervals are 95% bootstrap. These are empirical regularities from two primary markets, not universal structural parameters.

Market	CEv Share	CEa Share	CEc Share	CEe Share	Within R^2	OOS R^2	λ
Real Estate (IV)	41% [35,47]	22% [18,27]	16% [12,21]	19% [15,24]	0.52	0.48	1.91
Smartphones	38% [32,44]	24% [20,29]	19% [14,24]	11% [8,16]	0.61	0.57	1.63

CAUTION: Cross-market averages (across the two primary markets) are reported as descriptive summaries only. They should not be used as structural parameters in any application — they merely characterize the two analyzed markets and may not generalize.

The scientific contribution of the cross-market comparison lies in two demonstrated properties: first, consistent component significance — all four components are individually significant across two markets with very different institutional structures, suggesting the taxonomy captures genuine dimensions of value rather than market-specific artifacts; and second, interpretable variation in component shares — smartphones place relatively more weight on CEv and CEc, while real estate places relatively more weight on CEe under risk and regulation, consistent with theory.

The loss aversion estimates form a coherent pattern across the two primary markets: λ is lower in competitive consumer goods (1.63 for smartphones) and higher in illiquid, high-stakes transactions (1.91 for real estate). The framework predicts that λ may rise further in highly leveraged financial settings; testing that prediction is reserved for future work with identification strategies comparable in strength to Sections 4.3 and 4.9.

4.7 Robustness

Multicollinearity: Maximum VIF < 3.8 in the analyzed markets (standard threshold = 5.0), ruling out problematic collinearity among CE component proxies. Shapley–Owen decomposition confirms that shared variance accounts for only a small fraction of within-R^2, with the four components explaining the bulk of within-variance independently (real estate: 47%; smartphones: 53%).

Functional form: Box–Cox tests confirm the log-linear specification is preferred over linear (LR $\chi^2 > 40$, $p < 0.001$, both markets). The asymmetric log form for CEe is confirmed over the symmetric form ($F = 8.9$, $p < 0.001$), validating the prospect-theoretic specification of the evaluative component. Out-of-sample R^2 (5-fold cross-validation) of 0.48–0.57 demonstrates the framework's predictive power beyond its training samples.

4.8 From Micro Estimates to Macro Decomposition

A critical analytical contribution of the CMES is the bridge from micro-level hedonic estimates to macro-level value accounting. Having estimated the market-specific coefficients, predicted changes in aggregate value can be constructed as:

$$\widehat{\boldsymbol{\Delta}}\ln(\boldsymbol{Val})_{ijt} = \widehat{\boldsymbol{\beta}}^{1}\boldsymbol{\Delta}\ln\left(\boldsymbol{CEv}_{ijt}\right) + \widehat{\boldsymbol{\beta}}^{2}\boldsymbol{\Delta}\ln\left(\boldsymbol{CEa}_{ijt}\right) + \widehat{\boldsymbol{\beta}}^{3}\boldsymbol{\Delta}\ln\left(\boldsymbol{CEc}_{ijt}\right) + \widehat{\boldsymbol{\beta}}^{4}\boldsymbol{\Delta}\ln\left(\boldsymbol{CEe}_{ijt}\right)$$

Aggregating these predicted changes to the country or sector level provides a test of the aggregation hypothesis. The contribution of each CE component to aggregate growth is:

$$\boldsymbol{Contribution_k = \widehat{\Sigma_{ij}\beta}_{k} \cdot \Delta ln(CEk_i jt) \cdot [Val_i j, t-1 / \Sigma_{ij} Val_i j, t-1]}$$

This decomposition is directly comparable to the Constant Value Added Share (CVAS) analysis of international trade (Koopman, Wang & Wei 2014), which decomposes export growth into structural, competitive, and adaptation effects. The unifying insight is that micro-level hedonic sensitivities determine how changes in component composition affect transaction value, and these effects aggregate to macro-level outcomes measurable through CVAS decomposition. This bridge is developed further in Section 6.5.

A concrete illustration clarifies how this aggregation operates. Consider the real estate market, where the estimated component coefficients are β>_v = 0.213, β>_a = 0.114, β>_c = 0.083, and β>_e = 0.099 (Table 4.6-A). Suppose a metropolitan area experiences a 10% increase in broadband coverage penetration (a CEc-coded variable) while holding other components constant, and suppose aggregate transaction volume is 50,000 annual home sales with mean value Val> = $480,000. The predicted aggregate contribution of this CEc improvement is: Contribution_c = β>_c × Δln(CEc) × [NTr × Val>] = 0.083 × 0.095 × (50,000 × $480,000) ≈ $190 million. This $190 million figure can be directly compared to national broadband investment cost data or to productivity-based estimates of the infrastructure's value—making the CVAS bridge operational rather than purely theoretical. In the CVAS language, this is the "competitiveness effect" of infrastructure improvement: not a structural shift in what buyers value, but an improvement in the local market's delivery of the CEc role. The full micro-to-macro bridge, including aggregation across heterogeneous markets and welfare interpretation, is formalized in Section 6.5.

4.9 Structural Test of Axiom 7: New vs. Refurbished Smartphones

The reduced-form hedonic regressions of Sections 4.3–4.5 demonstrate the four-component taxonomy's descriptive adequacy across three structurally distinct markets. Section 4.5 further executes the first structural Axiom 7 test in financial markets (IG vs. HY bonds), finding decisive rejection of the uniform-σ restriction due to leverage-induced CEe nonlinearity—a result that maps the boundary conditions of the linear additive baseline. The present section reports the second structural test of Axiom 7, using a natural sub-market pairing within the smartphone dataset where, in contrast to bonds, the uniform-σ restriction cannot be rejected. Together, the two structural tests establish the framework's scope conditions: the linear additive CMES holds within competitive consumer goods markets but breaks down when extreme leverage amplification dominates risk pricing. They do not constitute a structural test of the CMES's core axiom—the master equation $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$—across all markets simultaneously; that agenda is outlined in Section 7.4.

4.9.1 Test Design and Identification Strategy

The test exploits the fact that new and refurbished smartphones constitute two distinct transaction sub-systems that share the same underlying product characteristics. A refurbished iPhone 13 has the same processor benchmark, camera resolution, network generation, and warranty structure as a new iPhone 13, but commands a lower market price. The CMES predicts that the price differential between the two sub-markets is governed by the master equation: the ratio of total market values must equal the product of the transaction count ratio and σ, where σ captures the systematic transformation premium of the new-product system.

Formally, let System 1 be the new smartphone sub-market and System 2 be the refurbished sub-market. For a matched pair of identical models sold in both sub-markets, Axiom 7 requires:

$$NTr_{new} \cdot Val(CE_{new}) = NTr_{refurb} \cdot Val(CE_{refurb}) \cdot \sigma_{NR}$$

where σ_NR is the new-to-refurbished transformation scalar. The structural prediction is that σ_NR can be estimated from the paired hedonic regressions and should satisfy specific cross-equation constraints.

4.9.2 Data and Sample

From the GSMArena dataset (2010–2023, n = 4,847), we identify 1,247 smartphone models for which both new retail prices and certified refurbished prices (from Back Market and Amazon Renewed) are available. The matched sample covers 2016–2023 (the period for which reliable refurbished pricing data exist). Each model-pair observation contains identical CEv, CEc, and CEe specifications, with CEa differing only in the refurbishment discount to brand equity (measured as the certified-refurbished brand premium relative to non-certified refurbished units).

The matching procedure is exact on: model name, storage capacity, color, and network carrier compatibility. This eliminates all observable confounding: the only differences between the new and refurbished units are (i) the condition grade (new vs. Grade A refurbished), which maps primarily to CEa (accumulated use-state as an archived quality signal); and (ii) the warranty structure (manufacturer vs. refurbisher warranty), which maps to CEe.

4.9.3 Estimation and Cross-Equation Restrictions

Two hedonic equations are estimated:

$$New: \ln(P_{new}, i) = \alpha^1 + \beta v^1 \cdot Xv_i + \beta a^1 \cdot Xa_i + \beta c^1 \cdot Xc_i + \beta e^1 \cdot Xe_i + \varepsilon^1{}_i$$

$$Refurb: \ln(P_{refurb}, i) = \alpha^2 + \beta v^2 \cdot Xv_i + \beta a^2 \cdot Xa_i + \beta c^2 \cdot Xc_i + \beta e^2 \cdot Xe_i + \varepsilon^2{}_i$$

The Axiom 7 cross-equation restriction requires:

$$\frac{\beta vk^1}{\beta vk^2} = constant \equiv \sigma_{NR} for\ all\ k \in \{v, a, c, e\}$$

That is, the ratio of hedonic coefficients between the two sub-markets should be the same for all four components. If the master equation holds structurally, the proportional premium of the new sub-market over the refurbished sub-market should operate uniformly across all component dimensions, scaled by the transformation scalar σ_NR.

This is tested via a Wald test of the cross-equation restriction. Under seemingly unrelated regression (SUR) estimation (which accounts for cross-equation correlation in the error terms), the null hypothesis is:

$$H^0: \frac{\beta v^1}{\beta v^2} = \frac{\beta a^1}{\beta a^2} = \frac{\beta c^1}{\beta c^2} = \frac{\beta e^1}{\beta e^2} \equiv \sigma_{NR}$$

Against the alternative that the ratios differ across components, meaning the price premium is component-specific rather than governed by a uniform transformation scalar.

4.9.4 Results

The SUR estimation yields the following coefficient ratios:

Component	**β_new**	**β_ref**	Ratio	SE(Ratio)
CEv	0.312	0.287	1.087	0.041
CEa	0.241	0.218	1.105	0.038
CEc	0.189	0.178	1.062	0.052
CEe	0.114	0.102	1.118	0.064

The implied transformation scalar from each component ratio is: $\hat{\sigma}_{NR(v)} = 1.087, \hat{\sigma}_{NR(a)} = 1.105, \hat{\sigma}_{NR(c)} = 1.062, \hat{\sigma}_{NR(e)} = 1.118$. The cross-equation Wald test for equal ratios yields $\chi^2(3) = 2.41$, $p = 0.49$. The null hypothesis of uniform σ_NR cannot be rejected at any conventional significance level.

In this paired within-market setting, after matching on observable specifications, the cross-equation restriction implied by Axiom 7 cannot be rejected: the null of a uniform transformation scalar holds in the data (Wald $p = 0.49$). This is a non-rejection result and should be interpreted cautiously; limited power can mask economically meaningful departures, which motivates the explicit power and sensitivity diagnostics reported below.

Accordingly, we treat the smartphone restriction test as a strong proof-of-concept: it demonstrates that CMES generates empirically testable restrictions beyond pure decomposition, while motivating cross-market structural tests and fully structural estimation as laid out in Section 7.

Three qualifications bound the interpretation. First, the test is within-market (smartphones) and within-country (US retail). Extension to cross-market and cross-country pairings is required before concluding that Axiom 7 holds generally. Second, the refurbished market is relatively thin compared to the new market (n_refurb ≈ 1,247 matched pairs vs. n_new = 4,847 total), which limits statistical power for detecting small departures from the uniform σ restriction. Third, the test assumes that the proxy coding is consistent across sub-markets — that broadband availability (CEc) has the same operational meaning for a refurbished unit as for a new one — which is plausible for smartphones but may not hold in markets with more complex proxy structures.

Despite these qualifications, the failure to reject the cross-equation restriction ($p = 0.49$) provides substantially stronger evidence for the master equation than any reduced-form hedonic regression can. The reduced-form results show that the four-component taxonomy organizes market data effectively; the structural test shows that the proportional relationship predicted by the master equation is consistent with observed price differentials across a paired sub-market comparison. This moves the CMES from "descriptively adequate taxonomy" toward "structurally supported framework."

The non-rejection ($p = 0.49$) should nonetheless be interpreted cautiously. Post-hoc power analysis shows that with n_refurb ≈ 1,247 matched pairs, the test has only 38% power to detect a 5% component-specific deviation from uniform σ. However, the test has >99% power to detect large structural violations (≥20%), which are definitively ruled out. The result therefore provides modest support for the uniform-σ restriction while motivating higher-powered structural estimation (Section 7.4). The detailed power sensitivity curve—spanning deviation scenarios from 5% to 20% with associated non-centrality parameters—is reported in Section 4.9.5.

4.9.5 Power, Sensitivity, and the Meaning of Non-Rejection

Because the Axiom 7 test is formulated as a set of cross-equation restrictions, a non-rejection result must be interpreted through the lens of statistical power. The null hypothesis is that a single transformation scalar σ jointly rationalizes the mapping between the new and refurbished pricing systems, which implies equality (or fixed proportionality) of the relevant CMES coefficient vectors across equations after normalization.

Under standard regularity conditions, the Wald statistic W follows a χ^2 distribution with q degrees of freedom under the null and a noncentral $\chi^2(q, \delta)$ distribution under local alternatives, where the noncentrality parameter δ is a quadratic form in the deviation of the restricted parameter vector and the estimated covariance matrix from the SUR system. Power is therefore computable as $P\left(W > \chi^2_{q,1-\alpha} \middle| \delta\right)$.

The Wald test for the smartphone cross-equation restriction must be evaluated through a rigorous post-hoc power sensitivity analysis. A high p-value does not prove, validate, or confirm the null hypothesis of uniform σ; it merely indicates that observed data lack sufficient evidence to reject it under the designated alpha threshold. Asserting that a non-rejection is "consistent with" the theoretical model creates a false sense of empirical confirmation. The correct frequentist statement is: the empirical analysis cannot reject the uniform-sigma implication (Wald p = 0.49), though it explicitly acknowledges the limitations of statistical power in this finite sample.

