Operational Locality in Open Quantum Systems via Stable Optimal Factorization and Distinguishability Gap

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Abstract

We introduce an operational criterion for locality in open quantum systems based on (i) a distinguishability gap measured by relative entropy to an optimally factorized product state, and (ii) stability of the optimal partition under protocol perturbations and along trajectories. The protocol yields reproducible L/C/N regime maps, detects critical windows as partition-competition regions, transfers to holdout channels without recalibration, and exposes clear failure domains (near-pure random ensembles). MI-graph geometry is treated as a secondary, ablation-dependent diagnostic layer rather than a universal definition of criticality.

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1 Introduction

A central difficulty in discussing *locality* in open quantum systems is the dependence on prior geometric assumptions: which subsystems count as "near", what interaction graph is presumed, and which coarse-graining is accepted. We propose an operational alternative: define locality through (a) how well a state is approximated by a product over an *optimally chosen* partition, and (b) how *stable* that optimal partition is under protocol variations and along parameterized trajectories. Criticality is characterized as competition among nearly-optimal factorizations (flat objective landscape and/or switching), rather than via a presumed correlation length.

2 Definitions

Fix an access level (fixed subsystem / observable algebra) and represent the accessible state by a density operator ρ with $\rho \geq 0$ and $\operatorname{Tr}\rho = 1$. Let $\mathcal{P}$ be a partition class (baseline: 1D contiguous blocks). Define

$$J_\eta(P;\rho) = \Phi(P;\rho) + \eta\,\Omega(P), \qquad P^*(\rho) = \arg\min_{P\in\mathcal{P}} J_\eta(P;\rho),$$

and the flatness proxy $\Delta J(\rho) = J_\eta(P_{(2)};\rho) - J_\eta(P_{(1)};\rho)$. Define the distinguishability gap

$$TP^*(\rho) = D\left(\rho \,\middle\|\, \bigotimes_{X\in P^*(\rho)} \rho_X\right), \quad D(\rho\|\sigma) = \operatorname{Tr}\big[\rho(\log\rho - \log\sigma)\big].$$

On trajectories $\rho(\lambda)$, define

$$\text{SwitchRate} = \frac{1}{T}\sum_{t=0}^{T-1} \mathbf{1}[P^*(\rho(\lambda_{t+1})) \neq P^*(\rho(\lambda_t))]\,.$$

3 Main Result (Theorem A2)

Theorem 1 (Theorem A2 (Operational Locality Criterion; empirical-protocol))**.** ***Fix (protocol inputs):*** *(1) an access level (current pack: ρ_{vis}); (2) a partition class $\mathcal{P}$ (baseline: 1D contiguous blocks); (3) a protocol Π returning top-k candidates and selecting $P^*(\rho) = \arg\min_{P\in\mathcal{P}} J_\eta(P;\rho)$ with tie/flatness handling; (4) the distinguishability gap $TP^*(\rho)$; and (5) stability metrics: $\Delta J(\rho)$, $PS(\rho)$, and SwitchRate on trajectories.*

Ref-only calibration rule: *thresholds are calibrated only on D_{ref} by a preregistered quantile rule, then frozen.*

Claim: *the labels $\mathbf{L}/\mathbf{C}/\mathbf{N}/\mathbf{U}$ are reproducible on D_{ref}, transferable to D_{hold} without recalibration, and detect critical windows via elevated competition (SwitchRate↑ and/or $\Delta J\downarrow$). Domain statements: D_{ref} mixed/structured; D_{hold} holdout channels; D_{fail} near-pure random ensembles. Geometry clause: no universal "C $\Rightarrow$ increased geometry-variance" claim; MI-graph geometry is secondary and ablation-dependent.*

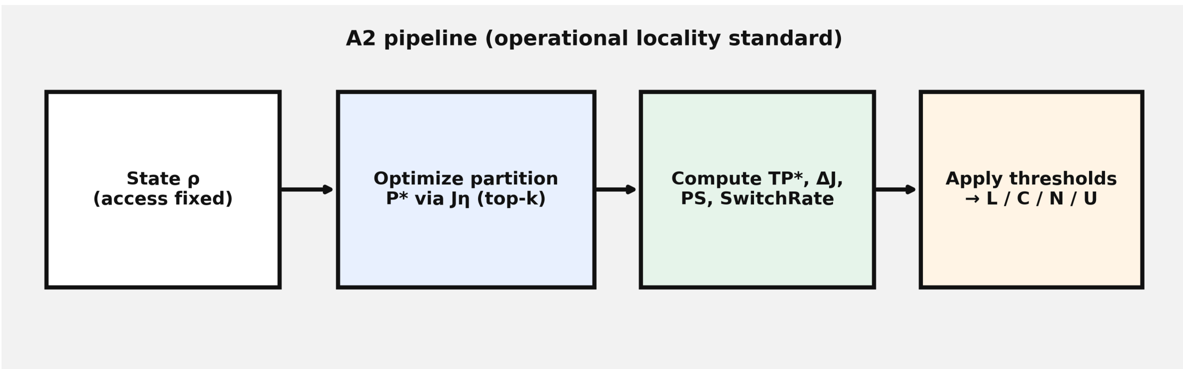

Figure 1: A2 pipeline.

