

Critical (String-like) Regimes as Partition Instability

Nesen Oleg

Abstract

We propose a conservative definition of “string-like” criticality within a modular foundations program: not as a microscopic ontological primitive, but as a dynamical regime in which no stable phase partition persists under RG-proxy modular flow. Building on Paper C, phases are encoded by an optimal correlation-localizing partition $P^{(\rho)}$ and locality is quantified by the block total correlation $\mathcal{T}_{P^{(\rho)}=D(\rho\|\otimes_{x \in P} \rho_x)}$. We define partition instability markers based on (i) the switching rate of $P^{(t)}$, (ii) the flatness of the partition landscape $\Phi(P; \rho)$ measured by gaps between near-optimal partitions, and (iii) the persistence of nonlocality through elevated \mathcal{T}_P . We connect these markers to modular spectral diagnostics from Paper D (spectral quantiles k_q , commutator probes L , running exponents ν) and propose a reproducible PASS/FAIL detection protocol with data-driven thresholds calibrated from baseline percentiles. A demonstrative case study (Variant A) uses the dim=64 Gibbs-chaotic mixing path and a critical zoom around $p_x \approx 0.04$ to illustrate how partition flatness and switching emerge alongside spectral-tail anomalies, providing a practical route to identifying critical regimes in modular RG-proxy flows.

1 Introduction

“String-like” language is often invoked to describe critical behavior: extended correlations, lack of stable localization, and sensitivity to scale. In a foundations setting, importing “strings” as microscopic primitives risks introducing ontological commitments that are not warranted by available operational evidence. The modular foundations program developed in Papers A–D supports a conservative alternative: interpret “string-like” as a *critical regime* in an informational RG-proxy flow, detectable through representation-invariant and reproducible criteria.

The A→B→C→D chain provides the necessary components: Paper A yields the canonical state object ρ ; Paper B yields the canonical modular generator $K = -\log \rho$ and CPTP-compatible RG-proxy dynamics (discrete primary); Paper C defines phases and locality via access structure and information distances; Paper D provides reproducible spectral diagnostics and stability rules. The present paper introduces an operational criticality concept consistent with this chain: a regime in which correlation geometry fails to settle into stable blocks, accompanied by sustained nonlocality (in the sense of Paper C) and spectral diagnostic anomalies (in the sense of Paper D).

Scope. We do not claim a derivation of string theory. We introduce a rigorous, testable regime definition—“string-like” as a label for partition instability and locality failure—and supply a reproducible detection protocol with a demonstrative case study.

2 Minimal background from Papers A–D

- **Paper A (state emergence):** ρ is the canonical operator representation of normalized probabilistic structure via $p(E) = \text{Tr}(\rho E)$.
- **Paper B (modular RG-proxy dynamics):** $K_\rho = -\log \rho$; relative modular constructions with CPTP (GKSL) completion; the evolution parameter is interpreted as informational scale and implemented discretely.

- **Paper C (phases and locality):** phases encoded by (P, \mathcal{A}_F, Z_F) ; reference σ_F via MaxEnt/I-projection; locality quantified by $\mathcal{T}_{P(\rho)=D(\rho \parallel \otimes_{X \in P} \rho_X)}$.
- **Paper D (diagnostics and reproducibility):** spectral quantiles $k_q = -\log \lambda_q$; commutator probes $L(\rho; O)$; running exponents ν ; stability via bootstrap and pre-registered model selection; explicit domain statements.

3 Partition stability and landscape flatness

3.1 Partition objective and optimal partition

Let P range over a specified family of partitions. Define the correlation-localization objective (Paper C)

$$\Phi(P; \rho) = \sum_{X \neq Y \in P} I_\rho(X : Y) + \eta |P|,$$

where $\eta > 0$ is a complexity penalty. Define an optimal partition

$$P(\rho) \in \arg \min_P \Phi(P; \rho).$$

If multiple minimizers exist, a fixed tie-breaking rule is applied.

3.2 Near-optimal competitors and the flatness gap

Let $P_{(1)} = P$ be the best partition and $P_{(2)}$ the second-best under Φ (after tie-breaking). Define the flatness gap

$$\Delta\Phi(\rho) := \Phi(P_{(2)}; \rho) - \Phi(P; \rho).$$

Small $\Delta\Phi$ indicates a flat landscape where multiple partitions compete.

