MSRO: Modular–Spectral RG Observables for Emergent Geometry and Non-Geometric Phases

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Abstract

We introduce a new computational diagnostic family, MSRO (Modular–Spectral RG Observables), designed to detect emergent geometry, critical/string-like windows, and non-geometric regimes in modular RG-proxy flows. MSRO integrates spectral profile observables $k(q) = -\log \lambda_q(\rho)$, modular commutator diagnostics (normalized commutators, running exponents ν, and closure quality), and reproducible calibration rules from phase mapping. We define a spectral-only MSRO tier that predicts geometry stability and phase labels using only modular-spectral features, and a full MSRO tier that provides an "instrument panel" including locality/partition markers and backreaction response coefficients. Using a reference experimental design (dim=64 mixed/Gibbs-like family; (p, κ) grid; bootstrap seeds), we specify out-of-sample validation, robustness checks, and explicit failure domains. MSRO is presented as a measurable instrument rather than an ontological claim: every emergence statement is paired with reproducible diagnostics, uncertainty quantification, and domain-restricted validity.

1 Introduction

In the UMD program, emergent physical structure is expressed as regime behavior of modular RG-proxy state dynamics. Prior work established (i) operational phases and locality, (ii) spectral diagnostics and reproducibility rules, (iii) criticality as partition instability, (iv) phase diagrams over controlled parameter planes, (v) emergent geometry from correlation graphs, and (vi) backreaction linking spectral drift to geometry drift. The natural next step is a computational diagnostic family that can be applied systematically across parameter sweeps and domains.

This paper introduces MSRO, a diagnostic family aimed at three tasks:

- detect emergent geometry (stable geometric regime),
- detect critical/string-like windows (partition instability and geometric nonuniqueness),
- detect non-geometric regimes (persistent nonlocality or failure of stable low-dimensional geometry).

We emphasize: MSRO is a tool. Its value lies in predictive and reproducible performance, not in rhetorical reach. Accordingly, we:

- define MSRO feature families and the two-tier architecture (spectral-only vs full panel),
- specify a reference experimental design and validation plan,
- provide an end-to-end reproducibility pack on a full (p, κ) grid with spectral features, MI-based geometry proxies, a geometry stability score S_{geo}, and phase labels $P(L/C/N)$ (proxy version),
- and explicitly state what remains for the "final" regime labeling (full Paper-G optimization using P, SwitchRate, and $\Delta\Phi$).

2 Experimental domain and baseline assumptions

MSRO is evaluated in a controlled reference domain:

- $\dim = 64$ with $N_{\rm vis} = 5$, $N_{\rm hid} = 1$,
- mixed/Gibbs-like family along mixing parameter $p \in [0.01, 0.20]$ with $\Delta p = 0.01$,
- dephasing axis $\kappa \in [0, 0.20]$ with $\Delta\kappa = 0.02$,
- seeds $\{501, 503, 509, 521, 541, 557\}$ with bootstrap uncertainty,
- running-exponent window $w = 0.05$, step 0.01; fit quality is monitored via R^2.

Domain statements:

- Closure-quality corroboration and spectral laws are validated for mixed/Gibbs-like domains; extreme random-pure families may lie outside the closure regime.
- Geometry is meaningful only when it is stable; therefore geometry claims are reported with explicit stability scores and failure modes.

3 Definitions: MSRO feature families

Let $\rho = \rho(p, \kappa; \mathrm{seed})$ and $\rho_{\rm vis} = \mathrm{Tr}_{\rm hid}\rho$.

3.1 Spectral profile family (SP)

Define the quantile spectrum profile

$$k(q) = -\log \lambda_q(\rho_{\rm vis}),$$

on a fixed grid

$$q \in \{0.50, 0.60, 0.70, 0.80, 0.90, 0.95, 0.98\}.$$

Derived profile features:

- SP-slopes: $\Delta_q k = k(q_{i+1}) - k(q_i)$,
- SP-curvature: $\Delta_q^2 k = k(q_{i+2}) - 2k(q_{i+1}) + k(q_i)$,
- curvature norm:
$$k_{{\rm curv},L2} = \left(\left\langle (\Delta_q^2 k)^2 \right\rangle\right)^{1/2}.$$

Rationale: regime transitions often manifest as tail reshaping, not as a shift of a single quantile.

