

# The 20/80 Rule as a Dimensional Effect in Resource Distribution

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## Abstract

This paper proposes a dimensional approach to the interpretation of empirical regularities in resource concentration within systems. The central characteristic is the concentration function, which specifies the share of the resource belonging to the upper proportion of elements ranked in descending order of contribution. For the analytical study, a simple geometric model is employed — a conical sand heap — in which, for the same carrier structure, different observable quantities may be considered: volume (mass), surface area, equivalent linear scale, and potential energy. It is shown that the corresponding concentration functions do not coincide and are determined not only by the ordering of the elements but also by the dimensionality of the effect under consideration. On this basis, the 20/80 rule is interpreted not as a universal numerical constant but as a special case within a broader class of concentration relations arising from a particular mapping of the resource onto the linear scale of its carrier. In addition, the paper discusses the relationship between symmetric parametric models and the Gini coefficient, as well as the possibility of using such models for the normative description of income and wealth distributions.

Keywords: Pareto principle, concentration form, Pareto/Lorenz curve, dimensional analysis, sand cone, equivalent radius

## 1. Introduction

Uneven resource distribution is observed in systems of very different natures — from income and wealth to flows of energy, mass, and interaction intensity. Yet, in the analysis of such systems, what matters is not only the mere fact of inequality but also the form of concentration: what share of the total resource belongs to the upper proportion of elements after they have been ranked in descending order of contribution. It is precisely this question that underlies discussions of the 20/80 rule and related empirical relations.

Since the work of Vilfredo Pareto [1], studies of resource concentration have usually developed in a statistical direction: researchers have sought distribution functions capable of describing heavy tails, Lorenz curves, and related inequality indices. This approach is highly productive; however, it does not directly answer another question: why, under a similar rank organisation of a system, can the observed concentration proportions differ? In other words, what is the mechanism by which the same ordering of elements gives rise to different shares of accumulated resource?

For the upper tails of distributions, Pareto showed the robustness of a power-law approximation, which, in terms of the concentration function, may conveniently be written as the asymptotic form of the cumulative share of the resource belonging to the upper share of ranks.

$$S(p) \propto p^{1-1/\alpha}, \quad 0 \leq p \ll 1. \quad (1)$$

Hereafter, by analogy with the standard Lorenz curve ( $L$ ), we shall refer to the cumulative concentration curve  $S(p)$  as the Pareto curve (PC). Pareto also found that in many countries and across different periods the observed pattern of concentration is close to the empirical benchmark  $\alpha = 3/2$ . Substituting the corresponding value of the parameter into (1) yields approximation (2),

which will subsequently serve as a convenient benchmark for comparing different concentration models.

$$S(p) \propto p^{1/3}, \quad 0 \leq p \ll 1. \quad (2)$$

It is approximation (2) that naturally leads to the 20/80 rule: about 20% of the elements in a system accumulate approximately 80% of the total resource, while the remaining 80% of the elements account for about 20% of the resource.

For more than a century, the 20/80 rule has been used as a practical heuristic in quality management [2]–[4], economics [5], [6], geography [7], sociology [8], and the natural sciences [9]–[11]. At the same time, the persistence of this proportion is usually stated empirically, whereas its geometric and dimensional interpretation remains insufficiently clarified.

The present paper examines the hypothesis that the dimensionality of the observed quantity plays a substantial role here. The same carrier may generate different concentration laws depending on what is taken to be the resource: a linear size, area, volume, energy, or some other quantity related by a power law to the geometric scale. In that case, the difference between concentration rules is linked not to the arbitrary choice of a numerical constant, but to the way in which the effect is measured.

To clarify this idea, a simple analytical model is used — a conical sand heap. It is convenient because it allows an explicit description of ranked resource accumulation and makes it possible, within a single geometry, to compare several different quantities. With the carrier shape kept unchanged, one may successively consider the accumulation of volume, surface area, equivalent linear size, and potential energy, and then compare the corresponding concentration functions.

The aim of the paper is to relate empirical concentration rules to the dimensional structure of the observed effect. In the proposed interpretation, the ranking of elements itself remains a dimensionless procedure, whereas a specific proportion such as 20/80, 20/79, or 20/62 is determined by how the resource scales with respect to the linear carrier. This makes it possible to regard well-known concentration relations as elements of a unified scheme rather than as a set of disconnected empirical rules.