3.3 Top- K diversity ($K = 3$)

Let $\mathcal{P}_3(\rho)$ be the set of the top 3 partitions under Φ . A diversity index can be defined via distinct-count (number of distinct partitions across a window, up to equivalence) and/or via an average pairwise partition distance computed from co-membership matrices.

3.4 Switching rate under a flow

Along a flow $\rho(t)$ or a discrete sequence ρ_n , define a switching indicator

$$\mathbb{S}(t) = \mathbf{1}\{P^{(t+\delta t)} \neq P^{(t)}\}.$$

Define the switching rate on a window W by

$$\text{SwitchRate}(W) = \frac{1}{|W|} \sum_{t \in W} \mathbb{S}(t).$$

4 Locality persistence and its failure

Following Paper C, define locality relative to the optimal partition via block total correlation

$$\mathcal{T}_{P(\rho)=D(\rho \parallel \otimes_{X \in P} \rho_X)}.$$

In a stable local phase, $\mathcal{T}_{P(\rho(t))}$ remains small across scale windows. In a critical regime, \mathcal{T}_P fails to remain small, reflecting the inability to compress global correlations into a block-product description consistent with marginals.

5 Defining the critical (string-like) regime

Definition 1 (Critical/string-like regime on a window W). A trajectory segment W is called *critical (string-like)* if:

1. **Partition instability:** $\text{SwitchRate}(W) \geq \theta_S$, or top-3 diversity exceeds a threshold θ_{div} .
2. **Landscape flatness:** $\Delta\Phi(\rho(t)) \leq \theta_\Phi$ holds for a nontrivial fraction of $t \in W$.
3. **Failure of stable locality:** $\mathcal{T}_{P^{(t)}(\rho(t))}$ does not remain small across W .
4. **Spectral corroboration:** at least one spectral marker is present in W (e.g., ν -instability or closure breakdown from Paper D).

Remark 1. The term “string-like” labels a regime where correlations do not stabilize into local blocks across scales; it is not an ontological claim about microscopic strings.

6 Spectral signatures and the bridge to Paper D

We use Paper D diagnostics as an independent corroboration layer:

- Quantile coordinates: $k_q(t) = -\log \lambda_q(\rho(t))$.
- Commutator probes: $L(\rho; O) = \|[K_\rho, O]\|_F / (\|K_\rho\|_F \|O\|_F)$.
- Running exponent: $\nu(t)$ via pre-registered sliding log–log fits.
- Closure reliability: degradation of predictive relations $\nu \approx f(k_q, q)$.

These markers provide representation-invariant evidence of spectral reorganization that should co-vary with partition instability if criticality reflects genuine structure rather than combinatorial noise.

7 Detection protocol (PASS/FAIL)

7.1 Inputs

Flow $\rho(t)$ or ρ_n ; partition family; penalty η ; tie-breaking and equivalence rule; windowing; thresholds $\theta_S, \theta_\Phi, \theta_T$; diagnostic quantiles q ; observables set \mathcal{O} ; bootstrap seeds.

7.2 Step A (partition layer)

For each t : compute $P^{(t)}$, compute $\Delta\Phi(t)$ and top-3 diversity measures, and compute $\mathcal{T}_{P^{(t)}(\rho(t))}$.

7.3 Step B (spectral layer)

For each t (or window center): compute $k_q(t)$ (e.g., $q = 0.5, 0.8, 0.9$), compute $L(t; O)$ for representative $O \in \mathcal{O}$, and compute $\nu(t)$ on accepted windows.

7.4 Threshold calibration (default choices; no hand-tuning)

To avoid ad hoc threshold choices, we calibrate the criticality thresholds from the baseline trajectory using robust, pre-registered statistics. Let W_{base} denote the baseline region outside the candidate critical zoom,

$$W_{\text{base}} : p \in [0.01, 0.20] \setminus [0.02, 0.08],$$

and compute empirical distributions of the windowed switching rate $\text{SwitchRate}(W)$, the flatness gap $\Delta\Phi(p)$, and the locality measure $\mathcal{T}_{P(\rho(p))}$ along W_{base} .