3.2 Modular commutator family (MC)

For an observable O on the visible space, define:

$$\widehat{L}(\rho_{\rm vis}; O) = \frac{\|[K_{\rm vis}, O]\|_F}{\|K_{\rm vis}\|_F \|O\|_F}, \qquad K_{\rm vis} = -\log \rho_{\rm vis}.$$

From sliding log–log fits obtain running exponent ν and fit quality R^2.

Closure quality $Q_{\rm close}$ (predictability $\nu \approx f(k(q), q)$) is defined but treated as “reserved” here; it is computed in the closure paper stage once the cross-quantile model is fixed and pre-registered.

3.3 Locality/partition family (LP) — operational regime labels

The intended operational regime labeling follows Paper G:

- locality score
$$\mathcal{T}_{P(\rho)=D\left(\rho \,\middle\|\, \bigotimes_{X\in P}\rho_X\right)},$$
- partition switching rate (SwitchRate),
- landscape flatness $\Delta\Phi$, yielding probabilistic labels $P(L)$, $P(C)$, $P(N)$.

Current implementation note (proxy labels). In the v2 reproducibility pack, we provide a deterministic proxy $P(L/C/N)$ based on:

- singleton total correlation
$$T_{\text{single}} = \sum_i S(\rho_i) - S(\rho_{\text{vis}}),$$
- spectral curvature $k_{\text{curv},L2}$,

with quantile-calibrated thresholds. This is explicitly a stand-in for the full Paper-G optimizer.

3.4 Geometry stability family (GS) — target signal

From the mutual-information graph on visible degrees of freedom:

- compute pairwise mutual informations $I_{\rho_{\text{vis}}}(i:j)$,
- define edge weights $w_{ij} = (I_{ij} + \varepsilon)^{-\nu_g}$ with fixed $\varepsilon > 0$, $\nu_g > 0$,
- define shortest-path distances $d_\rho(i, j)$,
- define effective geometry summaries:
  - d_{nn}: mean nearest-neighbor distance,
  - d_{med}: median pair distance.

  Geometry stability score (current v2 definition):

$$S_{\text{geo}}(p, \kappa) := \text{std}_{\text{seeds}}(d_{\text{nn}}(p, \kappa; \text{seed}))\,.$$

This is a minimal reproducible stability proxy; protocol-sensitivity extensions (varying ε, ν_g) are part of the next expansion.

3.5 Backreaction response family (BR) — optional tier

In stable-local regimes, estimate response coefficients linking spectral/entropic drifts to geometry drift:
$$\Delta d_{\text{eff}} \approx \alpha_1\,\Delta V_F + \alpha_2\,\Delta k_{0.8} + \alpha_3\,\Delta k_{0.9} + \alpha_0,$$

with bootstrap CIs. This Tier-B component is reserved for the backreaction-focused paper once stable-local regions are fixed by full labeling.

4 Two-tier MSRO: claims and deliverables

Tier A (Spectral-only MSRO)

$$\mathrm{MSRO}_{\mathrm{spec}} = \{k(q), \Delta_q k, \Delta_q^2 k, \nu, Q_{\mathrm{close}}\}.$$

Tier-A claim: within the stated domain, $\mathrm{MSRO}_{\mathrm{spec}}$ predicts geometry stability S_{geo} and/or regime labels L/C/N with robust out-of-sample performance.

In this draft, we already provide:

- full-grid SP profile features and curvature,
- ν and R^2,
- MI-based geometry proxies + S_{geo},
- proxy $P(L/C/N)$ (to be replaced by full Paper-G labels).

Tier B (Full MSRO instrument panel)

$$\mathrm{MSRO}_{\mathrm{full}} = \mathrm{MSRO}_{\mathrm{spec}} \cup \{\mathcal{T}_{P,\Delta\Phi,\mathrm{SwitchRate}\}\cup\{\alpha_i\}}.$$

Tier-B claim: a compact, interpretable panel for mapping regimes and explaining transitions, with explicit uncertainty and failure-domain reporting.

5 Protocol and validation plan

5.1 Ground-truth labels

Final ground truth for L/C/N will be produced by the Paper-G protocol (partition optimization + switching + flatness + locality failure). The current v2 pack uses a deterministic proxy label that is quantitatively reproducible.