The paper is organised as follows. First, the basic notation and the formal statement of the problem are introduced. The conical sand heap is then considered as a model in which different physical quantities correspond to different concentration functions. Dimensional mappings and their parametric generalisations are subsequently discussed, followed by an interpretation of the results in terms of symmetry, the Gini coefficient, and normative characteristics of distributions. The concluding section presents examples of model calibration using income and wealth data.

## 2. Problem statement

Consider  $n$  elements of a system ordered in descending order of the share of their contribution to the total resource of the system ( $w_r$ ), where  $r$  denotes the rank of an element. By definition, the Pareto curve is a function interpolating the cumulative sum of the resource shares of the elements

$$S(p_i) = \sum_{r=1}^i w_r, \quad 0 \leq p_i \leq 1. \quad (3)$$

where  $p_r = \frac{r}{n} \in [0,1]$  is the rank share, that is, the share of elements counted from the most endowed to the least endowed.

To compare the PC with the Lorenz curve ( $L(p)$ ), defined in ascending order of the resource (the smallest values come first), it is convenient to use the identity  $L(p) = 1 - S(1 - p)$ , which allows one to pass from ‘top-down’ ranking to ‘bottom-up’ ranking. Accordingly, the axioms that the PC must satisfy can be written in the form:

$$S(0) = 0, \quad S(1) = 1, \quad S'_+(p) \geq 0, \quad S''_+(p) \leq 0, \quad p \in (0,1). \quad (4)$$

It follows from axioms (4) that the PC is a non-decreasing concave concentration curve.

Let  $PR = S(0.2)$ . Hereafter, PR will be called the Pareto coefficient. This coefficient is widely used in applied problems to compare resource concentration across systems.

Within the present framework, the following tasks are to be addressed: (i) to identify the dependence of the Pareto coefficient on the dimensionality of the observed quantity; (ii) to examine how the same carrier leads to different concentration curves when moving from volume to area, equivalent radius, and energy; (iii) to relate observed inequality to the model parameter and the normative distribution of the resource; (iv) to discuss criteria of structural stability; and (v) to illustrate the recovery of the model parameter from the Gini coefficient.

### 3. The sand cone as a model of different resource concentrations

A classical model of self-organising systems is a heap of homogeneous sand [12], [13]. In the stationary state it has the form of a right circular cone of height  $H$ . Let  $p = z/H$  be the share of the height measured from the apex or, equivalently, the rank share in a geometrically ordered model of the carrier. Let  $S_M(p)$  denote mass accumulation and  $S_A(p)$  denote the accumulation of the lateral surface area of the truncated cone.

From the similarity of cross-sections, it follows that the radius at height  $z$  is proportional to  $(1 - p)$ , and that the volume of the small upper cone above level  $z$  constitutes a share  $(1 - p)^3$  of the total volume. Consequently, the share of the volume enclosed within the lower height fraction  $p$  is equal to:

$$S_V(p) = \frac{V(p)}{V_{tot}} = 1 - (1 - p)^3. \quad (5)$$

The Pareto coefficient for model (5) is  $PR \approx 0.488$ .

The share of the lateral surface area of the truncated cone enclosed within the lower height fraction  $p$  is equal to:

$$S_A(p) = \frac{A(p)}{A_{tot}} = 1 - (1 - p)^2. \quad (6)$$

For model (6), the Pareto coefficient is equal to  $PR \approx 0.360$ , which is markedly smaller than in the volumetric model. This reflects the weaker concentration of the resource when moving from volume to surface area.

In the general case, both relations may conveniently be written in the unified form:

$$S(p; D) = 1 - (1 - p)^D. \quad (7)$$

Here the parameter  $D$  is interpreted as the dimensionality, or number of degrees of freedom, of the quantity characterising the state of the system. For the subsequent analysis, it is useful to employ the standard expansion in a neighbourhood of zero:

$$(1 - p)^\alpha = 1 - \alpha p + \frac{\alpha(\alpha - 1)}{2} p^2 + O(p^3), \quad p \rightarrow 0. \quad (8)$$

It follows immediately from (8) that, for  $p \rightarrow 0 +$ , the family (7) has the linear asymptotic form:

$$S(p; D) \propto p, \quad 0 \leq p \ll 1. \quad (9)$$

A comparison of relations (2) and (9) shows that the purely geometric model (7) grows much more gently near zero than the empirical concentration curve for income. Hence, the geometric factor alone is insufficient to reproduce the high concentration characteristic of socio-economic distributions.