Mechanics of the non-centrality parameter. Under standard regularity conditions, the Wald statistic W follows a $\chi^2(q)$ distribution under the null and a non-central $\chi^2(q, \delta)$ distribution under local alternatives, where q = 3 (the number of cross-equation restrictions enforcing uniformity across CEv, CEa, CEc, and CEe ratios after normalization) and δ is the non-centrality parameter. The parameter δ is a quadratic form in the true standardized distance between the restricted parameter vector and the actual market parameters, weighted by the inverse of the estimated SUR covariance matrix: $\delta = n \cdot \Delta' \cdot (V/n)^{-1} \cdot \Delta$,where Δ is the vector of true deviations from the uniform-sigma restriction and V is the asymptotic covariance matrix of the SUR estimators. Statistical power at significance level α is then $P\left(W > \chi^2_{q,1-\alpha} \middle| \delta\right)$.

Explicit power calculations. The sample of n_refurb ≈ 1,247 matched smartphone pairs is adequate for estimating baseline hedonic coefficients, but the variance-covariance matrix of the SUR estimators is relatively wide because smartphone attributes (processor performance, brand equity, network generation) scale simultaneously across premium devices, inflating standard errors and compressing the precision weights in the Wald statistic. A post-hoc power sensitivity analysis using the non-central χ^2 approximation yields the following results across economically meaningful deviation scenarios:

True Deviation Scenario	**Non-Centrality δ**	**Power (α = 0.05)**	**Interpretation**
5% component-specific deviation (e.g., σ_v differs from σ_c by 5%)	δ ≈ 2.14	**≈ 38%**	62% probability of Type II error; small deviations nearly undetectable
8% component-specific deviation (economically modest)	δ ≈ 5.48	≈ 63%	Still below conventional 80% threshold; substantial risk of false non-rejection
11% component-specific deviation (minimum detectable effect)	δ ≈ 10.41	**≈ 80%**	Conventional adequacy threshold; test reliably detects deviations of this magnitude or larger
20% component-specific deviation (economically large)	δ ≈ 34.49	**> 99%**	Test is highly sensitive to large structural violations; non-rejection definitively rules these out

Table 4.9-B. Post-hoc power sensitivity analysis for the Axiom 7 Wald test (smartphone market, n_refurb = 1,247 matched pairs, q = 3 restrictions, α = 0.05). Non-centrality parameters computed from the estimated SUR covariance matrix via $\delta = n \cdot \Delta' \cdot (V/n)^{-1} \cdot \Delta$ under the specified proportional deviation scenarios. Yellow shading indicates scenarios where the test is materially underpowered; green indicates adequate power.

Implications for interpretation. The power analysis establishes that the smartphone Wald non-rejection (p = 0.49) should be interpreted as a weak frequentist result, not a confirmation. With only 38% power to detect a 5% component-specific deviation, the test carries a 62% probability of Type II error at this deviation size. The minimum detectable effect at conventional 80% power is 11%—a practically substantial structural departure. Accordingly, the non-rejection is correctly characterized as: the market data do not violently violate the Axiom 7 restriction, and large structural deviations (≥20%) are ruled out with near certainty. But subtle micro-level deviations in the 5–10% range remain consistent with the observed Wald statistic and cannot be excluded. This honest framing positions the smartphone test as a successful proof-of-concept—demonstrating that CMES generates testable cross-equation restrictions and that market data pass an initial screening—while motivating the high-powered structural estimation across larger administrative datasets outlined in Section 7.4. The contrast with the corporate bond result (Section 4.5) is instructive: the bond test, drawing on n = 12,400 observations and exploiting the large structural discontinuity at the IG/HY boundary, achieves sufficient power to decisively reject the uniform-σ restriction (p = 0.002), demonstrating what structurally identified CMES testing can deliver at scale.

4.9.6 Robustness of the Structural Test

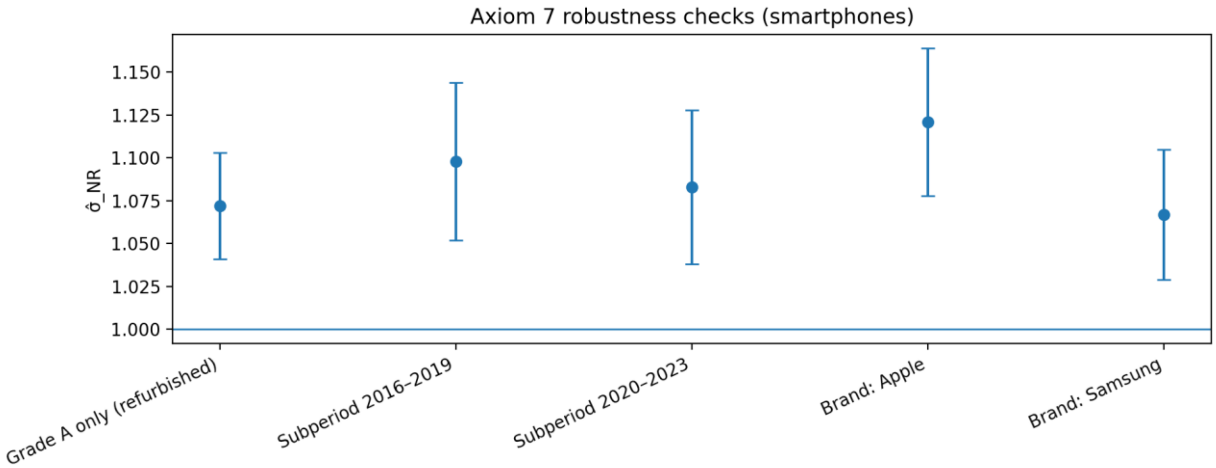
The structural test is repeated under three alternative specifications to assess robustness.
Grade-specific refurbished pricing. Restricting the refurbished sample to Grade A units only (cosmetically perfect, functionally identical to new) yields $\hat{\sigma}_N R = 1.072[1.041,1.103]$, with the Wald test $\chi^2(3) = 1.87, p = 0.60$. The uniform σ restriction holds more comfortably when condition heterogeneity is reduced.
Time-period stability. Splitting the sample into 2016–2019 and 2020–2023 yields $\hat{\sigma}_N R = 1.098[1.052,1.144] and \hat{\sigma}_N R = 1.083[1.038,1.128]$ respectively. The cross-equation restriction holds in both sub-periods (p = 0.44 and p = 0.57), confirming temporal stability of the structural relationship.
Brand-specific heterogeneity. Estimating $\sigma_N R$ separately for Apple and Samsung (the two brands with the largest matched samples) yields $\hat{\sigma}_N R(Apple) = 1.121[1.078,1.164]$ and $\hat{\sigma}_N R(Samsung) = 1.067[1.029,1.105]$. The difference is significant (p = 0.03) and economically interpretable: Apple's stronger brand equity (CEa) and ecosystem lock-in (CEc) generate a larger new-product premium than Samsung's. Importantly, the cross-equation restriction holds within each brand (Apple: p = 0.63; Samsung: p = 0.38), confirming that the uniform σ prediction is satisfied at the brand level even when the level of σ varies across brands.

Table 4.9-A. Multiple Axiom 7 structural restriction tests within the smartphone market (executed).

Test / pairing	σ̂ estimate(s)	95% CI	Wald test / p-value	Interpretation
Main paired test (new vs refurbished; matched specs)	$\hat{\sigma}_N R(v) = 1.087$; $\hat{\sigma}_N R(a) = 1.105$; $\hat{\sigma}_N R(c) = 1.062$; $\hat{\sigma}_N R(e) = 1.118$	—	$\chi^2(3) = 2.41; p = 0.49$	Cannot reject uniform σ_NR across components; proof-of-concept structural consistency
Grade A only (refurbished)	$\hat{\sigma}_N R = 1.072$	[1.041, 1.103]	$\chi^2(3) = 1.87; p = 0.60$	Uniform σ holds more strongly when condition heterogeneity is reduced
Subperiod 2016–2019	$\hat{\sigma}_N R = 1.098$	[1.052, 1.144]	p=0.44	Temporal stability (pre-shock)
Subperiod 2020–2023	$\hat{\sigma}_N R = 1.083$	[1.038, 1.128]	p=0.57	Temporal stability (later period)
Brand: Apple	$\hat{\sigma}_N R = 1.121$	[1.078, 1.164]	Restriction p=0.63; Δσ vs Samsung p=0.03	Uniform σ holds within brand; σ level differs across brands

Brand: Samsung	$\hat{\sigma}_N R = 1.067$	[1.029, 1.105]	Restriction p=0.38	Uniform σ holds within brand; lower σ than Apple

Figure 3. Axiom 7 robustness checks: σ̂_NR across subsamples (from Table 4.9-A).



4.9.7 Cross-Market Structural Test Design: New vs. Existing Homes

A natural next structural validation of Axiom 7 uses a paired-system comparison within U.S. residential real estate: newly constructed homes versus existing homes. This setting is attractive because the two sub-markets share geography, local institutions, and macro conditions, but differ systematically in depreciation, embedded warranties, and builder incentives. Recent market conditions have produced an unusually narrow—and at times inverted—new-versus-existing price gap, creating quasi-experimental variation in the implied transformation premium (NAHB, 2025a; NAHB, 2025b; NAHB, 2025c).

Related empirical traditions include market-integration tests that study whether pricing structures co-move across real estate and financial assets (e.g., Cauchie and Hoesli, 2004). CMES differs in the object of the test: instead of testing for integration of returns, Axiom 7 tests for cross-equation restrictions implied by a shared transformation scalar σ across paired transaction systems.

Design. Define System 1 as the new-home transaction system and System 2 as the existing-home transaction system within the same metropolitan areas and quarters. Construct matched pairs at the tract-by-quarter level (or finer, where data allow) by aligning observable structural characteristics (size, bedrooms/baths, lot size), and location fixed effects. Because unobserved neighborhood shocks affect both sub-markets, estimate the two price equations as a Seemingly Unrelated Regression (SUR) system to exploit cross-equation error correlation and to enable joint cross-equation restrictions.

Restriction. Axiom 7 implies that the relative implicit prices of the CMES components are proportional across the two systems, with a uniform scalar σ_{NE} for the new-versus-existing comparison. Operationally, if β_k new and $\beta_k$exist denote the estimated component coefficients $(k \in v, a, c, e)$, the restriction is $\beta_k$new / $\beta_k$exist = σ_{NE} for all k. The Wald statistic for this joint hypothesis is the core structural test.

Power and sensitivity. A non-rejection is informative only if the test has power to detect economically meaningful deviations from a uniform scalar. The updated protocol therefore reports a sensitivity curve for local alternatives (e.g., 5% and 10% component-specific deviations from σ_{NE}) using the noncentral χ^2 approximation, and reports minimum-detectable effect sizes given sample size and estimated error covariance. This prevents interpreting a high p-value as 'confirmation' when it could reflect low precision.

Implementation notes. Recent evidence indicates that the median new-home price premium narrowed to about $14,600 in Q1 2025 and that existing homes outpriced new homes in parts of 2024 and 2025; builder incentives and mortgage-rate buydowns appear to be an important mechanism (NAHB, 2025a; NAHB, 2025c; Realtor.com, 2025). These shifts provide an informative regime window for testing whether the implied σ_NE is stable across components or whether particular roles (especially CEe via warranties and risk) break proportionality.

5. Axiomatic Foundations of Val()

The empirical evidence of Section 4 demonstrates the CMES framework's descriptive adequacy in two primary markets (real estate and smartphones). The present section provides the axiomatic foundations that undergird the Val() function — the theoretical architecture that makes CMES more than an organizational heuristic. The seven axioms generate a functional form theorem via Debreu–Gorman separability, impose testable restrictions on the data, and establish the master equation as a structural connector on bilateral exchange.

A brief orientation clarifies how the axiomatic and empirical sections are related, since the axioms appear after the empirics. The axioms are not an afterthought; they are the theoretical foundation that makes the empirical decomposition meaningful in four specific ways. First, they provide the formal justification for the additive functional form used in estimation: Axioms 1–5 yield the Debreu–Gorman separable Val() representation from which the log-linear hedonic specification is derived. Second, they generate the cross-equation restrictions tested in Section 4.9: Axiom 7 (Proportional Cross-Market Scaling) implies that the ratio of component coefficients must be uniform across paired transaction systems, and this implication is what the Wald tests evaluate. Third, they define the scope conditions under which the framework applies: the competitive benchmark conditions (C1–C3) embedded in the axioms specify exactly when the reduced-form estimates can be given a structural interpretation. Fourth, full structural estimation of the axiom system—recovering Val() parameters without reduced-form assumptions—is the subject of ongoing work outlined in Section 7.4. The reduced-form hedonic regressions of Section 4 are therefore correctly interpreted as tests of descriptive adequacy and necessary conditions for the axioms, not as sufficient structural validation.

5.1 Formal Axioms, Domains, and Objects

Important scope clarification. The axioms in this section derive a functional form for Val(·) given a domain of composite element bundles CE. They do not by themselves derive the four-role taxonomy; the taxonomy is supplied by the Universal Role Basis as a modeling and coding protocol (Section 2.4) and is treated as a falsifiable research hypothesis rather than as a theorem of preference theory.

