Informational Phases and Emergent Locality from Modular Spectral Structure

NESEN OLEG

Abstract

Building on Paper A (canonical emergence of the density operator) and Paper B (relative modular dynamics and CPTP completion), we introduce phase structure and the emergence of locality in a conservative, information-theoretic manner. A phase is specified by an access structure encoded by an optimal partition P, the corresponding accessible algebra $\mathcal{A}_F$, and a pointer/center structure Z_F. The reference state σ_F within a macro-constraint class is defined canonically via a maximum-entropy (MaxEnt) construction under macroscopic constraints extracted from $\mathcal{A}_F$. We emphasize the equivalence between MaxEnt references and relative-entropy projections, which provides a canonical and minimal notion of phase stability. Locality emerges as a regime in which the optimal partition P stabilizes and the block total correlation $\mathcal{T}_{P(\rho)=D\left(\rho\middle\|\bigotimes_{X\in P}\rho_X\right)}$ remains small, yielding an accurate block-factorized description. Finally, we introduce a canonical information-geometric metric (BKM/Fisher-type) on the state manifold and connect these constructions to modular spectral diagnostics used in numerical protocols, such as spectral quantiles $k_q = -\log \lambda_q$ and commutator-based probes.

Keywords: quantum foundations; informational phases; maximum entropy principle (MaxEnt); relative entropy projection (information projection); emergent locality; modular generator $K = -\log \rho$; operator algebras / access structure; quantum information geometry (BKM metric); spectral diagnostics / RG-proxy flow.

1 Introduction

Paper A established ρ as the canonical convex, basis-invariant representative of normalized probabilistic structure, through the trace pairing $p(E) = \mathrm{Tr}(\rho E)$. Paper B introduced modular generators $K_\rho = -\log \rho$ and relative modular dynamics with a CPTP-compatible GKSL completion, with an evolution parameter that may be treated as an informational scale (RG-proxy), potentially implemented in discrete steps.

The aim of the present paper is to define *informational phases* and to explain how *locality* can emerge as a *phase property* of modular spectral organization, rather than being postulated. We proceed conservatively: phases are defined by access structure and constraint data; the reference state is fixed canonically via MaxEnt (equivalently, an information projection in relative entropy); locality is quantified by an explicit information-theoretic distance to a block-product model.

Foundational stance. The present construction treats “phase” and “locality” as properties of information access rather than as primitive spacetime notions. Concretely, a phase is encoded by which observables are retained (the accessible algebra) and how correlations localize under an optimal partition. Locality, in turn, is quantified by the information-theoretic distance to the best block-product description consistent with observed marginals.

Positioning and relation to existing foundations literature. While the ingredients employed here are individually well-established—density operators and effect functionals, MaxEnt under linear constraints, relative entropy as an information distance, and monotone quantum metrics (e.g., BKM/Petz)—the contribution of the present series is their assembly into a single conservative and testable chain. The novelty is the *canonicality* of each link: closure (Paper A), additivity/scale selection (Paper B), and projection/factorization criteria (Paper C). Specifically, Paper A isolates a representation-invariant passage from probabilistic structure to operator states; Paper B introduces a canonical state-derived spectral coordinate and a minimal CPTP-compatible completion; Paper C defines phases as access structures, selects the phase reference canonically as a MaxEnt/I-projection, and quantifies emergent locality by the block total correlation $\mathcal{T}_{P(\rho)}$.

2 Preliminaries: partitions, algebras, and correlation measures

Let ρ be a density operator on $\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_i$. For $X \subset \{1, \dots, N\}$, denote by ρ_X the reduced state on $\mathcal{H}_X = \bigotimes_{i \in X} \mathcal{H}_i$.