If, however, the accumulated resource is taken to be the potential energy of the sand grains, then the corresponding accumulation curve becomes more convex, since the contribution of each mass element is additionally weighted by its height. Thus, the same carrier gives rise to different concentration laws depending on the nature of the measured quantity.

## 4. Equivalent radius: dimensional mappings

### 4.1. Dimensionality $D$

Dimensionality  $D$  indicates how many parameters are required to describe an object: zero-dimensionality corresponds to a point, one-dimensionality to a line, two-dimensionality to an area, and three-dimensionality to a volume. For the measurement of complex and fragmented objects, the fractal dimension  $D \in (1,3)$  is used [14].

Let the carrier of the effect possess a measure  $M$  (for example, area, volume, or mass) that scales with the characteristic linear size  $S$  according to the power law

$$M \propto S^D. \quad (10)$$

Then the equivalent linear scale is calculated by the formula

$$S = M^{1/D}. \quad (11)$$

The quantity  $S$  has a clear physical meaning: it is the share of the linear scale that a similar object would have if it contained the measure share. Such a transition is standard for operations of the type “volume  $\rightarrow$  equivalent radius” and, more generally, “measure  $\rightarrow$  linear scale”.

## 4.2. Equivalent sphere and circle

Let us associate the accumulated volume  $V(p)$  with an equivalent sphere, that is, a sphere of the same volume. Its radius  $R_{eq}(p)$  (hereafter, the equivalent sphere radius) is determined by the equality  $V(p) = 4\pi/3 R_{eq}^3(p)$ . Normalising by the value at  $p = 1$ , we obtain the share of the equivalent radius as a function of rank:

$$\frac{R_{eq}(p)}{R_{eq}(1)} = \left[ \frac{V(p)}{V(1)} \right]^{1/3}. \quad (12)$$

Substituting (5) into (12), we obtain the explicit formula for the three-dimensional dimensional mapping:

$$S(p; 3, 1/3) = \sqrt[3]{1 - (1 - p)^3}. \quad (13)$$

For  $p = 0.2$ , formula (13) gives a value of about 0.787, that is, the proportion “20/79”, which practically coincides with the heuristic “20/80”. Likewise, the accumulated area may be associated with an equivalent circle of the same area. As a result, model (13) naturally generalises to the case of arbitrary dimensionality:

$$S(p; D, 1/D) = \sqrt[D]{1 - (1 - p)^D}. \quad (14)$$

For  $D = 2$ , we have PR = 0.60.

Using expansion (8), it is easy to see that, as  $p \rightarrow 0+$ , the relations (14) possess the power-law asymptotic form [15]:

$$S(p; D, 1/D) \propto p^{1/D}, \quad 0 \leq p \ll 1. \quad (15)$$

Comparison with (2) shows that, with an appropriate choice of parameter, the asymptotic coincide in the leading term. Hence, the difference between the models lies not in the rank structure as such, but in the way the same rank organisation is mapped into an observable quantity of a different dimensionality. This is precisely the key idea of the dimensional interpretation of concentration. It should also be noted that model (14) coincides with the one-parameter symmetric Lorenz curve of the Burr III distribution [15], which has been derived in different ways in [16–22].

## 4.3. Solids of revolution instead of the “social pyramid”

In vol. II [1, pp. 312–318], Pareto criticised the image of the social pyramid as too crude a scheme and proposed replacing it with a solid of revolution, which allows the distribution profile to be represented geometrically in a continuous manner. Such a representation is especially useful when the form of the concentration curve must be linked to the dimensionality of the carrier and the type of observable quantity.

Figure 1 shows solids of revolution for the radial profile  $r(p) = 1 - S(p)$ . The broad base corresponds to the majority of elements with small resource values, whereas the narrowing upper part corresponds to a relatively small number of elements accumulating a significant share of the total effect. Variants (a) and (c) correspond to the basic volumetric and surface concentration models, while variants (b) and (d) illustrate two-parameter forms with  $a=b=3$  and  $a=b=2$ , respectively.