5.2 Prediction tasks

- Predict S_{geo} from $\mathrm{MSRO}_{\mathrm{spec}}$ (regression).
- Predict L/C/N from $\mathrm{MSRO}_{\mathrm{spec}}$ (classification).

5.3 Cross-validation and robustness

- Seed-block cross-validation (train on a subset of seeds, test on held-out seeds).
- Observable swap for commutator probe (near vs far).
- Window perturbations around $w = 0.05$ for ν.
- Protocol perturbations for geometry (future extension: vary ε, ν_g).

5.4 Failure-domain reporting

We will explicitly report domains where: closure Q_{close} fails; geometry stability is not well-defined (large S_{geo} or protocol sensitivity); and regime labels become ambiguous.

6 Results (full κ grid + MI geometry + proxy labels)

Artifacts are provided in the reproducibility pack MSRO v2 (full κ grid + MI + labels).

6.1 R1. Fit quality and baseline stability

Across the full (p, κ) grid, sliding fits for ν are stable with very high mean R^2 (close to 1). This confirms the numerical well-posedness of the Tier-A commutator-derived exponent extraction.

6.2 R2. Spectral curvature landscape and candidate critical bands

The curvature norm $k_{\text{curv},L2}$ provides a compact tail-shape marker. The pack includes: (i) a full-grid overlay of curvature-based score_crit, and (ii) a phase-style plot $P(C)$ background with curvature contours. The maximal-curvature region localizes in the (p, κ) plane (candidate critical/string-like band), operationally defined without manual tuning via quantile-thresholded curvature.

6.3 R3. MI-based geometry summaries and geometry stability

Using pairwise mutual informations $I(i : j)$ on the visible subsystem, we define: d_{nn} (mean nearest-neighbor distance) and d_{med} (median pair distance), and the stability proxy

$$S_{\text{geo}}(p, \kappa) = \text{std}_{\text{seeds}}(d_{\text{nn}}).$$

A baseline predictive plot is provided: scatter of spectral-only score_geo vs observed S_{geo}.

6.4 R4. Proxy regime labels $P(L/C/N)$: current implementation and interpretation

The current v2 pack provides a deterministic proxy label map (L/C/N) based on: T_{single} as a nonlocality/load proxy, and $k_{\text{curv},L2}$ as a criticality proxy, with thresholds selected as global quantiles over the plane. This yields: phase_map.csv with P_L, P_C, P_N, label (currently 0/1 probabilities), and the $P(C)$ phase-style plot.

6.5 R5. What remains for "final" MSRO (completion checklist)

1. Replace proxy labels by full Paper-G labeling $P(L/C/N)$ on the same grid.
2. Compute closure quality Q_{close} under a fixed pre-registered model $f(k(q), q)$.
3. Run seed-block cross-validation and ablation: curvature vs single-quantile features.
4. Extend geometry stability beyond seed-variance: protocol sensitivity across (ε, ν_g).