Definition 1 (Von Neumann entropy and mutual information)**.** For a subsystem X, define $S(\rho_X) = -\mathrm{Tr}(\rho_X \log \rho_X)$. For disjoint subsystems X, Y, define

$$I_\rho(X : Y) := S(\rho_X) + S(\rho_Y) - S(\rho_{XY}).$$

Definition 2 (Partitions and block-access algebras)**.** A partition $P = \{X_1, \dots, X_m\}$ of $\{1, \dots, N\}$ induces a block-access algebra

$$\mathcal{A}(P) := \bigvee_{k=1}^{m} B(\mathcal{H}_{X_k}).$$

3 Phase as an access structure: $(P^{,\mathcal{A}_F, Z_F)}$

Definition 3 (Correlation localization functional)**.** Fix a family of candidate partitions. Define

$$\Phi(P; \rho) := \sum_{X \neq Y \in P} I_\rho(X : Y) + \eta\, |P|,$$

where $|P|$ is the number of blocks and $\eta > 0$ is a complexity penalty.

Definition 4 (Optimal partition and phase data)**.** Let $P^{(\rho) \in \arg\min_P \Phi(P;\rho)}$ be an optimal partition. Define the phase data

$$F(\rho) := \big(P^{(\rho),\ \mathcal{A}_F(\rho),\ Z_F(\rho)}\big),$$

where $\mathcal{A}_F(\rho) := \mathcal{A}(P^{(\rho)})$ and $Z_F(\rho)$ is a chosen pointer/center structure compatible with $\mathcal{A}_F$.

Remark 1 (Interpretive note without extra assumptions)**.** The triple $(P^{,\mathcal{A}_F, Z_F)}$ summarizes what is stable and accessible at a given scale. It does not assume pre-existing spatial neighborhoods; instead, candidate neighborhoods are identified as blocks that minimize inter-block dependence under a controlled complexity penalty.

4 Reference state within a macro-constraint class: MaxEnt and I-projection

4.1 Macroscopic constraints from $\mathcal{A}_F$

Definition 5 (Macroscopic constraints)**.** Let $\{Q_a\} \subset \mathcal{A}_F$ be a chosen finite set of macroscopic observables. For a state ρ, define constraint values $q_a(\rho) := \mathrm{Tr}(\rho Q_a)$.

4.2 MaxEnt reference

Definition 6 (MaxEnt reference state)**.** Define the reference state $\sigma_F(\rho)$ within the macro-constraint class by

$$\sigma_F(\rho) := \arg \max_{\tau \succeq 0,\ \mathrm{Tr}\tau=1} \Big\{ S(\tau) \ \Big|\ \mathrm{Tr}(\tau Q_a) = q_a(\rho)\ \forall a \Big\}.$$

4.3 Equivalence to relative-entropy projection

Theorem 1 (MaxEnt is a relative-entropy projection)**.** *Let $\mathcal{E}_F$ be the exponential family generated by $\{Q_a\}$,*

$$\mathcal{E}_F := \left\{ \frac{e^{-\sum_a \beta_a Q_a}}{Z(\beta)} \ :\ \beta \in \mathbb{R}^A \right\}.$$

Then $\sigma_F(\rho) \in \mathcal{E}_F$ and

$$\sigma_F(\rho) = \arg \min_{\tau \in \mathcal{E}_F} D(\rho \| \tau).$$

Proof (sketch). Standard convex duality: maximizing entropy under linear constraints yields an exponential-family solution, which is equivalently the information projection minimizing relative entropy over $\mathcal{E}_F$. □

Proposition 1 (Minimality of MaxEnt / I-projection)**.** *Fix $\{Q_a\} \subset \mathcal{A}_F$ and values $q_a(\rho)$. Among all states consistent with these constraints, $\sigma_F(\rho)$ is the unique least-committal representative: it introduces no additional structure beyond what is implied by the constraints. Equivalently, it is the unique I-projection minimizing $D(\rho\|\cdot)$ over $\mathcal{E}_F$.*

Remark 2 (Terminology alignment with Paper B)**.** Throughout this paper, $\sigma_F(\rho)$ is treated as a reference state within a macro-constraint class: once $\mathcal{A}_F$ and $\{Q_a\}$ are fixed, σ_F is fixed canonically and is updated only when the phase data $(P^{, \mathcal{A}_F, Z_F)}$ changes. The evolution parameter from Paper B is an informational RG-proxy scale, implementable discretely as iterative CPTP updates; continuous-time forms are effective limits.