4 Protocol / preregistration

For $N_{\text{vis}} \leq 5$, we enumerate all cut patterns (2^{N-1} partitions) and keep top-k (here $k = 5$). Thresholds are calibrated only on D_{ref} using a preregistered quantile rule, then frozen and transferred to D_{hold} without recalibration.

5 Results

5.1 Reference regime map (D_{ref})

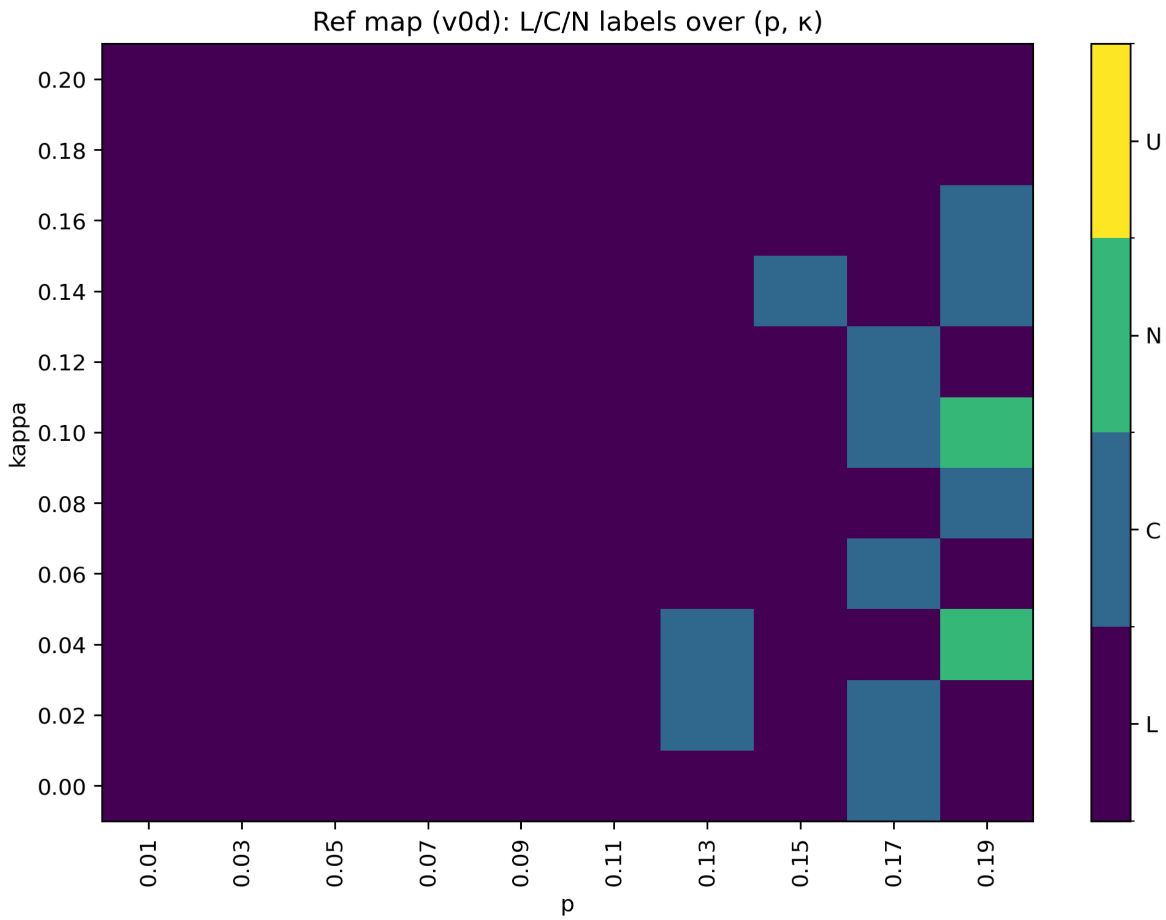

Figure 2: Reference map.