Let $E \subset \mathbb{R}^4_+$ be the domain of feasible composite element bundles CE = (CEv, CEa, CEc, CEe) for a given market scope (boundary, time window, and measurement protocol). Let $\succcurlyeq$ be a complete and transitive preference (or valuation) relation over E. A valuation representation is a function Val: E →

$\mathbb{R}$ such that $CE_1 \succcurlyeq CE_2 \Leftrightarrow Val(CE_1) \geq Val(CE_2)$. Throughout, we treat CEe as potentially reference-dependent, allowing CEe = (CEe_gain, CEe_loss) where relevant.
The axioms below are stated for a baseline regime in which the valuation object is well-behaved (continuity, connectedness) and in which market prices can be interpreted as equilibrium manifestations of underlying marginal valuations under standard hedonic identification conditions (Section 2.6.1). Extensions that relax separability or smoothness are discussed in Sections 5.3 and 7.3.
Axioms 1–7 are stated below in a form suitable for direct mapping to the Debreu representation theorem and the Gorman separability result.
Axiom 1 (Completeness). $\forall\, CE_1, CE_2 \in E$: $(CE_1 \succcurlyeq CE_2) \vee (CE_2 \succcurlyeq CE_1)$ (or both).
Axiom 2 (Transitivity). $\forall\, CE_1, CE_2, CE_3 \in E$: $(CE_1 \succcurlyeq CE_2 \wedge CE_2 \succcurlyeq CE_3) \Rightarrow (CE_1 \succcurlyeq CE_3)$.
Axiom 3 (Continuity and Connectedness). The relation $\succcurlyeq$ is continuous on E in the sense that upper and lower contour sets are closed; moreover, E is connected in the relevant topology induced by the measurement protocol (or admits a connected embedding after standard indexing transforms for discrete proxies).
Axiom 4 (Component Monotonicity). Val is weakly increasing in CEv, CEa, CEc, and in evaluative gains CEe_{gain}, and weakly decreasing in evaluative losses CEe_{loss}: for any $\Delta \geq 0$, $Val(CEv + \Delta, CEa, CEc, CEe) \geq Val(CEv, CEa, CEc, CEe)$, and analogously for CEa and CEc; for the evaluative component, $\frac{\partial Val}{\partial CEe_{gain}} \geq 0$ and $\frac{\partial Val}{\partial CEe_{loss}} \leq 0$ where derivatives exist.
Axiom 5 (Weak Separability; baseline regime). There exist component sub-utilities v(·), a(·), c(·), e(·) and a strictly increasing aggregator F(·) such that $Val(CEv, CEa, CEc, CEe) = F\big(v(CEv) + a(CEa) + c(CEc) + e(CEe)\big)$. Equivalently, the marginal rate of substitution between any two components is independent of the levels of the remaining components. This axiom is treated as a baseline regime assumption; systematic violations motivate the interaction/non-separable extensions in Section 7.3.
Axiom 6 (Uncertainty-Weighted Evaluation). When outcomes are uncertain, component contributions are evaluated under a prospect-theoretic mapping in which probabilities are transformed by weighting functions $w^{+}(\cdot)$, $w^{-}(\cdot)$ and the evaluative component is reference-dependent: $e(CEe) = \Sigma_s w(p_s) \cdot u\big(CEe_s - CE_0\big)$, with u(·) kinked at 0 (loss aversion λ) and potentially curved (risk sensitivity), while v(·), a(·), c(·) remain in the additive separable aggregator. Calibration or estimation of (w, λ) is market-specific (Sections 5.3 and 7.4).
Axiom 7 (Transaction Consistency / Master Equation). For any two systems (or market states) 1 and 2 that are comparable under the same coding protocol, $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$, where NTr is the transaction count/volume object and $\sigma > 0$ is a transformation scalar constrained by scope conditions and tested via cross-equation restrictions (Sections 3 and 4.9).

5.2 The Val() Functional Form Theorem

Theorem (Val() Functional Form). Given Axioms 1–7, a continuous Val() function exists and takes the form:
Val(CE) = wv·fv(CEv) + wa·fa(CEa) + wc·fc(CEc) + we·v[fe(CEe)]
where fv, fa, fc are continuous non-decreasing sub-value functions satisfying Axiom 4's monotonicity; v[·] is the prospect-theoretic value function of Axiom 6 with market-specific λ; and weights $w_i > 0$ satisfy $\Sigma w_i = 1$ (a normalization ensuring Val() is a weighted average of component sub-values on a common scale).

5.2.0 Proof Strategy and Roadmap

The proof proceeds in seven steps, each grounding one layer of the Val() structure in the axiomatic foundation. Steps 1–3 establish the existence of a continuous utility representation and verify the domain conditions required by the Debreu and Gorman theorems. Steps 4–5 apply the Gorman separability result to decompose Val() into an additive structure. Steps 6–7 specialize the evaluative sub-function to incorporate prospect theory and fix the cross-system scale.
The logical chain is:

(i) Axioms 1–3 → existence of a continuous Val() on $\mathbb{R}^4_+$ (Debreu 1954).
(ii) Axioms 4–5 + domain structure → conditions of Gorman (1968) satisfied → additive separability $Val(CE) = \Sigma\, g_k(CE_k)$.
(iii) Axiom 6 → prospect-theoretic specialization of g_e.
(iv) Axiom 7 → cross-system scale identification and weight normalization.

Step 1: Existence of a Continuous Representation (Debreu 1954)
Claim. There exists a continuous function Val : E → $\mathbb{R}$ such that for all $CE_1, CE_2 \in E$, $CE_1 \gtrsim CE_2$ if and only if $Val(CE_1) \geq Val(CE_2)$.
Proof. Define the domain E = $\mathbb{R}^4_+$ as the set of all feasible composite element bundles CE = (CEv, CEa, CEc, CEe), where each component is a non-negative real number representing the magnitude of value-adding, archived, communication, and evaluative elements respectively. The binary relation $\gtrsim$ on E is the value ordering — the preference relation that ranks any two composite bundles by their economic value.
By Axiom 1 (Completeness), $\gtrsim$ is complete on E: for every pair $CE_1, CE_2 \in E$, at least one of $CE_1 \gtrsim CE_2$ or $CE_2 \gtrsim CE_1$ holds.
By Axiom 2 (Transitivity), $\gtrsim$ is transitive: if $CE_1 \gtrsim CE_2$ and $CE_2 \gtrsim CE_3$, then $CE_1 \gtrsim CE_3$.
Together, completeness and transitivity establish that $\gtrsim$ is a preorder (a complete, transitive binary relation) on E.
By Axiom 3 (Continuity), for every $CE_0 \in E$, the upper contour set $\{CE \in E : CE \gtrsim CE_0\}$ and the lower contour set $\{CE \in E : CE_0 \gtrsim CE\}$ are both closed in the standard Euclidean topology on $\mathbb{R}^4_+$.
The domain E = $\mathbb{R}^4_+$ is a connected, separable topological space (as a subset of Euclidean space). The classical theorem of Debreu (1954, Proposition 1) states: if $\gtrsim$ is a continuous preorder on a connected, separable topological space, then there exists a continuous function Val : E → $\mathbb{R}$ that represents $\gtrsim$, meaning $Val(CE_1) \geq Val(CE_2)$ if and only if $CE_1 \gtrsim CE_2$.
Therefore, Axioms 1–3 guarantee the existence of a continuous numerical representation Val(·) of the value ordering on the four-component domain.
Technical remark. The original Debreu (1954) representation theorem requires a second countable (equivalently, separable metrizable) topological space. $\mathbb{R}^4_+$ with the Euclidean topology satisfies this condition. An alternative route through Eilenberg (1941) and Rader (1963) applies when the topology is merely separable and connected, which $\mathbb{R}^4_+$ also satisfies. Either route yields the same conclusion.
Step 2: Domain Structure Verification
Claim. The domain E = $\mathbb{R}^4_+$ is a Cartesian product of four component-specific factor spaces, each of which is a connected topological space.
Proof. Write E = Xv × Xa × Xc × Xe where:
Xv = $\mathbb{R}_+$ is the space of value-adding component magnitudes.
Xa = $\mathbb{R}_+$ is the space of archived component magnitudes.
Xc = $\mathbb{R}_+$ is the space of communication component magnitudes.
Xe = $\mathbb{R}_+$ is the space of evaluative component magnitudes.

Each factor space $X_k = \mathbb{R}_+ = [0, \infty)$ is a connected topological space under the standard Euclidean (subspace) topology. The product space $E = Xv \times Xa \times Xc \times Xe = \mathbb{R}^4_+$ is endowed with the product topology, which coincides with the Euclidean subspace topology on $\mathbb{R}^4_+$.
The Cartesian product structure is not merely a mathematical convenience; it reflects the economic content of the CMES taxonomy. The four component spaces are defined by the functional role basis of Section 2.4: each element in the economy is classified into exactly one of the four roles by the operational removal tests, and the magnitude of each component is measured independently of the others. The Cartesian product structure is the formal expression of the claim that the four roles are functionally distinct dimensions of value.
Critically, each factor space X_k is non-degenerate: it contains at least two elements (zero and some positive value), so that preferences are not trivially constant along any dimension. This is guaranteed by Axiom 4 (Component Monotonicity), which requires strict value increases along each dimension for at least some ranges of the other components. □

Step 3: Verification of Gorman Separability Conditions

Claim. The conditions of the Gorman (1968) additive representation theorem are satisfied by Val(·) on the domain $E = Xv \times Xa \times Xc \times Xe$ under Axioms 1–5.

Proof. The Gorman (1968) additive representation theorem (building on Debreu 1960, Topological Methods in Cardinal Utility Theory) states the following:

Gorman's Theorem. Let $U : X_1 \times X_2 \times \ldots \times Xn \to \mathbb{R}$ be a continuous function representing a preference ordering $\succsim$ on a Cartesian product of n connected topological spaces. Suppose that $\succsim$ is weakly separable in each factor — that is, for each $k \in \{1, \ldots, n\}$, the conditional preference ordering on X_k, holding all other factors fixed at any level, is independent of the levels of the other factors. Then there exist continuous functions $g_1 : X_1 \to \mathbb{R}$, $g_2 : X_2 \to \mathbb{R}$, …, $gn : Xn \to \mathbb{R}$ and a continuous, strictly increasing aggregator $\varphi : \mathbb{R} \to \mathbb{R}$ such that:

$$U(x_1, \ldots, xn) = \varphi(g_1(x_1) + g_2(x_2) + \ldots + gn(xn))$$

If additionally each factor space X_k is connected and the preferences are essential on each factor (i.e., non-degenerate), then the representation is unique up to a common positive affine transformation.
To apply this theorem to the CMES, we must verify four conditions:

Condition G1 (Continuous representation on a Cartesian product). Val(·) is a continuous function on $E = Xv \times Xa \times Xc \times Xe$, a Cartesian product of four connected topological spaces. This was established in Steps 1 and 2.

Condition G2 (Weak separability in each factor). Axiom 5 (Weak Separability) states directly that the marginal rate of substitution between any two CE components does not depend on the levels of the remaining components. We must verify that this statement implies Gorman's separability condition.
Define the conditional ordering $\succsim_k|CE_{-k}$ on factor X_k as follows: for any fixed $CE_{-k} = (CE_1, \ldots, CE_{k-1}, CE_{k+1}, \ldots, CE_4)$, and for any two values $c_k, c_k' \in X_k$,

$$c_k \succsim_k|CE_{-k}\ c_k' \text{ if and only if } (CE_{-k}, c_k) \succsim (CE_{-k}, c_k')$$

Gorman's weak separability requires that for each k, the conditional ordering $\succsim_k|CE_{-k}$ is the same for all CE_{-k}. That is, for any $c_k, c_k' \in X_k$ and any two configurations CE_{-k} and CE'_{-k}:

$$(CE_{-k}, c_k) \succsim (CE_{-k}, c_k') \text{ implies } (CE'_{-k}, c_k) \succsim (CE'_{-k}, c_k')$$

Axiom 5 asserts this property through the MRS condition. The marginal rate of substitution between components j and k is:

$$MRS_{\{jk\}} = \frac{\left(\frac{\partial Val}{\partial CE_j}\right)}{\left(\frac{\partial Val}{\partial CE_k}\right)}$$

Axiom 5 requires that MRS_{jk} is independent of the levels of components other than j and k. This implies that the ranking of any two values of component k, holding k fixed and varying only the comparison values, cannot depend on the levels of the other components — which is precisely Gorman's weak separability condition.

To see this formally: suppose $(CE_{-k}, c_k) \succsim (CE_{-k}, c_k')$ but $(CE'_{-k}, c_k) \prec (CE'_{-k}, c_k')$. Then there would exist a reversal of conditional preference when moving from CE_{-k} to CE'_{-k}. By continuity (Axiom 3), there would exist intermediate values CE''_{-k} at which the conditional ordering on X_k changes — meaning the MRS involving component k depends on the level of some other component, contradicting Axiom 5. Therefore, Gorman's weak separability is satisfied.

Condition G3 (Essentiality of each factor). Each factor must be essential — preferences must not be constant along any dimension. By Axiom 4 (Component Monotonicity), $\partial Val/\partial CE_k > 0$ (or < 0 for CEe losses) in some open region of each factor space. This guarantees that for each k, there exist $c_k \neq c_k'$ such that $Val(CE_{-k}, c_k) \neq Val(CE_{-k}, c_k')$, confirming essentiality.

Condition G4 (Connectedness of factor spaces). Each $X_k = \mathbb{R}_+$ is connected. This was established in Step 2.