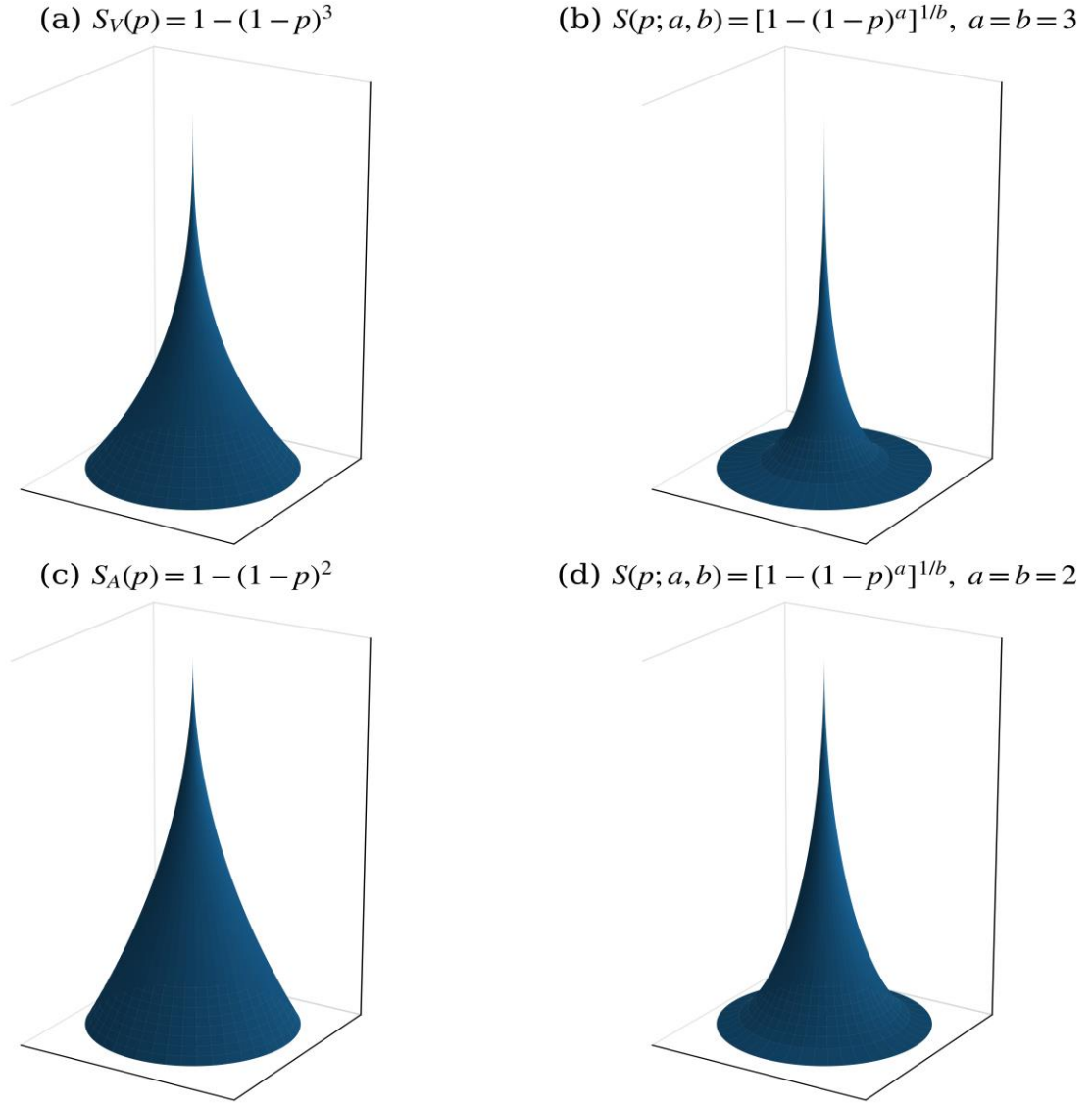


Figure 1 — Solids of revolution for Pareto curves (vertical axis:  $p$ , radius:  $1-S(p)$ )

## 5. Two-parameter PC

The one-parameter symmetric curve (14) is a special case of the more general two-parameter family [23]

$$S(p; a, b) = [1 - (1 - p)^a]^{1/b}. \quad (16)$$

where  $a$  and  $b$  are empirical shape parameters satisfying the conditions  $1 \leq a < \infty$  and  $1 \leq b < \infty$ . To describe the behaviour in a neighbourhood of zero, it is convenient to introduce the exponent  $\beta = 1/b$ . Then, from expansion (8), we obtain

$$S(p; a, b) \approx (ap)^\beta, \quad 1 \leq a < \infty, \quad 0 \leq \beta \leq 1, \quad p \rightarrow 0. \quad (17)$$