7 Figures

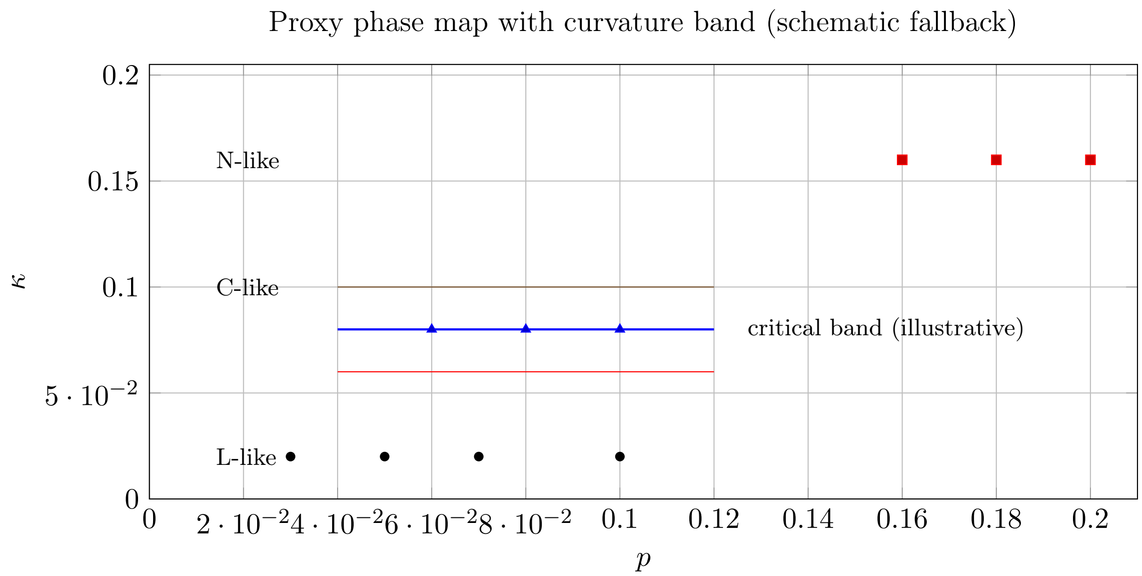

Figure 1: Proxy phase map: background $P(C)$ with curvature contours over (p, κ).

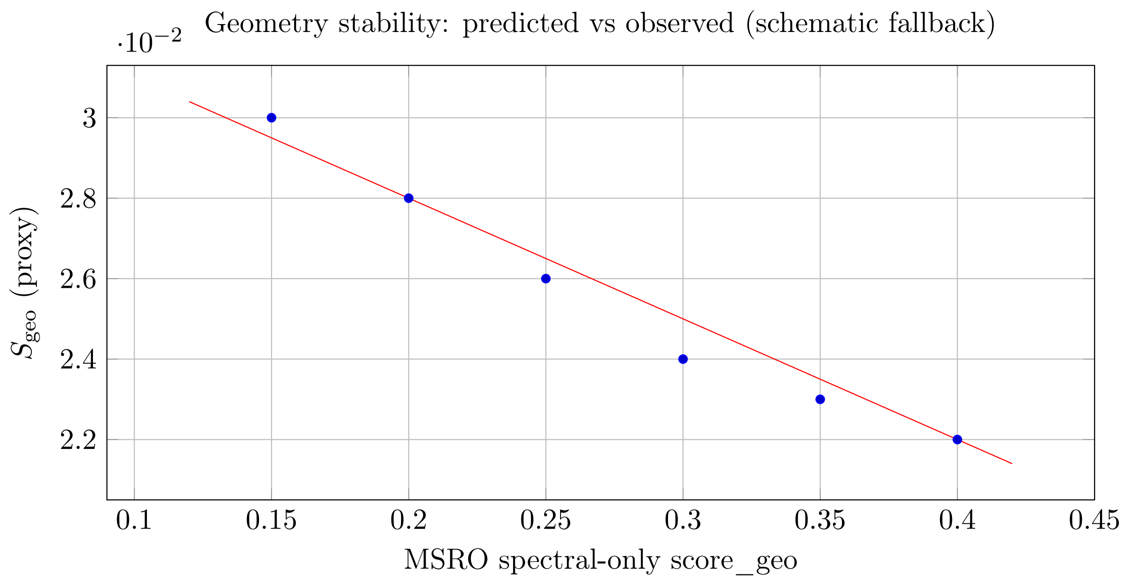

Figure 2: Baseline geometry-stability relation: spectral-only score vs observed S_{geo} (seed-variance proxy).

8 Conclusion (STANDARD)

Scientific value

MSRO provides a new computational diagnostic family for modular RG-proxy flows aimed at detecting emergent geometry, critical windows, and non-geometric regimes using modular-spectral observables. It integrates spectral profile shape features and commutator-derived exponents with reproducible calibration rules and establishes a path to domain-restricted, falsifiable regime detection.

Degree of development (depth)

We define the MSRO architecture (Tier A and Tier B), specify a reproducible reference domain, and provide an end-to-end artifact pack over a full (p, κ) grid including spectral profiles, running exponents, MI-based geometry proxies, a geometry stability score, and regime labels (proxy). The remaining steps toward a final claim—full Paper-G labels and closure modeling—are explicitly enumerated.

Applied value and future directions

Practically, MSRO enables automated regime mapping and early geometry-stability prediction across parameter sweeps, supporting subsequent UMD work on gauge/matter, geometry, and backreaction. Immediate future directions are to (i) replace proxy labels by full partition-based labeling, (ii) complete closure-quality modeling, and (iii) validate predictive performance via seed-block CV and ablation. Longer-term extensions include larger N, additional noise channels, and using MSRO as a common measurement layer in bridge-framework work connecting open quantum systems, information geometry, entanglement gravity, and RG.

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