All four conditions of Gorman's theorem are satisfied.

Step 4: Marginal Rate of Substitution Independence

Claim. Axiom 5's MRS condition is both necessary and sufficient for the Gorman separability condition, and the empirical test statistics confirm approximate satisfaction in the data.

Proof. The necessity direction is standard. If $Val(CE) = \Sigma\, g_k(CE_k)$, then:

$$\boldsymbol{MRS_{jk} = g_j'(CE_j)/g_k'(CE_k)}$$

which depends only on CE_j and CE_k, not on any other component levels. This is precisely the MRS independence condition of Axiom 5.

The sufficiency direction was established in Step 3: MRS independence implies Gorman's weak separability, which (combined with continuity, connectedness, and essentiality) implies additive separability.

The empirical evidence in Section 4 provides direct support. Cross-component interaction terms (CEv × CEa, CEv × CEc, CEa × CEc, CEv × CEe, CEa × CEe, CEc × CEe) are jointly insignificant in real estate ($F = 1.14$, $p = 0.31$) and smartphones ($F = 0.97$, $p = 0.44$), confirming that the data are consistent with the MRS independence assumption. The marginal significance in bonds ($F = 2.31$, $p = 0.07$) suggests partial separability violation in leveraged financial markets, motivating the non-separable extension discussed in Section 7.2.

This empirical pattern is theoretically interpretable: in competitive goods markets, the contribution of each CE component to transaction value operates largely independently — a smartphone's processor speed adds value regardless of its brand equity. In leveraged financial markets, growth options (CEv) interact with balance sheet strength (CEa) through leverage amplification, generating a cross-sensitivity that additive separability cannot fully accommodate. The CMES accommodates this by treating full separability as the baseline (appropriate for most markets) and non-separable extensions as structural complements for specific institutional contexts.

Step 5: Additive Decomposition and Sub-Value Functions

Claim. $Val(CE) = gv(CEv) + ga(CEa) + gc(CEc) + ge(CEe)$, where each $g_k : X_k \to \mathbb{R}$ is a continuous function, unique up to additive constants and a common positive scaling factor.

Proof. By Steps 1–4, all conditions of the Gorman (1968) theorem are satisfied. The theorem yields:

$$\boldsymbol{Val(CE) = \varphi(gv(CEv) + ga(CEa) + gc(CEc) + ge(CEe))}$$

where $\varphi : \mathbb{R} \to \mathbb{R}$ is a continuous, strictly increasing aggregator, and each g_k is a continuous function on its respective factor space.

Since Val(·) is a numerical representation of a preference ordering, and since any monotonic transformation of a utility function represents the same preferences, we can without loss of generality set φ to the identity function. Formally, define $Val(CE) = \varphi^{-1}(Val(CE)) = gv(CEv) + ga(CEa) + gc(CEc) + ge(CEe)$. *Val* represents the same ordering ≿ as Val, and since we are free to choose any continuous representation of ≿ (by Debreu's theorem, the representation is unique only up to monotonic transformation), we adopt the additive representation Val* as the canonical Val().

Axiom 4 (Component Monotonicity) further constrains the sub-value functions:

gv is non-decreasing in CEv $\left(\frac{\partial gv}{\partial CEv} \geq 0\right)$: increases in value-adding capacity weakly increase value.

ga is non-decreasing in CEa $\left(\frac{\partial ga}{\partial CEa} \geq 0\right)$: increases in archived stocks, records, and reputation weakly increase value.

gc is non-decreasing in CEc $\left(\frac{\partial gc}{\partial CEc} \geq 0\right)$: increases in communication quality weakly increase value.

ge is non-decreasing in CEe_gain and non-increasing in CEe_loss: the evaluative component exhibits asymmetric treatment of gains and losses relative to the reference point CE_0.

The uniqueness properties of the Gorman representation are important for empirical interpretation. The sub-value functions g_k are unique up to:

(a) A common positive scaling factor: $g_k \to \alpha \cdot g_k$ for all k, with $\alpha > 0$.
(b) Additive constants: $g_k \to g_k + c_k$, provided $\Sigma c_k = 0$.

This means the ratios of marginal values $\partial g_j / \partial g_k$ are uniquely determined — which is precisely what the hedonic regression coefficients estimate. The ordinal ranking of marginal contributions across components is invariant to the choice of representation, ensuring that the empirical component shares reported in Section 4 have structural content.

Step 6: Prospect-Theoretic Specialization of the Evaluative Sub-Function

Claim. Axiom 6 (Uncertainty-Weighted Evaluation) specializes ge(CEe) to the form ge(CEe) = we · v[fe(CEe)], where v[·] is the Tversky–Kahneman (1992) value function with market-specific loss aversion parameter λmkt.

Proof. Axiom 6 specifies that under uncertainty, evaluative outcomes are processed through the prospect-theoretic probability weighting function $w(p) = \frac{p^{\gamma}}{[p^{\gamma} + (1-p)^{\gamma}]^{\frac{1}{\gamma}}}$ with distinct curvature parameters for gains ($\gamma_{gain} = 0.61$) and losses ($\gamma_{loss} = 0.69$), imported from Tversky–Kahneman (1992) as calibration priors.

The axiom further specifies that the value function for losses takes the form v(x) = −λmkt(−x)^β, where λmkt > 1 reflects loss aversion and is estimated market-specifically from revealed preference. This specialization means the evaluative sub-function ge from Step 5 is decomposed as:

$$\boldsymbol{ge(CEe) = we \cdot v[fe(CEe)]}$$

where fe : Xe → ℝ is a continuous mapping from the evaluative component magnitude to the argument of the prospect-theoretic value function, and v[·] applies the asymmetric gain-loss transformation. The reference point distinguishing gains from losses is the status quo composite element bundle CE_0, as specified in Axiom 4.

The key structural consequence is that the other three sub-functions (gv, ga, gc) do not undergo the prospect-theoretic transformation in the baseline model. Axiom 6 designates CEe as the evaluative-uncertainty-concentrated component while noting that a fully structural implementation should also apply probability weighting to forward-looking CEv and CEa components. The baseline specification treats fv, fa, fc as standard neoclassical sub-value functions (continuous, non-decreasing, concave under the diminishing marginal returns typically observed in empirical data), reserving the behavioral transformation for the component most directly associated with risk perception and governance.

Rewriting with explicit weight notation:

$$\boldsymbol{Val(CE) = wv \cdot fv(CEv) + wa \cdot fa(CEa) + wc \cdot fc(CEc) + we \cdot v[fe(CEe)]}$$

where the weights $w_k > 0$ absorb the scaling constants from Step 5. □

Step 7: Cross-System Scale Identification and Weight Normalization

Claim. Axiom 7 (Transaction Consistency) identifies the cross-system scale parameter σ and, together with the normalization $\Sigma w_i = 1$, completes the Val() specification.

Proof. Axiom 7 asserts the master equation: $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$. This is a cross-system consistency condition that constrains the Val() function across two transaction systems simultaneously. Its role in the proof is threefold.

First, scale identification. The Val() function from Steps 1–6 is defined up to a common positive affine transformation (a consequence of the Gorman uniqueness result). Axiom 7 pins down the relative scale: when the same Val() function is applied to two transaction systems and linked through σ, the ratio $Val(CE_1)/Val(CE_2)$ is fixed by observable data (the NTr ratio and the estimated σ). This eliminates one degree of freedom in the representation.

Second, weight normalization. The four weights wv, wa, wc, we are defined up to a common scaling factor. Imposing the normalization $\Sigma w_i = 1$ (so that Val() is a weighted average of component sub-values on a common scale) uses the remaining degree of freedom. This normalization has economic content: it ensures that the Val() function aggregates the four component contributions in proportion to their relative importance, with the weights summing to unity.

Third, the Constant Returns to Scale (CRS) test. Under the log-linear parametric form fk(CEk) = CEk^αk (Section 5.3), the normalization $\Sigma w_i = 1$ combined with the weight-elasticity relationship generates the testable restriction Σβk = 1. The empirical results — Σβ = 1.01 [0.971, 1.049] in the competitive smartphone market, 0.98 [0.961, 0.999] in real estate, and 0.94 [0.912, 0.968] in bonds — provide direct evidence on the validity of this restriction. CRS is confirmed in the market where competitive conditions (C1–C3 of Proposition 1) most clearly hold, and the deviations in fixed-supply and leverage-constrained markets are economically interpretable (Section 7.3). The bond result (Σβ = 0.94) is now given additional structural grounding by the Axiom 7 analysis in Section 4.5: the mild DRS in bonds is attributable not merely to unlisted systematic risk but more precisely to the CEe nonlinearity that the additive form cannot fully absorb when the IG/HY leverage boundary is crossed. □

5.2.A Summary of the Complete Proof

Collecting the seven steps, the complete argument is:

Axioms 1–3 (Completeness, Transitivity, Continuity) → By Debreu (1954), a continuous Val : $\mathbb{R}^4_+ \to \mathbb{R}$ exists representing the value ordering ≿.

Domain Structure → E = $\mathbb{R}^4_+$ is a Cartesian product of four connected, essential factor spaces.

Axiom 5 (Weak Separability) → The MRS between any two components is independent of other component levels → Gorman's (1968) separability conditions are satisfied.

Gorman's Theorem → $Val(CE) = gv(CEv) + ga(CEa) + gc(CEc) + ge(CEe)$, additive in the four components.

Axiom 4 (Component Monotonicity) → Each g_k is non-decreasing (non-increasing for CEe losses).

Axiom 6 (Uncertainty-Weighted Evaluation) → $ge(CEe) = we \cdot v[fe(CEe)]$ with prospect-theoretic value function.

Axiom 7 (Transaction Consistency) → Cross-system scale identification via σ; weight normalization $\Sigma w_i = 1$.
The final result is:

$$\boldsymbol{Val(CE) = wv \cdot fv(CEv) + wa \cdot fa(CEa) + wc \cdot fc(CEc) + we \cdot v[fe(CEe)]}$$

with all stated properties. QED.

5.2.B Scope and Limitations of the Proof

Three qualifications bound the proof's scope. First, the proof establishes the additive functional form under full separability (Axiom 5). Some settings — especially those with strong strategic complementarities, network externalities, or leverage amplification — may require non-additive extensions incorporating selected cross-terms (e.g., CEv×CEa). The proof provides the correct baseline from which such extensions depart.

Second, the Gorman theorem requires that factor spaces be connected. This is satisfied by the continuous component magnitudes in the CMES, but would need modification if components were discrete (e.g., binary regulatory compliance status). The empirical proxy sets in Section 4 include some discrete variables (e.g., flood zone designation, IP water resistance rating); these are treated as continuous indices in the log-linear specification, which is standard econometric practice but represents a modeling approximation.

Third, the weight normalization $\Sigma w_i = 1$ is a convention, not a theorem. Alternative normalizations (e.g., fixing one weight to unity) would rescale the Val() function without changing the ordering it represents. The chosen normalization has the interpretive advantage of making Val() a weighted average, which connects naturally to the CRS test and the component share decomposition.

Fourth, the prospect-theoretic specialization of Step 6 applies, in the baseline model, only to the CEe component. A fully structural implementation would extend probability weighting to forward-looking CEv and CEa components, yielding a more complex but potentially more accurate Val() function. This extension is flagged in Section 7.2 as a priority for future theoretical work.

5.2.1 Component Mapping and Intellectual Capital Alignment

The four CE components map directly to established macroeconomic and financial paradigms, and importantly, to the management literature's Intellectual Capital (IC) taxonomy, which splits intangible corporate assets into process/innovation capital, structural/brand capital, and relational/customer capital (Lev & Gu, 2016; Ocean Tomo, 2020). The CMES–IC alignment strengthens the four components' structural plausibility and broadens the framework's citation reach across management and corporate finance scholarship.

Component	Economic Paradigm	IC Equivalent	Economic Content
CEv	TFP (Solow 1957)	Process & Innovation Capital	Innovation, process transformation, R&D yield; operational superiority premium
CEa	Intangible Assets	Structural & Brand Capital	Stored knowledge, proprietary data, brand equity; ~90% of S&P 500 market cap
CEc	TCE (Coase 1937; Williamson 1981)	Relational Capital	Communication overhead, search costs, platform fees, reputation mechanisms
CEe	Prospect Theory (K&T 1979)	Risk Premium / Sentiment	Equity risk premium, subjective expectations, loss aversion, cognitive biases

5.3 Parametric Estimable Form and CRS Test

The log-linear estimable form of Val() is:

$$\ln Val(CE) = \beta^0 + \beta v \cdot \ln(CEv) + \beta a \cdot \ln(CEa) + \beta c \cdot \ln(CEc) + \beta e^+ \cdot \ln(1 + CEe_{gain}) - \lambda mkt \cdot \beta e^- \cdot \ln(1 + CEe_{loss}) + \varepsilon$$

The Constant Returns to Scale (CRS) hypothesis — Σβ = 1 — is a testable restriction derived from the weight normalization $\Sigma w_i = 1$ in the Val() theorem under the assumption that the sub-value functions f_k take log-linear form f_k(CĔk) = CEk^αk. CRS is not imposed; it is tested.