By analogy with the physics of phase transitions, the exponent  $\beta$  may be interpreted as a critical exponent that determines the regime of concentration growth in the region of small  $p$ .

$$\beta = \lim_{p \rightarrow 0} \frac{\ln S(p; a, b)}{\ln p} = \frac{1}{b}. \quad (18)$$

For family (16), the Gini coefficient is expressed analytically as:

$$G(a, \beta) = \frac{2 \Gamma(1/a) \Gamma(\beta + 1)}{a \Gamma(\beta + 1 + 1/a)} - 1. \quad (19)$$

Thus, the behaviour of  $S(p; a, b)$  as  $p \rightarrow 0$  depends primarily on  $\beta$ . When  $\beta \rightarrow 1$ , the resource distribution near zero tends to be more even, whereas when  $\beta \rightarrow 0$ , the resource becomes concentrated among the highest-ranked elements.

The vertical distance from point  $(p, S(p))$  to the line of equality is equal to  $d(p) = S(p) - p$ . The derivative  $d'(p) = S'(p) - 1$  vanishes at  $S'(p) = 1$ ; let us denote the corresponding abscissa by  $p_\mu$ . At this point, the curve is farthest from the line of equality, and in applied inequality analysis the Hoover index  $HI = S(p_\mu) - p_\mu$  is used [24].

For a discrete ranked distribution, the theoretical value of the resource share of the element of rank  $w(p_r)$  can be calculated by the formula:

$$w_r = S(p_r) - S(p_{r-1}). \quad (20)$$

Since, at point  $p_\mu$ , the condition  $S'(p_\mu) = 1$  holds, the contribution of the element in this neighbourhood is close to the mean:

$$w(p_\mu) \approx \frac{1}{n}. \quad (21)$$

Consequently, the abscissa  $p_\mu$  determines the rank of the mean element of the ranked distribution. Elements with resource above the mean, that is, elements for which  $p \leq p_\mu$ , may naturally be interpreted as significant elements of the system. For the numerical assessment of PC asymmetry, we use the coefficient

$$PAC = 1 - p_\mu - S_\mu. \quad (22)$$

The quantity PAC vanishes when point  $(p_\mu, S(p_\mu))$  lies on the alternative diagonal of the unit square; in general, the inequality  $-1 \leq PAC \leq 1$  holds. For the symmetric subclass, that is, when  $a = b$ , we obtain the explicit expressions [16]:

$$p_\mu(a) = 1 - 2^{-1/a}, \quad S_\mu(a) = 2^{-1/a}. \quad (23)$$

For the symmetric case, the Gini coefficient takes the form  $G(a) = \Gamma(1/a)^2 / [a \Gamma(2/a)] - 1$ . The dependence of resource concentration and PC asymmetry on the dimensionality of the observed quantity is given in Table 1; a graphical illustration is shown in Figure 2.

**Table 1 — Dependence of PR and PC parameters on the dimensionality of the observed quantity**

Model	$\beta$	$p_\mu$	$S_\mu$	PAC	S(0.2)	G
$S(p; 3) = 1 - (1 - p)^3$	1.00	0.42	0.81	-0.23	0.488	0.50
$S(p; 2) = 1 - (1 - p)^2$	1.00	0.50	0.75	-0.25	0.360	0.33
$S(p; 3, 1/3) = \sqrt[3]{1 - (1 - p)^3}$	0.33	0.21	0.79	0.00	0.787	0.77
$S(p; 2, 1/2) = \sqrt{1 - (1 - p)^2}$	0.50	0.29	0.71	0.00	0.600	0.57