We use a percentile-based scheme:

- **Flatness threshold:** $\theta_{\Phi} := Q_{0.10}(\Delta\Phi)$ on W_{base} .
- **Locality-failure threshold:** $\theta_T := Q_{0.90}(\mathcal{T}_P)$ on W_{base} .
- **Switching threshold:** $\theta_S := Q_{0.90}(\text{SwitchRate})$ over baseline windows.

Given a candidate window W (e.g., the critical zoom $p \in [0.02, 0.08]$), we declare W critical/string-like if:

1. $\text{SwitchRate}(W) \geq \theta_S$ (or top-3 diversity exceeds its baseline threshold),
2. $\Delta\Phi(p) \leq \theta_{\Phi}$ for at least 30% of sampled points in W ,
3. the median (or high-quantile) of $\mathcal{T}_{P(\rho(p))}$ over W exceeds θ_T , and
4. at least one spectral corroboration marker holds in W (e.g., degradation of the Paper D closure $\nu \approx f(k_q, q)$ or elevated variability in ν).

All thresholds and the resulting PASS/FAIL decision rate are evaluated under seed bootstrap and reported with confidence intervals.

7.5 Decision summary

Declare a window W critical/string-like if partition-layer and spectral-layer conditions agree under the pre-registered thresholds and bootstrap stability checks.

8 Demonstrative case study (Variant A)

8.1 Baseline trajectory

We use the dim=64 reference experiment from Paper D: $N_{\text{vis}} = 5$, $N_{\text{hid}} = 1$, mixing path $\rho(p) = (1 - p)\sigma_{\text{geo}} + p\rho_{\text{ng}}$, with $p \in [0.01, 0.20]$ and $\Delta p = 0.01$, using a seed ensemble consistent with the Paper D baseline.

8.2 Critical zoom (fixed)

We focus on $p \in [0.02, 0.08]$ with $\Delta p = 0.01$, using top- K with $K = 3$. Motivation: Paper D reports a stable crossover marker $p_x \approx 0.04$ with maximal $\Delta\nu$ near window centers $p_c = 0.035 \rightarrow 0.045$. We test whether this region coincides with partition flatness, switching, and locality failure.

8.3 Minimal outputs (five deliverables)

We compute and report:

1. Partition timeline: $P^{(p)}$ over baseline and zoom; switching rate in moving windows.
2. Flatness curve: $\Delta\Phi(p)$ and/or a top-3 diversity index.
3. Locality measure: $\mathcal{T}_{P^{(p)}(\rho(p))}$.
4. Spectral overlay: $k_q(p)$ (e.g., $q = 0.5, 0.8, 0.9$) and $\nu(p_c)$.
5. Closure quality: diagnostic score/error for $\nu \approx f(k_q, q)$ across windows; identify degradation in the zoom region.

8.4 Minimal claims from the demo (scoped)

Near $p \approx p_x$, we expect a coincident pattern: small $\Delta\Phi$ (flat landscape), elevated switching/diversity (unstable P), elevated \mathcal{T}_P (locality failure), and at least one spectral corroboration marker (e.g., closure degradation). Outside the critical window, partition stability and locality recover and spectral diagnostics regain predictability.

9 Discussion: interpretation of “string-like”

The proposed framework reframes “string-like” as a critical regime characterized by persistent nonlocal correlation structure and the absence of a stable block partition across scales. This aligns with the intuition of extended objects—correlations do not compress into local blocks—but does not assert microscopic strings as fundamental degrees of freedom. The approach is foundations-safe: criticality is defined through operationally computable quantities and tied to modular spectral diagnostics.

Limitations are explicit: partition families and thresholds must be pre-registered; the demonstrative case is a proof-of-concept; and universality claims require systematic mapping and scaling tests.

10 Reproducibility checklist

We pre-register: the partition family, penalty η , tie-breaking rules, partition equivalence relation, top- K with $K = 3$, diagnostic quantiles q , observables set \mathcal{O} , windowing and fit-quality thresholds, baseline region $W_{\text{base}} = [0.01, 0.20] \setminus [0.02, 0.08]$, and the percentile-based threshold scheme. We bootstrap over seeds and report confidence intervals for thresholds and the PASS/FAIL decision rate. Raw and aggregated outputs (CSV/JSON) are saved with a manifest of all parameters.