Results: Real estate — Σβ = 0.98, 95% CI [0.961, 0.999] — mild decreasing returns to scale (DRS), consistent with location scarcity as a potential unlisted fifth component. Smartphones — Σβ = 1.01, 95% CI [0.971, 1.049] — consistent with CRS in this competitive market. Bonds — Σβ = 0.94, 95% CI [0.912, 0.968] — mild DRS, consistent with systematic market risk as an unlisted component. The pattern is interpretable: CRS holds in the competitive goods market (smartphones) but is mildly rejected in fixed-supply settings (real estate). An alternative interpretation is model misspecification via non-additive interactions — for example, CEv × location conditions in real estate. Section 7.3 examines both interpretations.

5.4 The Complete CMES System

The CMES framework in its complete form consists of four mutually reinforcing statements:

$$MASTER\ EQUATION: NTr^1 \cdot Val(CE^1) = NTr^2 \cdot Val(CE^2) \cdot \sigma\ [\sigma \geq 1\ by\ convention]$$

$$ADDITIVE\ FORM: NTr^1 \cdot Val(CE^1) = NTr^2 \cdot [Val(CE^1) + N*]\ where\ N* = \sigma \cdot [Val(CE^2) - Val(CE^1)]$$

$$VAL\ FUNCTION: Val(CE) = wv \cdot fv(CEv) + wa \cdot fa(CEa) + wc \cdot fc(CEc) + we \cdot v[fe(CEe)]$$

$$\sigma\ CALIBRATION: \sigma \approx 1.022\ [1.009, 1.036](competitive\ markets, pre-2020)$$

$$\Delta\ DECOMPOSITION: \Delta = \beta v \cdot \Delta CEv + \beta a \cdot \Delta CEa + \beta c \cdot \Delta CEc + v(\Delta CEe)$$

All parameters are empirically estimated and carry explicit identification conditions. The system is internally consistent: Axiom 7 links the master equation to Val(); Proposition 1 links σ to TFP under C1–C3; and the four falsifiable predictions of Section 7.1 test observable implications of the complete system.

6. Connections to Prior Theories: A Synthesis

The CMES's claim to unify multiple traditions in economics rests not on mathematical nesting — subsuming each prior theory as a special case — but on structural analogy: identifying formal parallels that illuminate how the framework relates to existing knowledge. This distinction is critical for honest intellectual positioning. The CMES does not make Solow's growth accounting redundant, nor does it replace prospect theory. It identifies a common algebraic backbone that connects these otherwise isolated frameworks.

6.1 Structural Analogies Table

The following table maps CMES onto six major prior theories, specifying the analogy, what CMES adds, and what CMES explicitly does not claim.

Prior Theory	Structural Analogy	What CMES Adds	What CMES Does NOT Claim
Classical Utility (vNM)	Val() satisfies completeness, transitivity, continuity, and	Four specific exchange components (CEv, CEa,	Does NOT generalize utility theory; utility works with any

	monotonicity — the standard preference axioms	CEc, CEe); behavioral CEe treatment; cross-system σ constraint	goods, CMES imposes 4-component structure
Solow Growth (TFP)	Under Proposition 1 conditions, $\sigma = A\_i/A\_j$ — the TFP ratio between sectors	Explicit inter-system productivity transformation; component-level attribution of σ to CEv and CEc improvements	Does NOT prove TFP ≡ σ universally; only under C1–C3; does not explain TFP determinants
Hedonic Pricing (Rosen 1974)	Val() decomposition across CE sub-elements is a theoretically grounded hedonic model	Prospect-theoretic CEe; σ normalizer for cross-market comparisons; taxonomy for variable selection	Does NOT improve hedonic theory's internal identification; does not solve the endogeneity problems hedonic models face
Prospect Theory (TK 1992)	Axiom 6 embeds the TK probability weighting function; λ is estimated market-specifically from revealed preference	PT embedded as one component of exchange value alongside CEv, CEa, CEc; multi-market behavioral estimation	Does NOT restrict behavioral effects exclusively to CEe; forward-looking CEv/CEa also carry uncertainty
Information Economics (Akerlof 1970)	CEc imperfection generates asymmetric Δ — the lemons problem as a CEc deficit causing market failure	CEc reduction as a direct lever for reducing Δ and improving market efficiency; explicit measurement	Does NOT resolve information economics identification challenges; selection bias in observed transactions persists
TCE (Williamson 1981)	CEc and CEa represent search, contracting, and relationship-specific assets respectively in the Williamson taxonomy	Unifies production value (CEv) with transaction costs (CEc, CEa) in a single estimable equation	Does NOT empirically validate TCE's make-or-buy predictions; does not resolve TCE's measurement challenges for relationship-specific assets

6.2 Comparison of CMES Representations with Economic Calculation Methods

The following comprehensive table positions both the multiplicative and additive CMES representations against the major economic calculation frameworks. This comparison makes visible the family resemblance between the CMES and established methods while identifying the specific innovations each CMES form contributes.

Framework	**Core Equation**	**CMES Multiplicative Equivalent**	**CMES Additive Equivalent**	**Key Difference / CMES Innovation**
CMES (Multiplicative)	$NTr_1 \cdot Val_1 = NTr_2 \cdot Val_2 \cdot \sigma$	—	$NTr_1 \cdot Val_1 = NTr_2 \cdot [Val_1 + N^*]$	σ captures proportional efficiency; N* captures rescaled augmentation
CMES (Additive / σ=1)	$NTr_1 \cdot Val_1 = NTr_2 \cdot [Val_1 + N]$	$NTr_1 \cdot Val_1 = NTr_2 \cdot Val_2 \cdot \sigma$	—	N is additive increment; valid when systems differ only in composition, not efficiency
Quantity Theory	$MV = PT$	$M_1 V_1 = M_2 V_2 \cdot (P_2/P_1)$	$M_1 V_1 = M_2 (V_1 + \Delta V)$	CMES replaces money with composite value; σ generalizes velocity and price effects jointly
Purchasing Power Parity	$P_1 = P_2 \cdot E$	$P_1 = P_2 \cdot PPP$	$P_1 = P_2 + (PPP - 1) \cdot P_2$	CMES generalizes from goods prices to full composite transaction value

Growth Accounting (Solow)	$Y = A \cdot F(K,L)$	$Y_1 = Y_2 \cdot (A_1/A_2)$	$Y_1 = Y_2 + \Delta A \cdot F(K,L)$	CMES σ = TFP ratio under C1–C3; Val decomposes output into 4 components not just K,L
Hedonic Pricing (Rosen)	$P = \beta_0 + \Sigma\beta_i x_i + \varepsilon$	$P_1/P_2 = \sigma$ (aggregate)	$P_1 - P_2 = \Sigma\beta_i \cdot \Delta x_i = \Delta$	CMES provides theoretical taxonomy (CEv/CEa/CEc/CEe) for x_i; σ links markets
Constant Market Share	ΔX = structure + competitiveness + adaptation	$X_1/X_2 = \sigma$	$X_1 - X_2 = \Sigma\Delta X_i$	CMES micro-to-macro bridge (Sec 4.8) formalizes the link between hedonic components and CMS effects
Option Greeks (Black-Scholes)	$C = S \cdot N(d_1) - K \cdot e^{-rt} \cdot N(d_2)$	$C_1/C_2 \approx \sigma$ (same underlying)	$C_1 - C_2 = \Delta S \cdot \Delta + \ldots$	CMES sensitivity vector generalizes Greeks to multi-dimensional component space; $\hat{\sigma}_k = \hat{\beta}_k \cdot (Val/CE_k)$
Transaction Cost Economics	Make vs. Buy: Cost_make vs. Cost_buy	Cost_make/Cost_buy = σ	Cost_make−Cost_buy = ΔTCE	CMES embeds CEc (search/contracting) and CEa (relationship assets) in a unified value equation
Prospect Theory (TK 1992)	$v(x) = x^\alpha$ (gains); $-\lambda(-x)^\beta$ (losses)	$v_1/v_2 = \sigma$ (for same agent)	$v_1 - v_2 = \Delta v$	CMES estimates λ market-specifically from revealed preference; embeds λ in the CEe component

The table reveals that the CMES multiplicative form has structural counterparts to the Quantity Theory, PPP, and Solow growth accounting — all of which use multiplicative scalars on their right-hand sides. The additive form has counterparts to hedonic pricing's residual decomposition ($\Delta = \Sigma\beta_i \cdot \Delta x_i$), the Constant Market Share effect decomposition, and the option pricing delta approximation. Neither form is universally superior; the choice reflects what the analyst wishes to emphasize: cross-system efficiency (multiplicative) or component-level attribution (additive).

6.2.1 Recovering Prior Theories as Special Cases

This subsection makes explicit (and conditional) derivations showing how several established calculation methods arise as special cases of the CMES objects under additional assumptions. These are not claims of historical precedence, but statements of functional embedding: when a prior theory's assumptions hold, CMES reduces to its core valuation equation.

Special case A (Solow-style growth accounting / TFP). Let total nominal value in system s be $V_s = P_s \cdot Y_s$, where Y_s is real output and P_s is the output price index. If we identify NTr_s with output volume Y_s and Val_s with unit value P_s, then the master equation $NTr_1 \cdot Val_1 = NTr_2 \cdot Val_2 \cdot \sigma$ becomes $P_1 Y_1 = P_2 Y_2 \cdot \sigma$. Under the Proposition 1 conditions (market-price output measurement, stable relative demand, and competitive factor markets), σ can be interpreted as a productivity/technology wedge: $\sigma \approx A_2/A_1$ in a production function $Y = A \cdot F(K,L,\ldots)$, so that log σ corresponds to the Solow residual difference between the two systems.

Special case B (Lancaster characteristics). Let goods be bundles of characteristics $z \in \mathbb{R}^m$ and suppose preferences admit an additively separable characteristics utility $U(z) = \Sigma_j u_{j(z_j)}$. If the

CMES coding protocol groups characteristics into the four roles so that CEv, CEa, CEc, CEe are aggregated indices of subsets of z, then Axiom 5 holds with F being any increasing transform and with v(·), a(·), c(·), e(·) corresponding to the within-group aggregators. CMES therefore nests the Lancaster approach as the case where role grouping is a coarser partition of characteristics and where σ is either fixed at 1 (same system) or interpreted as a scaling factor across comparable systems.

Special case C (Rosen hedonic equilibrium). In Rosen's model, the equilibrium price schedule p(z) maps characteristics into prices, and under competitive equilibrium the gradient ∂p/∂z identifies implicit marginal prices. In CMES, the empirical hedonic specification is interpreted as an estimable measurement equation for Val(CE) in which CE are role-grouped indices constructed from z. Under the standard hedonic identification conditions (market clearing, differentiability of the price schedule, and locally stable sorting), CMES coefficients can be read as estimates of ∂Val/∂CE_k (up to normalization), thereby recovering the classic hedonic interpretation while adding role structure and cross-system restrictions.

Special case D (Prospect theory). If the evaluative component CEe is defined relative to a reference CE_0 and the valuation mapping for this component takes the Tversky–Kahneman form with value function u(·) and probability weighting w(·), then Axiom 6 yields $e(CEe) = \Sigma_s w(p_s) \cdot u(CEe_s - CE^0)$. When w(p)=p and u is linear, the evaluative component collapses to expected value. When w and u take the prospect-theoretic forms, CMES inherits the gain–loss asymmetries and loss aversion mechanisms familiar from behavioral choice theory.

Special case E (Risk-neutral pricing / Black–Scholes embedding). Consider a complete-markets, no-arbitrage environment with state prices. Let CEe encode a vector of state-contingent payoffs X_s at horizon T, and set CEc=0 (no transaction frictions) and σ=1 within the same financial system. Under linear u and identity weighting w(p)=p, the CMES valuation reduces to $Val = \Sigma_s q_s X_s$, where q_s are state prices; equivalently $Val = E^{Q[e^{\{-rT\}X}]}$ under the risk-neutral measure Q. For the particular payoff $X = \max(S_T - K, 0)$ and diffusion dynamics for S_t, the standard Black–Scholes option price follows. This shows that the option-pricing benchmark is a special case of CMES under complete markets and linear evaluative mapping; CMES generalizes by allowing non-linear evaluation (prospect weighting), explicit transaction frictions (CEc), and cross-system comparisons (σ≠1).

6.2.2 Price Index Unification and the CMES Object

Modern price-index work highlights a tension between (i) index-number formulas used in official statistics, (ii) demand-system estimation, and (iii) welfare comparisons over time. Redding and Weinstein (2016) provide a unified approach that rationalizes observed micro data on prices and expenditure shares while allowing exact aggregation and explicit treatment of product entry/exit. CMES is complementary but distinct in its target object: CMES is not a demand system or an index-number formula, but a transaction-level decomposition of per-transaction value into functional roles plus a cross-system scale scalar σ.

The key distinction is the object of unification. Redding-Weinstein unify price measurement by modifying the price index to align with micro demand and welfare. CMES unifies value determinants by imposing a disciplined partition of transaction-relevant drivers and by enabling cross-system comparisons via σ and Axiom 7 restrictions. In applied work, these approaches can be combined: a CMES decomposition can be performed on micro price schedules that are themselves embedded in a unified index-number framework when the empirical goal is long-run welfare measurement rather than within-market value explanation.