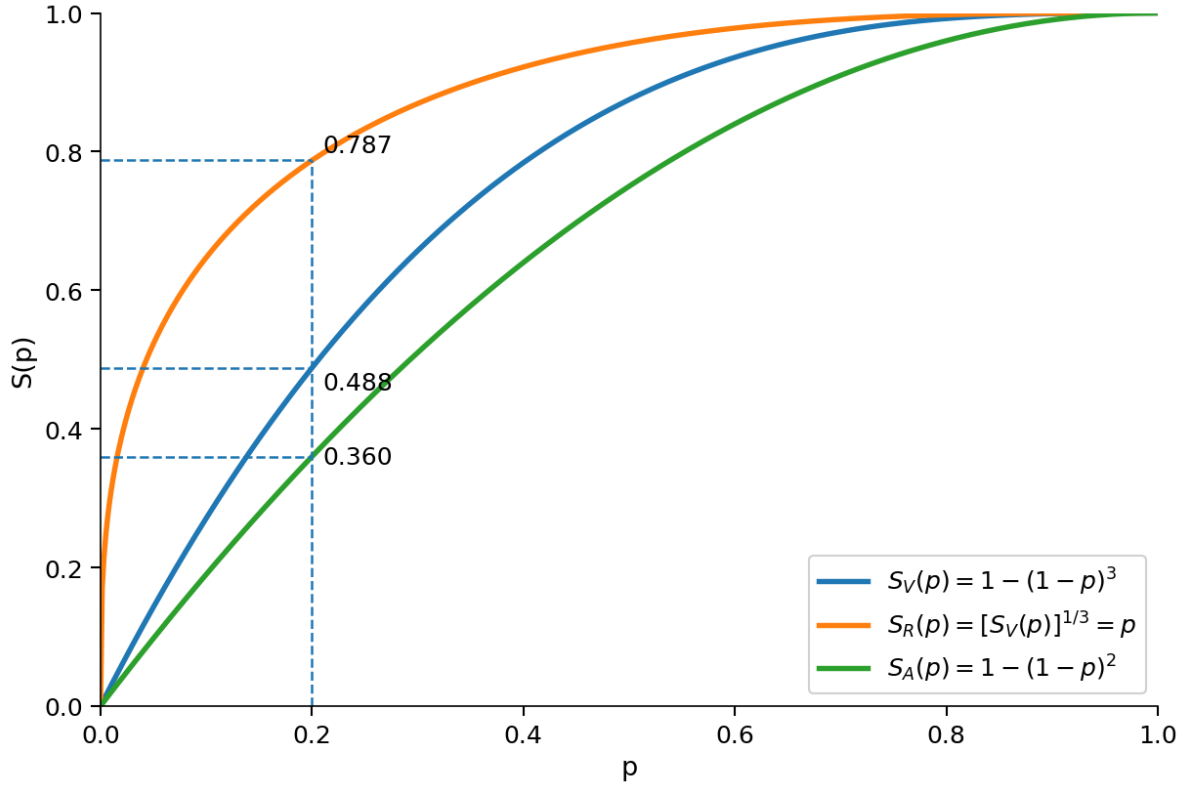


Figure 2 — Concentration functions for cone volume and dimensional mappings

The data in Table 1 show that the diversity of concentration rules is not accidental: it reflects the same ranking, but different dimensionalities of the observed quantity. For the same dimensionality, symmetric curves generally exhibit higher concentration than the asymmetric basic models.

Systems for which  $\beta = 1$  may be treated as summative: the internal linkage between elements is weaker here than external influences, and the concentration curve describes predominantly the additive summation of partial contributions. An alternative is provided by integrative systems, in which interactions among the elements within the system exceed external influences; in this case, the concentration curve reflects not simple summation but the cooperative amplification of contributions, which manifests itself in greater concentration and different asymmetry.

## 6. Norm of system ordering

### 6.1. Centre-of-mass coordinate and the Gini coefficient

By the norm of ordering, we shall henceforth mean such a distribution of the resource that, within the class of models under consideration, endows the system with the greatest structural stability. In the proposed geometric interpretation, this condition is naturally linked to minimising the normalised height of the centre of mass of the distribution of element contributions to the aggregate effect.

Using formula (20), we write the centre-of-mass coordinate in the form

$$p_c = \int_0^1 p \, dS(p) = \int_0^1 p S'(p) \, dp. \quad (24)$$

Since  $S(0) = 0$  and  $S(1) = 1$ , integration by parts gives

$$p_c = 1 - \int_0^1 S(p) dp. \quad (25)$$

On the other hand, the Gini coefficient for the concentration curve is equal to

$$G = 2 \int_0^1 S(p) dp - 1. \quad (26)$$

Hence,

$$p_c = \frac{1 - G}{2}. \quad (27)$$

Thus, in the present model, the Gini coefficient acquires a clear geometric interpretation: it specifies the position of the centre of mass of the contribution distribution. The smaller  $p_c$ , the lower the centre of mass and the more concentrated the resource distribution.