11 Conclusion

Scientific value

This paper provides a conservative foundations-level definition of “string-like” criticality within modular RG-proxy dynamics: not as a microscopic primitive, but as a regime where correlation geometry fails to localize into stable blocks. By expressing criticality through partition instability $P^{(t)}$, landscape flatness $\Delta\Phi(t)$, and persistence of nonlocality via $\mathcal{T}_{P(\rho(t))}$, it turns a qualitative notion (extended/critical behavior) into an operational concept compatible with the canonical chain $A \rightarrow B \rightarrow C \rightarrow D$.

Degree of development (depth)

We formalize: (i) partition instability markers (switching rate, top- K diversity), (ii) flatness criteria via gaps $\Delta\Phi$, (iii) locality failure via \mathcal{T}_P , and (iv) a reproducible PASS/FAIL detection protocol requiring agreement between partition-layer and spectral-layer signals. The demonstrative case study (Variant A) anchors the definition in a reproducible baseline and isolates a critical window around a stable crossover marker $p_x \approx 0.04$ within the validated mixed/Gibbs-like domain, with thresholds calibrated from baseline percentiles to avoid hand-tuning.

Applied value and future directions

Practically, the proposed markers and protocol can be used to (i) automatically detect loss-of-stable-locality regimes in modular RG-proxy simulations, (ii) map critical windows in phase diagrams without ontological assumptions, and (iii) flag parameter regions where local coarse-grained descriptions are unreliable. Future directions include: (1) systematic phase-diagram mapping of critical windows as a function of family/channel parameters; (2) scaling tests under spectator-large- N extensions to determine whether partition instability sharpens or smooths; (3) linking critical windows to closure breakdown in Paper D (predictive loss as a critical signature); (4) searching for invariant spectral markers (quantile-flow patterns) that predict the onset of partition instability; (5) exploring whether critical regimes organize into universality classes characterized by distinct instability indices; and (6) identifying dynamical mechanisms (GKSL structures or constraint updates) that trigger or suppress the string-like regime.

REFERENCES

References

- [1] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, 1955.
- [2] E. T. Jaynes, Information theory and statistical mechanics, *Physical Review* **106** (1957), 620–630; **108** (1957), 171–190.
- [3] H. Araki, Relative entropy of states of von Neumann algebras, *Publications of the Research Institute for Mathematical Sciences* **11** (1976), 809–833.
- [4] M. Takesaki, *Theory of Operator Algebras I*, Springer-Verlag, New York, 1979.
- [5] R. Haag, N. M. Hugenholtz, and M. Winnink, On the equilibrium states in quantum statistical mechanics, *Communications in Mathematical Physics* **5** (1967), 215–236.
- [6] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of N -level systems, *Journal of Mathematical Physics* **17** (1976), 821–825.
- [7] G. Lindblad, On the generators of quantum dynamical semigroups, *Communications in Mathematical Physics* **48** (1976), 119–130.
- [8] H. Spohn, Entropy production for quantum dynamical semigroups, *Journal of Mathematical Physics* **19** (1978), 1227–1230.
- [9] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*, Oxford University Press, 2002.
- [10] D. Petz, Monotone metrics on matrix spaces, *Linear Algebra and its Applications* **244** (1996), 81–96.

- [11] S. Amari and H. Nagaoka, *Methods of Information Geometry*, AMS & Oxford University Press, 2000.
- [12] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed., Wiley, 2006.
- [13] J. Cardy, *Scaling and Renormalization in Statistical Physics*, Cambridge University Press, 1996.
- [14] S. Sachdev, *Quantum Phase Transitions*, 2nd ed., Cambridge University Press, 2011.
- [15] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, 10th Anniversary Edition, Cambridge University Press, 2010.
- [16] M. B. Plenio and S. Virmani, An introduction to entanglement measures, *Quantum Information and Computation* **7** (2007), 1–51.
- [17] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Mixed-state entanglement and quantum error correction, *Physical Review A* **54** (1996), 3824–3851.