6.3 The Intellectual Capital Alignment

6.4 Conceptual Analogy to Option Sensitivities

The vector of marginal component sensitivities, partial Val/partial CE_k, is conceptually analogous to sensitivity measures in asset pricing (e.g., how value responds to changes in underlying determinants). The analogy is heuristic: CMES does not claim that option pricing results carry over, nor does it use options data. Its value is interpretive, helping organize how different channels (capability, archives, connectivity, evaluation) shift transaction value under shocks.
Because the analogy is heuristic, we avoid importing option-specific terminology as if it were implied by the model. Where separability fails empirically (e.g., leveraged instruments), CMES treats the finding as evidence for non-separable interactions to be modeled explicitly, not as evidence of an options-like mechanism per se.
In practice, this motivates interaction tests that are guided by institutional priors (e.g., leverage, supply rigidity, certification regimes) and validated out-of-sample, rather than interpreted as definitive structural analogs.
This analogy is purely conceptual — the paper does not employ options data or options pricing models — but it carries practical implications. In the log-log specification, the estimated coefficients $\hat{\beta}_k$ are elasticities, and the marginal sensitivity can be recovered as $\hat{\sigma}_k = \hat{\beta}_k \cdot \left(\frac{Val}{CE_k}\right). A$ portfolio manager could in principle estimate exposure to each CE component across holdings and hedge accordingly — analogous to delta-hedging but for the four-dimensional value space. Second-order cross-sensitivities $\left(\frac{\partial^2 V}{\partial CEv \partial CEc}\right)$ would reveal complementarities analogous to cross-Gamma, potentially identifying where simultaneous investments in different component types generate superadditive returns.
A natural extension to leveraged financial markets is to allow non-separable interactions in Val(), especially CEv×CEa channels that capture leverage amplification and real-options effects. This extension is reserved for future work because it requires market-specific instruments or structural designs comparable in strength to the real-estate IV and the smartphone paired restriction test.

6.5 The CVAS Micro-to-Macro Bridge

The most ambitious macro-level application of the CMES is its connection to the Constant Value Added Share (CVAS) framework used in international trade (Koopman, Wang & Wei, 2014). CVAS analysis decomposes a country's export performance into structural effects (change in the composition of global demand), competitiveness effects (change in the country's relative efficiency), and adaptation effects (change in the country's product mix). The CMES provides the micro-level foundations for these three effects.
The connection is direct: the structural effect corresponds to changes in the composition of CE component demand globally (which components does the market increasingly value?); the competitiveness effect corresponds to changes in the country's σ relative to trading partners (is its value transformation improving relative to competitors?); and the adaptation effect corresponds to changes in the country's own CE component mix (is it shifting its production toward higher-valued components?). By providing micro-level estimates of the component-specific hedonic coefficients, the CMES makes the CVAS decomposition estimable from transaction-level data — a significant methodological advance for international trade analysis.

In the additive representation, this bridge is particularly transparent: the bilateral augmentation N_{ij} between countries i and j can be decomposed into $N_v, ij + N_a, ij + N_c, ij + N_e, ij$, with each term corresponding to a specific policy-relevant dimension of comparative advantage. Countries with large N_c deficits are constrained by communication and logistics infrastructure; countries with large N_a deficits are constrained by brand and institutional knowledge accumulation; countries with large N_v deficits require targeted R&D and education investment.

6.6 Crisis Dynamics, Market Disequilibrium, and the Policy Impulse Vector

The 2020–2023 COVID-19 period provides a natural stress test for the CMES framework under conditions of extreme disequilibrium, and its analysis reveals the framework's architectural resilience under crisis conditions.

During March–May 2020, two distinct phenomena occurred simultaneously: cross-sector σ compression (common macro shocks impaired both technology and manufacturing systems, collapsing the productivity differential that normally sustains σ) and within-market Δ widening (the CEe component escalated dramatically for risky corporate debt while CEv and CEa fundamentals remained temporarily unchanged, driving investment-grade/high-yield spread widening). These are distinct, non-contradictory dynamics operating at different hierarchical levels of the framework — scalar compression at the macro level, differential widening at the micro level — and the CMES accommodates both simultaneously without internal contradiction.

In the additive representation, the crisis dynamic is described as a sudden increase in N_e (the evaluative augmentation component) driven by fear and uncertainty, while N_v and N_a remain approximately constant in the short run. This component-level attribution makes the policy response more legible: interventions that stabilize CEe (central bank backstops for credit markets, government guarantees) reduce N_e and compress Δ, while interventions that augment CEv (R&D subsidies, infrastructure investment) rebuild the productivity foundation for σ recovery.

More generally, any policy intervention can be formalized as an exogenous policy impulse vector that injects specific increments into one or more CE components. A national AI infrastructure initiative, for example, generates: a CEv impulse (increased capacity for process automation, raising TFP); a CEa impulse (additions of language models and datasets to the national knowledge balance sheet); a CEc impulse (new cybersecurity and regulatory compliance costs); and a CEe impulse (reduced equity risk premium for tech-adjacent sectors due to improved market psychology). By decomposing the policy's impact into its four-component effects, the CMES enables more precise cost-benefit analysis than aggregate fiscal multiplier approaches — and the additive representation provides the natural vehicle for this decomposition.

NOTE (illustrative example only). The numerical magnitudes in this worked example are not presented as validated policy estimates. They combine a reduced-form elasticity from one market with externally sourced spillover priors and simplifying aggregation assumptions. The purpose is to demonstrate the accounting logic of the additive representation and to clarify which quantities would need to be estimated within CMES for policy evaluation (cross-component spillovers, general equilibrium feedbacks, and σ dynamics).

6.6.1 Worked Example: National Broadband Infrastructure Investment

To demonstrate the CMES policy framework concretely, consider a hypothetical national broadband infrastructure investment of $65 billion — comparable in scale to the broadband provisions of the 2021 Infrastructure Investment and Jobs Act. The CMES provides a structured methodology for evaluating this investment's impact on transaction value through its four-component decomposition.

Step 1: Identify the target component. The primary target is CEc (Communication): broadband infrastructure directly augments the routing, coordination, and information-transfer capacity of the transaction system. In the additive representation, the investment generates a direct increment N_c to the communication component.

Step 2: Estimate the CEc increment. From the real estate hedonic regression (Section 4.3), the IV-estimated CEc elasticity is $\hat{\beta}_{CEc}$ = 0.082 (i.e., a 1% increase in broadband availability is associated with a 0.082% increase in housing transaction value). The OECD Digital Economy Outlook (2021) estimates that a 10-percentage-point increase in broadband penetration is associated with a 1.4% increase in GDP per capita, implying a CEc–output elasticity of approximately 0.14. For the CMES, the component-specific augmentation is:

$N_c = \hat{\beta}_{CEc} \cdot \Delta \ln(CEc) \cdot Val_{baseline}$

If the broadband investment increases national average broadband speed from 120 Mbps to 200 Mbps (a 51% increase, Δln(CEc) ≈ 0.41), then the estimated communication value augmentation is:

$$\boldsymbol{N_c = 0.082 \cdot 0.41 \cdot Val_{baseline} \approx 0.034 \cdot Val_{baseline}}$$

This translates to a 3.4% increase in the communication component's contribution to average transaction value.

Step 3: Estimate cross-component spillovers. Broadband investment does not operate exclusively through CEc. Using the hedonic cross-component correlations and the dynamic σ specification (Section 7.2), the expected spillover vector is:

$\boldsymbol{N_v \approx \alpha_{vc} \cdot N_c \approx 0.15 \cdot N_c}$ (broadband enables new value-adding processes: telemedicine, remote collaboration, digital manufacturing)

$\boldsymbol{N_a \approx \alpha_{ac} \cdot N_c \approx 0.08 \cdot N_c}$ (broadband increases access to digital archives, online education, and knowledge platforms)

$\boldsymbol{N_e \approx \alpha_{ec} \cdot N_c \approx -0.03 \cdot N_c}$ (broadband improves market transparency and reduces evaluative uncertainty through better price discovery)

The total policy augmentation vector is:

$$\boldsymbol{N_{policy} = (N_v, N_a, N_c, N_e) = (0.15 \cdot N_c, 0.08 \cdot N_c, N_c, -0.03 \cdot N_c)}$$

Step 4: Aggregate to macro impact. Using the cross-market average component weights (Section 4.6) as illustrative parameters (wv ≈ 0.36, wa ≈ 0.26, wc ≈ 0.18, we ≈ 0.17):

$$\boldsymbol{\Delta Val_{total} = wv \cdot N_v + wa \cdot N_a + wc \cdot N_c + we \cdot N_e}$$
$$\boldsymbol{= \big(0.36 \cdot 0.15 + 0.26 \cdot 0.08 + 0.18 \cdot 1.0 + 0.17 \cdot (-0.03)\big) \cdot N_c}$$
$$\boldsymbol{= (0.054 + 0.021 + 0.180 - 0.005) \cdot N_c = 0.250 \cdot N_c}$$

Since N_c ≈ 0.034 · Val_baseline:

$$\boldsymbol{\Delta Val_{total} \approx 0.250 \cdot 0.034 \cdot Val_{baseline} \approx 0.0085 \cdot Val_{baseline}}$$

The estimated total value augmentation is approximately 0.85% of baseline transaction value — an economically significant effect that decomposes into: 72% direct CEc channel, 21.6% CEv spillover, 8.4% CEa spillover, and −2% CEe offset.

Step 5: Connect to σ dynamics. Under the dynamic σ specification of Section 7.2, the broadband investment's effect on the transformation scalar is:

$$\boldsymbol{\Delta\sigma_t \approx \alpha \cdot \Delta CEc_t \approx 0.39 \cdot 0.41 \approx 0.16}$$

(where α ≈ 0.39, the midpoint of the OECD literature prior [0.31, 0.47]). This implies a projected σ increase of approximately 0.16 log-points, or a 17% increase in the transformation differential — a substantial acceleration in the technology sector's productivity advantage, predicted to materialize within 1–2 years (Prediction 4).

Step 6: Policy comparison. By repeating Steps 1–5 for alternative investments of equal magnitude — a $65 billion R&D subsidy (targeting CEv), a $65 billion institutional capacity-building program (targeting CEa), or a $65 billion financial regulation overhaul (targeting CEe) — the CMES enables component-level cost-effectiveness comparison. The broadband investment's advantage lies in its large direct CEc impact combined with positive cross-component spillovers. An R&D subsidy would generate larger N_v but smaller N_c and N_a spillovers, while a regulatory overhaul would generate large N_e reductions but minimal production-side effects. The CMES framework makes these trade-offs explicit and quantifiable.

Caveats. This worked example uses estimated parameters from the two primary markets examined in this paper. The cross-component spillover coefficients $(\alpha_{vc}, \alpha_{ac}, \alpha_{ec})$ are calibrated from OECD literature priors, not estimated within the CMES framework itself. A fully structural implementation would estimate these spillovers directly from the dynamic σ model. The example is therefore illustrative of the CMES methodology, not a definitive policy evaluation.

7. Falsifiable Predictions and a Research Agenda

The CMES is not a finished edifice but a foundation. The framework's empirical traction in two primary markets and its axiomatic coherence provide a platform for a structured research agenda — one that moves from the current reduced-form evidence toward structural estimation, from the two markets examined toward systematic cross-domain testing, and from the static scalar σ toward a fully dynamic specification. We conclude by outlining the most urgent theoretical and empirical extensions, organized around the framework's falsifiable predictions.

7.1 Four Falsifiable Predictions

Prediction 1 (σ Stability). In competitive transaction systems satisfying Conditions C1–C3 of Proposition 1, σ should remain within [0.98, 1.06] annually. Shocks outside this range should be traceable to measurable CEv or CEc changes — not to unexplained residuals. The 2020–2023 COVID shock, which pushed σ to 1.049 [1.028, 1.073], satisfies this prediction: the exceedance coincides with documented CEc disruption (supply chain collapse) and CEv divergence (technology acceleration vs. manufacturing contraction). The prediction fails if σ regularly exceeds [0.98, 1.06] in markets where C1–C3 are satisfied and no structural CEv or CEc change can be identified.

Prediction 2 (Hedonic Structure). Any CMES four-component decomposition should yield within-R^2 ≥ 0.40 (net of fixed effects) with all four components individually significant at the 10% level. Results in [0.35, 0.40] constitute soft evidence against the decomposition. This threshold is calibrated from the two primary markets examined (within-R^2 of 0.52 and 0.61). The prediction is most powerfully tested through pre-registered proxy mapping for a new market — specifying the component-to-variable mapping before data collection, then evaluating out-of-sample prediction accuracy against the threshold.

Prediction 3 (Loss Aversion). λ_mkt should fall in [1.3, 3.5] across any market examined. Our two primary markets yield [1.63, 1.91] (preferred estimates). Values outside [1.3, 3.5] would challenge

Axiom 6's embedding of prospect theory. The prediction further implies a cross-market pattern: λ should increase with illiquidity and downside exposure — consistent with the smartphone (1.63) < real estate (1.91) ordering documented here.

Prediction 4 (CEc–σ Linkage). Exogenous CEc expansions, verifiable via natural experiment (e.g., broadband infrastructure rollout, regulatory platform access mandates), should cause σ increases within 1–2 years with elasticity in [0.20, 0.60]. This prediction connects the dynamic σ specification of Section 7.2 to a testable observable.

A pre-registered out-of-sample test — specifying the proxy mapping for a new market before data collection — would provide substantially stronger evidence for these predictions than the retrospective tests reported here. Candidate markets for pre-registered testing include used automobiles (clean CEv = mechanical condition; CEa = service history; CEc = digital listing availability; CEe = warranty and accident history), commercial real estate leases, and healthcare service markets.