5.2 Holdout transfer (D_{hold})

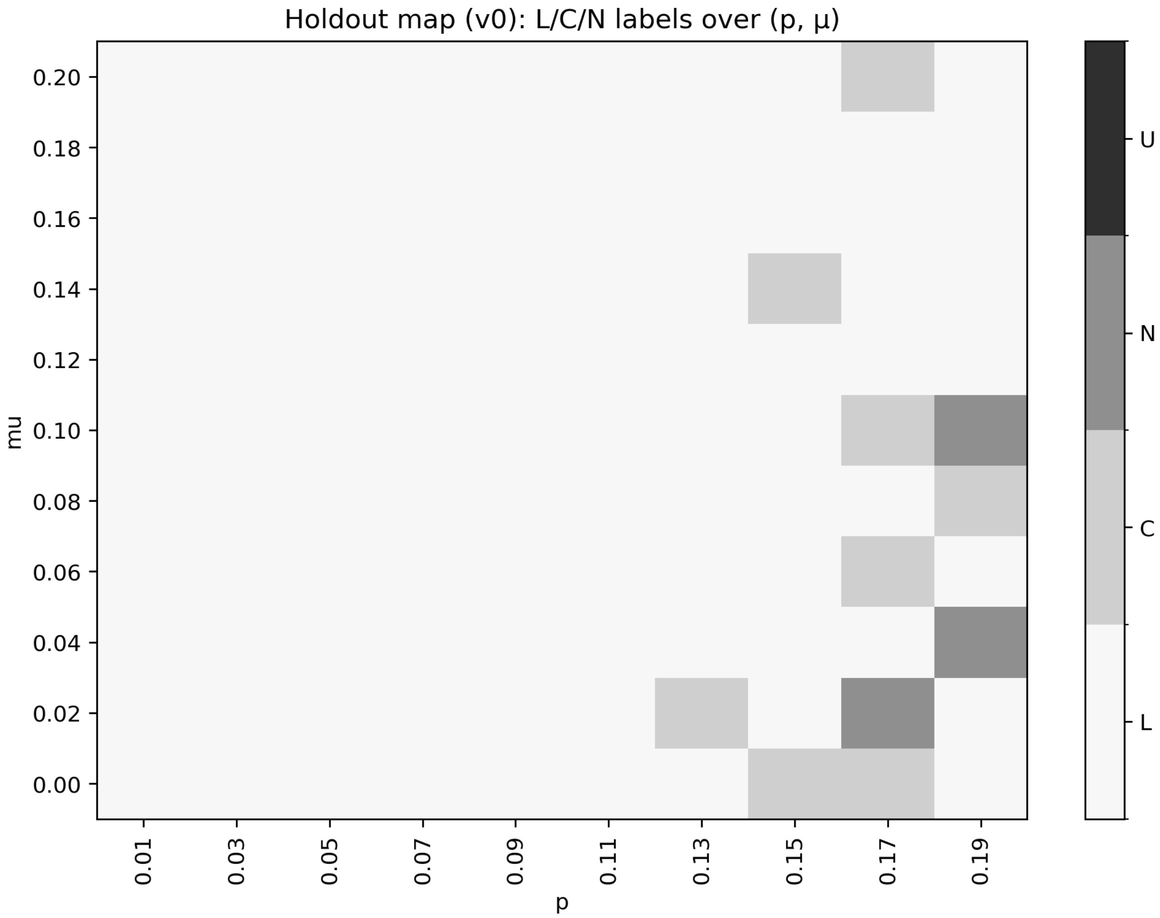

Figure 3: Holdout map.