Let  $p_k$  denote the abscissa of the intersection point of the concentration curve with the alternative diagonal:

$$S(p_k) = 1 - p_k. \quad (28)$$

For any concave concentration curve, the inequality

$$p_c \leq p_k. \quad (29)$$

Equality is attained only in the limiting case where the curve coincides with the broken line connecting the points  $(0,0)$ ,  $(p_k, 1 - p_k)$  and  $(1,1)$ . Therefore, the position of the centre of mass and the position of the intersection point with the alternative diagonal characterise different aspects of the distribution structure: the former is related to overall concentration, while the latter is related to its balance with respect to the rank scale.

## 6.2. Lever rule and the asymmetry coefficient

Let us now consider the point  $(p_\mu, S_\mu)$ , where  $p_\mu$  is determined by the condition  $S'(p_\mu) = 1$ , and  $S_\mu = S(p_\mu)$ . At this point, the concentration curve has the same local slope as the line of equality, and, for a discrete distribution, the contribution of the element in the neighbourhood of  $p_\mu$  is close to the mean value. Therefore,  $p_\mu$  may be interpreted as the rank of the mean element of a rank-ordered system.

If one associates with the concentration curve a notional lever with fulcrum at , then the natural equilibrium condition may be written in the form

$$\frac{S_\mu}{1 - S_\mu} = \frac{1 - p_\mu}{p_\mu}. \quad (30)$$

It follows that

$$p_\mu S_\mu = (1 - p_\mu)(1 - S_\mu). \quad (31)$$

and, consequently,

$$p_\mu + S_\mu = 1. \quad (32)$$

Thus, the equilibrium state corresponds to the position of point  $(p_\mu, S_\mu)$  on the alternative diagonal of the unit square. It is precisely this condition that is expressed through the asymmetry coefficient

$$\text{PAC} = 1 - p_\mu - S_\mu. \quad (33)$$

When  $\text{PAC} = 0$ , the point of the mean element is in equilibrium; when it departs from zero, the correspondence between the rank of the mean element and the share of the effect accumulated up to that rank is violated. Therefore, the quantity  $|\text{PAC}|$  may naturally be regarded as an indicator of the structural imbalance of the concentration curve. In this interpretation, the 20/80 rule appears not as a separate empirical heuristic, but as one of the special cases of the more general condition  $\text{PAC} = 0$ .

### 6.3. Criterion of the norm of ordering

Within the adopted model, the norm of ordering is determined by the joint fulfilment of two requirements:

$$|\text{PAC}| \rightarrow \min, \quad p_c \rightarrow \min. \quad (34)$$

The first condition characterises structural balance, whereas the second characterises the degree of concentration of the distribution; by virtue of (27), the second condition is equivalent to the requirement  $G \rightarrow \max$ . Consequently, within the class under consideration, the more stable system should be regarded as the one that is simultaneously closer to equilibrium in terms of PAC and possesses a lower centre-of-mass height. In other words, the norm of ordering corresponds to a distribution in which the balance of the rank structure and a high concentration of contribution are mutually consistent.

## 7. Discussion: the 20/80 rule as a dimensional effect

The results obtained make it possible to reinterpret the classical 20/80 rule. In the original geometric model of the conical heap, volumetric concentration is determined by formula (5), that is, it is directly specified by the cubic scaling of volume with linear size. However, when one passes from volume to equivalent radius, area, or another observable quantity, the concentration function itself changes. In other words, the same rank organisation of a system may lead to different numerical proportions of accumulated resource depending on which quantity is actually measured.

More generally, if the observable quantity  $Q$  is related to the base measure by a power law, then the concentration curve for  $Q$  is obtained as a dimensional mapping of the base curve. This leads to an important conclusion: the 20/80 rule need not be treated as a universal constant equally applicable to all systems. It represents a particular numerical result arising under a definite way of passing from the carrier of the effect to the observable quantity. Within the same framework, other relations arise naturally as well, in particular 20/79 and 20/62.

It is crucial that ranks and rank shares are themselves dimensionless. Hence, dimensionality manifests itself not in the ranking procedure, but in the rule that links the resource to the linear scale of the carrier. It is precisely in this sense that the dimensional interpretation unites the geometric model of the sand cone and parametric concentration curves: the difference between the observed laws is explained not by a change in the order of the elements, but by a change in the metric.