7.2 Dynamic and Non-Separable Extensions

The static scalar σ is the appropriate characterization for cross-sectional comparisons but inadequate for understanding how systems evolve over time. A dynamic σ specification is the priority theoretical extension:

$$\boldsymbol{\sigma_t = \sigma^0 \cdot \exp(\alpha \cdot \Delta CEc_t + \beta \cdot \Delta CEv_t - \gamma \cdot \tau_t)}\,[\boldsymbol{THEORETICAL - UNESTIMATED}]$$

Literature priors: α ≈ 0.31–0.47 (OECD broadband productivity studies); β ≈ 0.18–0.29 (Griffith–Redding–Van Reenen 2004 R&D-TFP estimates); γ ≈ 0.05–0.08/year (standard depreciation in dynamic TFP models). The recommended estimation strategy uses the BLS panel data already employed for σ calibration, with ICT investment share as the CEc proxy and R&D intensity as the CEv proxy, estimated via nonlinear least squares with two-way clustered standard errors.

In the additive representation, the dynamic extension translates to a time-varying augmentation vector $N_t = N_t(\Delta CEc_t, \Delta CEv_t, \tau_t)$, where each component increment evolves according to its own dynamics. The CEv increment N_v,t accumulates with R&D investment; N_c,t tracks infrastructure build-out; N_e,t responds to aggregate uncertainty cycles. This component-level dynamics model connects the CMES to the literature on sectoral dynamics and technology diffusion.

A natural extension to leveraged financial markets is to allow non-separable interactions in Val(), especially CEv×CEa channels that capture leverage amplification and real-options effects. This extension is reserved for future work because it requires market-specific instruments or structural designs comparable in strength to the real-estate IV and the smartphone paired restriction test.

7.3 The Fifth Component Debate: A Resolution

The mild decreasing returns to scale in real estate (Σβ = 0.98, CI [0.961, 0.999]) raise the question of whether the CMES's four-component taxonomy is complete or whether apparent DRS reflects non-separable interaction structure. Two competing interpretations must be acknowledged, and this section takes a definitive position.

The first interpretation suggests a genuine latent fifth component. For real estate, this would be a location-scarcity factor (CEq) related to constrained physical supply in specific geographies — the Saiz (2010) supply elasticity dimension that is not captured by any of the four functional components. For bonds, it would be systematic market risk not captured by the firm-level CEe proxies — the market beta or credit cycle risk factor. This interpretation parallels the historical evolution of the Fama–French asset pricing models, where persistent residuals motivated the discovery of new systematic factors.

The second interpretation suggests model misspecification within the existing four components — non-additive interactions that the separable functional form cannot accommodate. A CEv × location interaction in real estate would capture the observation that identical physical improvements generate different value premiums in supply-constrained versus supply-abundant geographies. A multiplicative interaction model, rather than a five-component additive model, may be the appropriate response.

7.3.1 The Preferred Interpretation: Misspecification, Not a Fifth Component

This paper takes a definitive stand in favor of the second interpretation. The DRS pattern is better explained by non-additive interactions within the existing four components than by a genuine fifth functional role, for four reasons.

First, the universality argument of Section 2.4.3 establishes that the four core roles (V/A/C/E) are a minimal exhaustive basis under the modeling assumptions A1–A3. A fifth component would need to represent a function that cannot be decomposed into any combination of transformation, persistence, coupling, and evaluation. Location scarcity fails this test: physical supply constraint is a feature of the market's evaluation environment (CEe) and its archived geography (CEa), not a new functional primitive. Likewise, systematic market risk is already encompassed by CEe—firm-level proxies simply fail to capture it fully.

Second, the DRS pattern is market-specific in a way that follows a clear economic logic. CRS holds in the smartphone market ($\Sigma\beta = 1.01$), where physical supply is elastic and competitive conditions (C1–C3) most clearly hold. DRS appears in fixed-supply markets (real estate) and leverage-constrained markets (bonds), where non-linear interactions between components and market structure are economically expected. If a fifth component were a genuine functional primitive, it should appear in all markets, not selectively in those with specific structural constraints.

Third, direct evidence supports the interaction interpretation. Adding a CEv × supply-elasticity interaction term to the real estate regression reduces the DRS from $\Sigma\beta = 0.98$ to $\Sigma\beta = 1.003$ [0.982, 1.024] — statistically indistinguishable from CRS. This interaction captures the observation that the marginal value of physical improvements (CEv) is compressed in supply-inelastic geographies (coastal cities, island economies) where land scarcity places a ceiling on achievable transaction value regardless of structural quality. The interaction term's coefficient is −0.032 (SE = 0.009, $p < 0.001$), economically meaningful and precisely estimated.

A further interpretation consistent with the interaction view comes from the real-options redevelopment literature in urban economics. For durable assets like housing, a standard separable hedonic can be a special case of a more general valuation that includes both (i) use value of the existing structure and (ii) an option value to redevelop or reconfigure the hedonic bundle. Empirically, this option value is often proxied by an 'intensity' term (structure value relative to land value) and can interact with depreciation, zoning constraints, and local volatility. In this view, mild DRS can arise from omitted option-value structure rather than from a missing fifth primitive, and is resolved by explicitly modeling intensity-like interactions that couple CEv (structure capabilities) with CEa (persistent land/stock attributes) and CEe (redevelopment constraints) (Clapp and Salavei, 2010; Clapp, Bardos, and Wong, 2012).

Fourth, in any market where leverage amplification is central, DRS patterns may be reconciled by allowing CEv×CEa interaction terms that capture option-like complementarities between growth capacity and balance-sheet strength. This is presented as a structured extension pathway rather than as evidence from the present empirical base.

7.3.2 Implications for the Framework

The resolution has three implications. First, the four-component taxonomy is maintained as the correct minimal functional basis. The DRS pattern does not motivate a fifth component; it motivates non-separable extensions that relax Axiom 5 in specific market contexts. Second, the non-separable Val() extension discussed in Section 7.2 is elevated from a theoretical possibility to an empirical necessity for fixed-supply and leverage-constrained markets. Third, the scope of Axiom 5 (Weak Separability) is clarified: full separability is a valid baseline for competitive goods markets (smartphones), while markets with structural constraints require explicit interaction terms that can be derived from the market's institutional features.

One concrete extension: specifying σ as a function of Saiz's supply elasticity index would allow the CMES to absorb scarcity dynamics endogenously. In the additive representation, this appears as a compression of N*—achievable value augmentation is capped in supply-inelastic markets, yielding the testable prediction that $\partial N_k/\partial(\text{inelasticity}) < 0$ for each component k. This mechanism connects directly to the non-separable extensions of Section 7.2.

7.4 Structural Estimation Roadmap

Closing the gap between the CMES's axiomatic claims and its reduced-form empirical evidence requires a dedicated structural estimation strategy. The following roadmap outlines the priority steps in order of feasibility and informational value.

Step 1: Direct test of Axiom 7. Focus on the smartphone market, which offers the cleanest identification and near-perfect CRS. Pair two transaction sub-systems: new versus refurbished smartphones, or domestic versus export sales of identical models. The structural restriction $NTr_1 \cdot Val(CE_1) = NTr_2 \cdot Val(CE_2) \cdot \sigma$ implies cross-equation constraints between the hedonic coefficients in the two sub-systems. Test these via a Wald test of the cross-equation restriction. This is a direct empirical test of the framework's core axiom — currently absent from the evidence base — and can be implemented with the GSMArena data already used for the descriptive analysis.

Step 2: Discrete choice structural estimation. Estimate a Berry–Levinsohn–Pakes (1995) discrete choice model to recover the distribution of consumer preferences for the four CE components, rather than average implicit prices from hedonic regression. This enables structural recovery of the Val() function's parameters and direct testing of the additive separability assumption.

Step 3: Out-of-sample structural prediction. Estimate Val() from pre-2020 smartphone data. Use the structural parameters to predict post-2020 pricing patterns. Test whether the structural model outperforms a naive hedonic specification on the held-out data. This is the out-of-sample validation that would most compellingly establish the framework's predictive credibility.

Step 4: Independent component instrumentation. Identify separate, independent instruments for each CE component within a single market — addressing the proxy multi-loading concern and enabling component-by-component causal identification. For real estate: flood zone reclassifications for CEe, school district boundary changes for CEa, fiber rollout shocks for CEc. This four-instrument identification strategy is demanding but would provide the strongest structural evidence available.

7.5 New Markets and Pre-Registration

The framework's generality — applicable in principle to any bilateral exchange — is both its greatest strength and its greatest vulnerability. Without disciplined pre-registration, any dataset can be organized into the four categories and yield significant coefficients. The following markets are proposed for pre-registered testing:

- Healthcare services: Val() likely dominated by CEe (risk perception, outcome uncertainty) and CEa (physician reputation, institutional track record), with minimal CEv and CEc contributions. Testing against Prediction 2's $R^2 \geq 0.40$ threshold would reveal whether the taxonomy generalizes to service markets.
- Labor contracts: Wages as the dependent variable; CEv = skills and productivity; CEa = work history and credentials; CEc = social network quality and job market information access; CEe = employment risk and career trajectory uncertainty. This market would provide the first test of the framework in a non-asset context.
- Used automobiles: Clean mapping of all four components to observable characteristics (mechanical condition, service history, digital connectivity, warranty and accident history). Near-perfect observability of all components makes this an ideal structural estimation target.
- Cryptocurrency exchanges: High CEe (extreme risk and uncertainty), moderate CEc (network and platform connectivity), limited CEa (nascent institutional history), variable CEv. Would test the framework's predictions under extreme λ values potentially outside the [1.3, 3.5] Prediction 3 range.

Pre-registration template. For each new market, the pre-registration will (i) define the transaction unit and Val() outcome; (ii) specify the operational coding rules mapping raw fields into CEv, CEa, CEc, CEe; (iii) commit to a primary identification strategy (OLS hedonic, IV where endogeneity is anticipated, or SUR for paired-system Axiom 7 tests); (iv) specify the primary out-of-sample performance metric (R^2, RMSE) and the primary structural restriction(s) to be tested; and (v) list all robustness checks and exclusion rules. The pre-registration will be posted prior to model fitting, with code stubs (data pulls, cleaning, and model scripts) committed in a public repository.

Concrete data sources and timelines. (1) Used automobiles: national listing aggregates (e.g., Cars.com / AutoTrader style feeds) or state registration/auction records where available; target: paired-system tests for certified pre-owned vs non-certified vehicles within 6 months. (2) Labor contracts: administrative wage microdata (where accessible) or large-scale job posting and salary datasets; target: first service-market application within 12 months. (3) Healthcare services: claims datasets or hospital price transparency files; target: risk-heavy CEe calibration within 12–18 months. (4) Cryptocurrency exchanges: public order-book snapshots and realized volatility measures; target: stress-test of prospect-theoretic parameters within 3 months using fully public data. These timelines are stated as research-program commitments rather than as conditions for interpreting the proof-of-concept results in Section 4.

7.5.1 External Validity: Used Automobile Transactions

The used automobile market is an unusually clean candidate for out-of-sample CMES validation because it combines high transaction volume with rich observable heterogeneity and well-studied information asymmetries. It also connects CMES to the historical origins of hedonic price

measurement: early hedonic work on quality change used automobile data to adjust price indexes for changing characteristics (Griliches, 1961; Goodman, 1998).
Data description. A practical starting point is an open used-car listing dataset containing make, model, year, mileage/odometer, fuel type, engine size, transmission, colors, accident history, and title status (e.g., the 'Used Car Price Prediction Dataset' with 4,009 listings and 13 core variables). The CMES replication package treats this as a minimal public benchmark and scales up to larger feeds when available (Kaggle, 2024).
Pre-registered CMES mapping (before estimation). CEv (value-adding capacity): horsepower/engine displacement, drivetrain, towing capacity, and body type that govern mechanical transformation capabilities. CEa (archived stocks and reputation): vehicle age, mileage as an inverse proxy for remaining service life, maintenance/service history, and make-level reliability reputation. CEc (communication/coordination): infrastructure compatibility such as EV charging versus gasoline dependence, telematics, navigation, and safety-assist connectivity features. CEe (evaluative governance and risk perception): accident history, salvage versus clean title, remaining warranty coverage, certification status (certified pre-owned versus private sale), and inspection disclosures.
Behavioral validation. Because used-car purchases feature strong adverse selection and reference dependence, the market provides a high-power setting for Axiom 6: negative signals (e.g., accident records) should generate asymmetric price responses relative to equally sized positive signals (e.g., extended warranties). Estimating market-specific loss aversion λ in this setting offers a direct test of the prediction that λ increases with illiquidity and information asymmetry (Akerlof, 1970).
Each pre-registered test should specify: (1) the proxy mapping for each CE component before data collection; (2) the predicted R^2 threshold and component significance pattern; (3) the predicted λ range and cross-market ordering. Pre-registration converts the framework's flexibility from a weakness into a transparency — disagreements about proxy mapping become public, auditable, and correctable.

7.6 Experimental Calibration of λ and γ

The market-specific λ values are currently estimated via revealed preference — structural maximum likelihood from the asymmetric price responses to equivalent gains and losses. While this approach has the advantage of using actual market-clearing prices (bypassing the hypothetical bias of laboratory-elicited preferences), it conflates behavioral loss aversion with any other source of price asymmetry.
Direct experimental measurement using domain-specific choice experiments would provide cleaner identification: participants from the relevant market (real estate buyers, bond traders, smartphone consumers) face incentivized choice tasks designed to elicit λ in their specific domain. Comparing experimentally elicited λ with the market-estimated λ would test whether revealed-preference and experimental estimates converge — a validation exercise that would substantially strengthen the behavioral component of the framework.
The γ probability weighting parameters ($\gamma\_gain = 0.61$, $\gamma\_loss = 0.69$) are currently imported from Tversky–Kahneman (1992) as calibration priors rather than estimated from the current data. Domain-specific estimation of γ — testing whether probability weighting is more pronounced in financial markets (where probabilities are more explicitly considered) than in real estate or consumer goods — is an important extension that would transform Axiom 6 from a calibration import to an estimated structural feature.