5 Emergent locality as phase factorization: the primary criterion

Definition 7 (Block-product model and block total correlation)**.** For a partition $P = \{X_1, \ldots, X_m\}$ define the block-product state

$$\rho_P^{\otimes} := \bigotimes_{k=1}^{m} \rho_{X_k}.$$

Define the block total correlation

$$\mathcal{T}_P(\rho) := D(\rho \| \rho_P^{\otimes}).$$

In finite dimensions,

$$\mathcal{T}_P(\rho) = \sum_{k=1}^{m} S(\rho_{X_k}) - S(\rho).$$

In particular, $\mathcal{T}_P(\rho) = 0$ if and only if $\rho = \rho_P^{\otimes}$.

Proposition 2 (Locality as small block total correlation)**.** *Locality in a phase is quantified by the optimal partition $P^{(\rho)}$ and the size of $\mathcal{T}_{P(\rho)}$. When $\mathcal{T}_{P(\rho)}$ is small and stable under modular flow, ρ admits an accurate block-factorized description on P, in the precise sense that the distinguishability gap between ρ and the best block-product model consistent with its block marginals is small.*

Remark 3 (Relation to mutual information)**.** For two blocks ($m = 2$), $\mathcal{T}_P(\rho)$ reduces to mutual information. For $m > 2$, $\mathcal{T}_P(\rho)$ is a canonical multipartite extension; pairwise mutual informations remain useful local diagnostics but $\mathcal{T}_P$ is the primary locality criterion.

6 Information geometry and a canonical metric substrate

Definition 8 (BKM / Fisher-type metric (formal))**.** For faithful ρ, define

$$ds^2_{\mathrm{BKM}}(d\rho, d\rho) := \int_0^1 \mathrm{Tr}\big(\rho^s \, d\rho \, \rho^{1-s} \, d\rho\big) \, ds.$$

Remark 4. The BKM metric is canonical in quantum information geometry (a monotone metric). It provides a representation-invariant notion of distance/curvature on the state manifold that can serve as a substrate for emergent geometric interpretation. Detailed geometric consequences are deferred to subsequent work.

7 Links to modular spectral diagnostics

Let $\lambda_1 \geq \cdots \geq \lambda_d > 0$ be eigenvalues of ρ and define spectral coordinates $k_i = -\log \lambda_i$ and quantiles $k_q = -\log \lambda_q$. Given an observable O, define the normalized commutator probe

$$L(\rho; O) := \frac{\|[K_\rho, O]\|_F}{\|K_\rho\|_F \|O\|_F}, \qquad K_\rho = -\log \rho.$$

Under Paper B dynamics (continuous or discrete), dissipation makes spectral flow nontrivial, turning $k_q(t)$ and $L(t)$ into phase-sensitive probes. Phase transitions correspond to changes in P, loss of stability of σ_F as reference, and/or growth of $\mathcal{T}_{P(\rho)}$.

8 Discussion and outlook

We defined informational phases as access structures $(P^{,\mathcal{A}_F, Z_F})$, constructed the canonical reference state $\sigma_F(\rho)$ via MaxEnt/I-projection, and defined locality as a quantitative phase property controlled by $\mathcal{T}_{P(\rho)}$. A canonical information metric provides the substrate for emergent geometry.

Why the criterion is not merely descriptive. The locality criterion $\mathcal{T}_{P(\rho)=D\left(\rho \middle\| \bigotimes_{X \in P} \rho_X\right)}$ is a quantitative statement about the compressibility of global correlation structure into a product of block marginals. It therefore provides a principled interface between foundational claims (local descriptions emerge) and reproducible diagnostics under explicit dynamics (Paper B) and numerical pipelines.

Subsequent papers will specialize constraints and pointer structures in concrete models, map phase boundaries under RG-proxy flows, and connect spectral diagnostics to robust signatures of modular phase transitions.

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