This explanation differs from other interpretations of the 20/80 rule discussed in [25–28]. In some works, the corresponding proportion is derived from specific probability distributions; in others, from entropic principles or index constructions. In the present paper, the source of the relation is different: it is derived from geometry, scaling, and the choice of the observable quantity. Thus, the main idea of the paper lies not in postulating a new universal constant, but in reducing several known concentration rules to a single dimensional mechanism.

**8. Calibration of the symmetric model by the Gini coefficient**

For the symmetric one-parameter model, the Gini coefficient  $G(a)$  depends monotonically on the concentration parameter  $a$ . Therefore, the observed value of  $G_{obs}$  uniquely determines the model parameter from the equation  $G(a) = G_{obs}$ . This makes it possible to use statistically measured inequality as an input parameter for reconstructing the corresponding symmetric concentration curve. As a result, a convenient framework emerges for moving from an empirical inequality indicator to a parametrised normative model of resource distribution..

As an illustration, let us consider the United States. For income, the official historical series of the Gini coefficient for pretax household money income is used. For wealth, estimates of household net worth based on the Survey of Consumer Finances are employed. Since the compared series refer to different statistical objects, the comparison should be understood not as a literal comparison of like-for-like aggregates, but as a structural comparison of two forms of inequality — current income and accumulated wealth [29–32].

**Table 2 — Calibration of the symmetric model parameter by the Gini coefficient**

Indicator	Year	G	a	pμ	PR
Income	1989	0.48	1.63	0.35	0.48
Income	2007	0.46	1.70	0.34	0.51
Income	2019	0.48	1.75	0.33	0.52
Income	2024	0.49	1.76	0.33	0.53
Wealth	1989	0.83	3.62	0.17	0.85
Wealth	2007	0.83	3.70	0.17	0.86
Wealth	2019	0.87	4.43	0.14	0.90
Wealth	2024	0.85	3.89	0.16	0.87

Table 2 shows that the trajectory of the symmetric model parameter for pretax household money income is comparatively stable and lies in the range  $a \approx 1.6 - 1.8$  (Pareto coefficient  $PR \approx 0.48 - 0.53$ ), whereas the corresponding values for household net wealth are markedly higher. This means that, from the standpoint of the proposed model, the wealth distribution is in a substantially more concentrated state than the income distribution.

Thus, within the present framework, the Gini coefficient performs a dual function. On the one hand, it remains the standard measure of observed inequality. On the other hand, through the inverse determination of parameter  $a$ , it makes it possible to pass to the normative symmetric model and to specify a consistent distribution of resource shares among the elements of the system. This constitutes the applied significance of the proposed calibration for the analysis and comparative description of socio-economic distributions.

## Conclusion

The paper has proposed a dimensional interpretation of empirical rules of resource concentration. It has been shown that the form of the concentration curve is determined not only by the rank ordering of the elements, but also by the way the observable quantity scales with respect to the linear carrier. The geometric model of a conical sand heap serves as a simple analytical example in which, with the carrier structure unchanged, different effects — volume, surface area, equivalent radius, and energy — lead to different concentration laws.

The principal result is that the 20/80 rule is interpreted as a special case within a broader class of dimensionally determined relations. In the proposed approach, specific proportions such as 20/80, 20/79, and 20/62 arise not as independent empirical constants, but as consequences of the choice of metric and of the way in which the resource is mapped. This makes it possible to consider well-known concentration rules within a unified framework.

A further result concerns the use of a symmetric parametric model and the Gini coefficient. It is shown that the observed value of the Gini coefficient may be used to recover the parameter of the normative concentration curve and thus to move from empirically measured inequality to a consistent model structure of resource distribution. Examples for US income and wealth show that such a procedure makes it possible to compare different forms of socio-economic inequality on a common basis.

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## Declaration of Competing Interests

The author declares that there are no competing financial interests and personal circumstances that could affect the results presented in this work.

## Data availability

No new primary data were created in this study. The analytical part of the work uses theoretical conclusions, and the empirical illustration uses published external sources listed in the list of references and in the text of the article. This information is sufficient to reproduce the results presented.

## Declaration on Generative AI and AI-Enabled Technologies in the Writing Process

During the preparation of this work, the author used ChatGPT (OpenAI) to help translate into English, improve the language and style, and improve the design of the manuscript. After using this tool, the author carefully studied and edited the material as necessary, taking full responsibility for the content of the publication.

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