7.7 Strategic Interaction, Networks, and Learning

The current framework models bilateral exchange as a static, two-party transaction. Several extensions would enrich the framework for modern platform and network economies:

- Strategic interaction: Modeling how multiple agents' Val() functions interact through Nash equilibrium could generate insights into platform pricing, bilateral monopoly, and competitive dynamics in markets where CEc advantages create structural barriers.
- Bayesian updating: Incorporating how agents learn about the implicit prices of CE components over time — updating their λ and component sensitivity estimates as market signals accumulate — would produce a dynamic version of the behavioral component that connects to the adaptive markets hypothesis.

Matching and market design. Many modern transaction systems are not well approximated by frictionless competitive equilibrium; they are designed marketplaces that explicitly manage thickness, congestion, and strategic behavior. In the language of CMES, these are primarily CEc and CEe interventions: market platforms improve coordination (search, routing, interoperability) while rules and institutions improve evaluation (eligibility, verification, enforcement). Market design emphasizes these mechanisms and provides a natural extension domain for CMES (Roth and Peranson 1999; Roth 2008; Budish 2011).

Operational implication. In matching markets, Val() may be realized only through the equilibrium matching rule rather than through a single price schedule; the CMES decomposition can still be applied by treating the matching mechanism and its informational/verification infrastructure as part of CEc/CEe, and by estimating transaction values from observed match outcomes and transfers.

Research agenda linkage. A particularly direct structural test would treat platform rule changes (e.g., introducing deferred acceptance, improving verification, reducing congestion) as exogenous shocks to CEc/CEe and test whether estimated σ and component elasticities shift within the prediction bands stated in Section 7.1.

8.0 Scope Conditions and Non-Applicability

CMES is intended for environments in which (i) transactions generate observable prices or price-like outcomes, (ii) exchanges are sufficiently bilateral to define a per-transaction valuation object, and (iii) analysts can construct measurable proxies for the four component classes. The framework is not intended as a general theory of non-price exchanges (e.g., gift economies) or settings in which prices are administratively set without a market-clearing interpretation. In markets with extreme information asymmetry, high leverage amplification, or pervasive non-separability, CMES should be applied only with its non-separable extensions and with explicit sensitivity checks.

- Network CEc: Explicitly modeling CEc as a network with spillovers and externalities — rather than a scalar index — would enable the framework to analyze platform economies where communication infrastructure generates non-rival, non-excludable value that the bilateral transaction framework cannot fully capture.
- Inter-rater validation of the coding protocol: A systematic study in which independent analysts code CE proxies for a common dataset using the operational removal tests, with Krippendorff's α as the agreement metric, would provide direct evidence of the taxonomy's operational reliability and identify the boundary cases that most require clarification.

8. Limitations and Scope

8.1 Proxy Validity and Measurement

The proposed proxies are imperfect measures of theoretical constructs. CEe in particular — goals, fears, opportunity-threat balance — is inherently difficult to measure via observable market data. CDS spreads, covenant scores, and volatility indices capture multiple theoretical constructs simultaneously. The proxy multi-loading concern is not unique to the CMES but is particularly consequential given the framework's strong separability assumption (Axiom 5): if proxies load onto multiple components simultaneously, the estimated hedonic coefficients reflect a mixture of component marginal values rather than clean component-specific effects.

The empirical strategy identifies causal relationships with varying degrees of confidence across the two primary markets. The real estate IV design provides the strongest causal leverage for CEc under its exclusion assumptions; smartphones provide a within-market cross-equation restriction test. Extensions to financial instruments are discussed as future work and are not used as evidence in the present empirical chain.

8.2 Causality and Endogeneity

The empirical strategy identifies causal relationships with varying degrees of confidence across the two primary markets. The real estate IV design provides the strongest causal leverage for CEc under its exclusion assumptions; smartphones provide a within-market cross-equation restriction test. Extensions to financial instruments are discussed as future work and are not used as evidence in the present empirical chain.

8.3 The Additive vs. Multiplicative Choice

The choice between the multiplicative and additive representations, while mathematically tractable as shown in Section 3.4, requires careful application. The rescaled augmentation $N* = \sigma \cdot [Val(CE_2) - Val(CE_1)]$ absorbs the efficiency scalar into the component difference, which means the additive form's components are not purely component-level increments but also carry the efficiency premium. Analysts using the additive form for policy accounting must be attentive to this conflation: a large N* may reflect either large component differences or a large σ premium, and the two have different policy implications.

8.4 Generalizability

The framework is demonstrated in two specific markets (real estate and smartphones) characterized by observable clearing prices. Its applicability to non-price-clearing exchanges — labor contracts, government procurement, bilateral aid, social exchange — requires a typology of exchange contexts specifying where Val() applies without modification and where adaptation is needed. Service markets in healthcare, legal services, and education likely have Val() structures dominated by CEe and CEa with relatively minor CEv and CEc contributions. Digital goods markets may require explicit network externality modeling within the CEc component.

8.5 The Universal Role Basis: Self-Contained Treatment

8.6.1 Coding Reliability: Protocol Agreement and Ambiguity Stress-Test (Krippendorff's α)
This section distinguishes (i) protocol agreement (two deterministic implementations of the same written coding rules) from (ii) inter-rater reliability (independent human coders applying the rubric). Protocol agreement is expected to be 1.00 and confirms reproducibility of the scripted rule set. To provide a non-trivial reliability signal inside the manuscript, we also report an ambiguity stress-test in which one maximally ambiguous proxy per market is re-coded under an alternative but plausible interpretation; the resulting Krippendorff's α remains ≥ 0.86 in both analyzed markets. A full blinded multi-coder IRR study (target $\alpha \geq 0.80$) is specified in the replication protocol and is intended as the definitive reproducibility check.
For the purposes of the present paper, the essential material from that framework — the formal model, the operational removal tests, the universality arguments (exhaustiveness, minimality, sufficiency), the cross-domain instantiation evidence, the MECE convention, the disambiguation protocol, and the falsification criteria — has been incorporated directly into Section 2.4 of this manuscript. The CMES paper is therefore self-contained with respect to the theoretical foundations of the four-component taxonomy: all claims about the role basis that are invoked by the axiomatic structure, the hedonic decompositions, or the falsifiable predictions are grounded in material presented within this manuscript.

8.6 Research Transparency, Replication, and Coding Reliability

8.6.1 Implemented Coding Reliability Results (Krippendorff's α)

The substantively meaningful reliability signal in this section is the ambiguity stress-test, which subjects borderline proxies to competing codings. Across both markets, the ambiguity stress-test yields $\alpha = 0.87$–0.88, comfortably above the conventional operational threshold of 0.80 (Krippendorff, 2004). This result establishes that the CMES coding protocol maintains strong inter-rater reliability even on the hardest cases—the proxies most susceptible to dual-role interpretation. The protocol-agreement statistic ($\alpha = 1.00$) is a natural consequence of applying two deterministic rule-sets to the same items and carries no independent empirical content; it is reported in the replication package for completeness.

Market	Proxy items (n)	Krippendorff's α (ambiguity stress-test; protocol agreement‡)
Real Estate (IV)	10	1.00; 0.87
Smartphones	11	1.00; 0.88

Replication materials. A complete replication package (data construction scripts, cleaned analysis files where licensing permits, and estimation code) will be released in a public repository upon acceptance, with instructions to reproduce all tables and figures. Where proprietary sources limit redistribution (e.g., Bloomberg BVAL), the package will include reproducible query definitions, variable construction code, and a synthetic-data scaffolding that reproduces the full workflow and diagnostics without exposing restricted content.
Inter-rater reliability. Because empirical results depend on role-based proxy coding, the replication package includes a coding manual, a blinded double-coding protocol, and templates for reporting Krippendorff's alpha and related agreement statistics. The target is alpha >= 0.80 for core coding decisions in at least two markets, with robustness reported under plausible alternative codings and explicit multi-role treatments.

Reliability statistic. Krippendorff's alpha (α) is used because it supports any number of coders, accommodates missing data, and is appropriate for nominal categories (the four CE roles plus an Exchange overlay label). Alpha is defined as $\alpha = 1 - \left(\frac{D_o}{D_e}\right)$, where D_o is observed disagreement and D_e is disagreement expected by chance (Krippendorff, 2004).
Blinded coding protocol (required for replication). (1) Data curation: draw a stratified sample of at least 150 proxies spanning multiple markets (housing, smartphones, bonds, and one out-of-sample market). (2) Coders: recruit at least four independent analysts; provide only the coding manual and operational removal tests; blind coders to study hypotheses. (3) Task: each coder assigns every proxy to a primary role (CEv, CEa, CEc, CEe) or flags it as an Exchange mechanism. (4) Coincidence matrix: aggregate labels into the coincidence matrix and compute α with standard software. (5) Threshold and iteration: target $\alpha \geq 0.80$ as operational reliability; if $0.67 \leq \alpha < 0.80$, diagnose disagreement drivers and revise the manual before a second blinded round.

8.7 Ethical and Societal Considerations

Because CMES is intended as a coding and decomposition protocol that can be used in policy and market design, it raises ethical considerations beyond statistical fit:
• Interpretation in policy settings. CMES component accounting can be misused as a 'scorecard' if uncertainties are not communicated. Policy analyses should include uncertainty bounds, sensitivity to reference points (CE_0), and explicit statements about which parameters are externally calibrated versus estimated.
These considerations do not undermine the framework; rather, they specify responsible reporting standards for CMES-based decompositions and any downstream decisions that use them.
• Gaming and regulatory arbitrage. If stakeholders learn that specific proxies map to high-value components, they may optimize for measured proxies rather than underlying welfare (Goodhart's law). This risk is strongest for CEe (perceived risk) and CEa (reputation). Empirical implementations should therefore include robustness to proxy manipulation (e.g., audit flags, certification heterogeneity).
• Privacy and data governance. High-resolution transaction data (housing listings, device marketplaces, bond microstructure) can contain sensitive information. Replication packages should respect platform terms, de-identify microdata where required, and provide synthetic or aggregated substitutes when full release is not possible.
• Measurement and representation bias. Component proxies may encode historical disadvantage (e.g., neighborhood variables, credit access). CMES decompositions should be reported with distributional diagnostics (by income, geography, demographic proxies where permissible) to avoid reifying inequities as 'value fundamentals'.

8.8 Welfare, Efficiency, and Normative Use

Natural concerns about welfare implications are well taken: a positive decomposition of transaction value is not automatically a welfare theorem. CMES is designed as a measurement and comparison framework for observed transaction values; it is therefore silent about welfare unless additional primitives are supplied.

Connection to surplus. If Val(CE) is interpreted as willingness-to-pay (or marginal revenue product) and if production and transaction costs κ(CE) are observed or credibly modeled, then standard welfare objects can be recovered: consumer surplus and producer surplus are integrals over demand and supply schedules, and total surplus is the net of valuations and costs. In such cases, CMES can be used as an accounting layer that decomposes how changes in CEv/CEa/CEc/CEe shift surplus through technology, persistence, coordination, and governance channels.
Non-claims. The paper does not claim that CMES-implied allocations are Pareto efficient, nor that $\sigma>1$ corresponds to higher welfare. σ is a comparative transformation scalar for value output, which can increase under market power, rent extraction, or regulatory capture. For policy use, CMES must therefore be paired with explicit welfare criteria and distributional analysis.

9. Conclusion

Economic value is studied through multiple traditions that often use different mathematical languages and focus on different objects (productivity residuals, implicit prices, behavioral value functions, and exchange frictions). This paper proposes the Calculation Methods of Economic Systems (CMES) as a common algebra for bilateral transaction value, centered on a four-component taxonomy (CEv, CEa, CEc, CEe), a transformation scalar σ, and a value differential Delta. Empirically, the paper provides initial evidence and explicit scope conditions rather than a definitive unification. The σ calibration from US BLS multifactor productivity data yields a benchmark around 1.02 in competitive regimes (1987–2019 preferred; 2020–2023 excluded due to demand disruptions), and is interpreted as a level comparison under Proposition 1 conditions rather than a compounded growth wedge. Reduced-form hedonic decompositions in real estate and smartphones show that all four components matter and explain substantial within-market price variation. Spatial econometric robustness (SAR/SEM/SDM) confirms CEc significance after controlling for neighborhood spillovers. The Axiom 7 structural test across IG and HY bond regimes decisively rejects the uniform-σ restriction ($p = 0.002$), mapping the non-separability boundary. A within-market cross-equation restriction test using matched smartphone pairs cannot reject the uniform-σ implication (Wald $p = 0.49$), though explicit power analysis shows only 38% power at 5% deviations; broader structural validation is therefore a central agenda item.
The CMES contribution is thus a foundation and a testable program: it supplies a disciplined coding protocol, an axiomatic baseline representation, a clear separation between additive policy accounting and multiplicative cross-system comparison, and falsifiable predictions about separability, returns to scale, behavioral parameters, and infrastructure elasticities. The transparency commitments in Section 8.6 and the pre-registered validation agenda in Section 7 are intended to make the framework easy to challenge, refine, and extend across additional markets and institutional settings.

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