5.3 Critical windows on trajectories

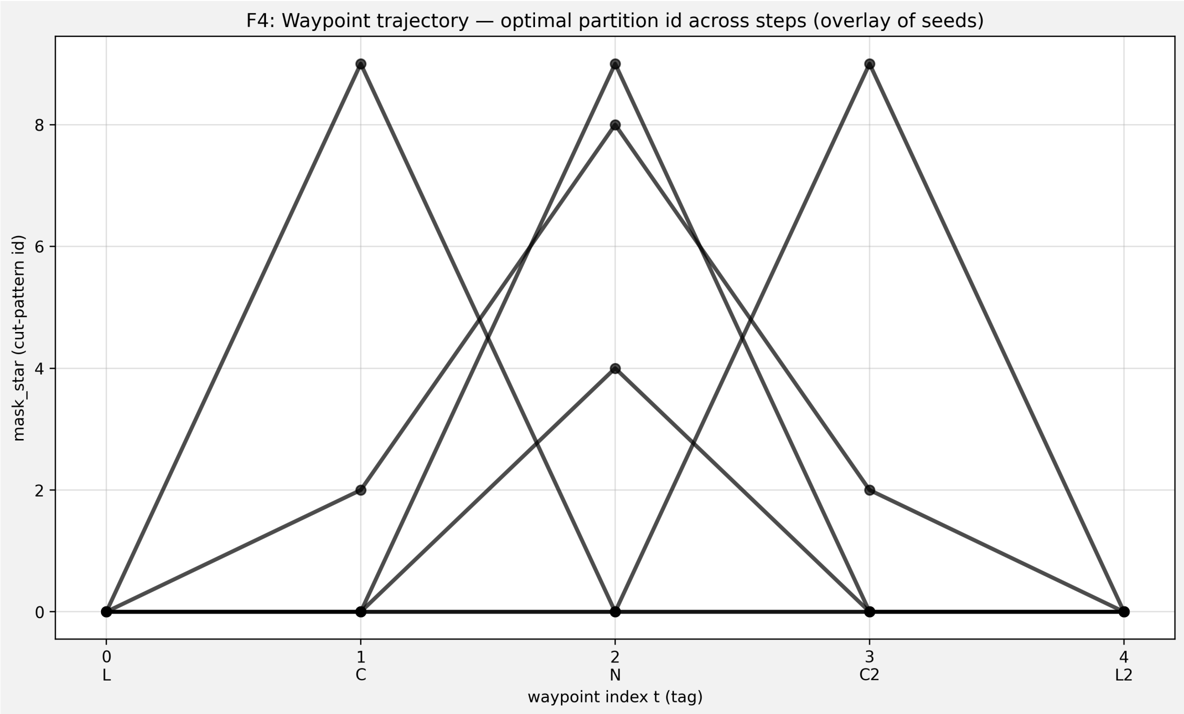

Figure 4: Waypoint trajectory: optimal partition id across steps (overlay of seeds).

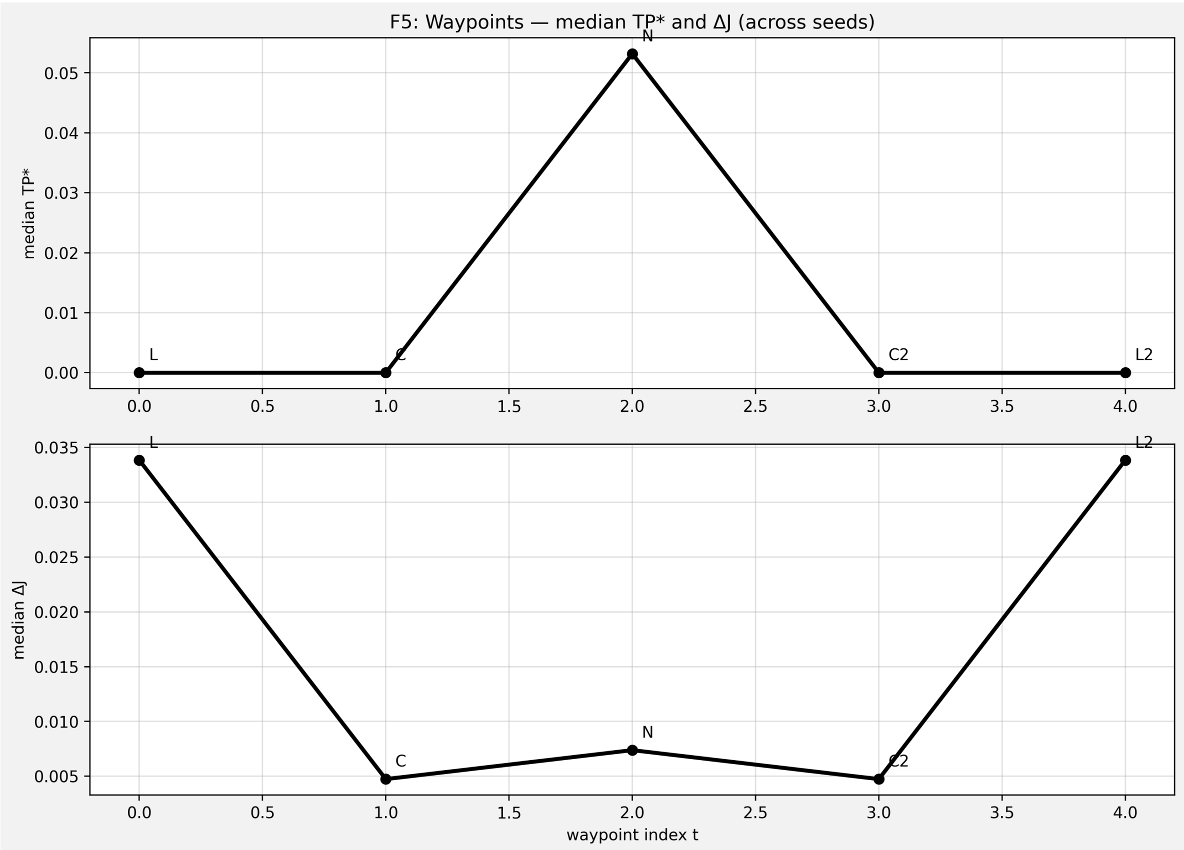

Figure 5: Waypoints: median TP^* and ΔJ across seeds.

5.4 Failure domain (D_{fail})

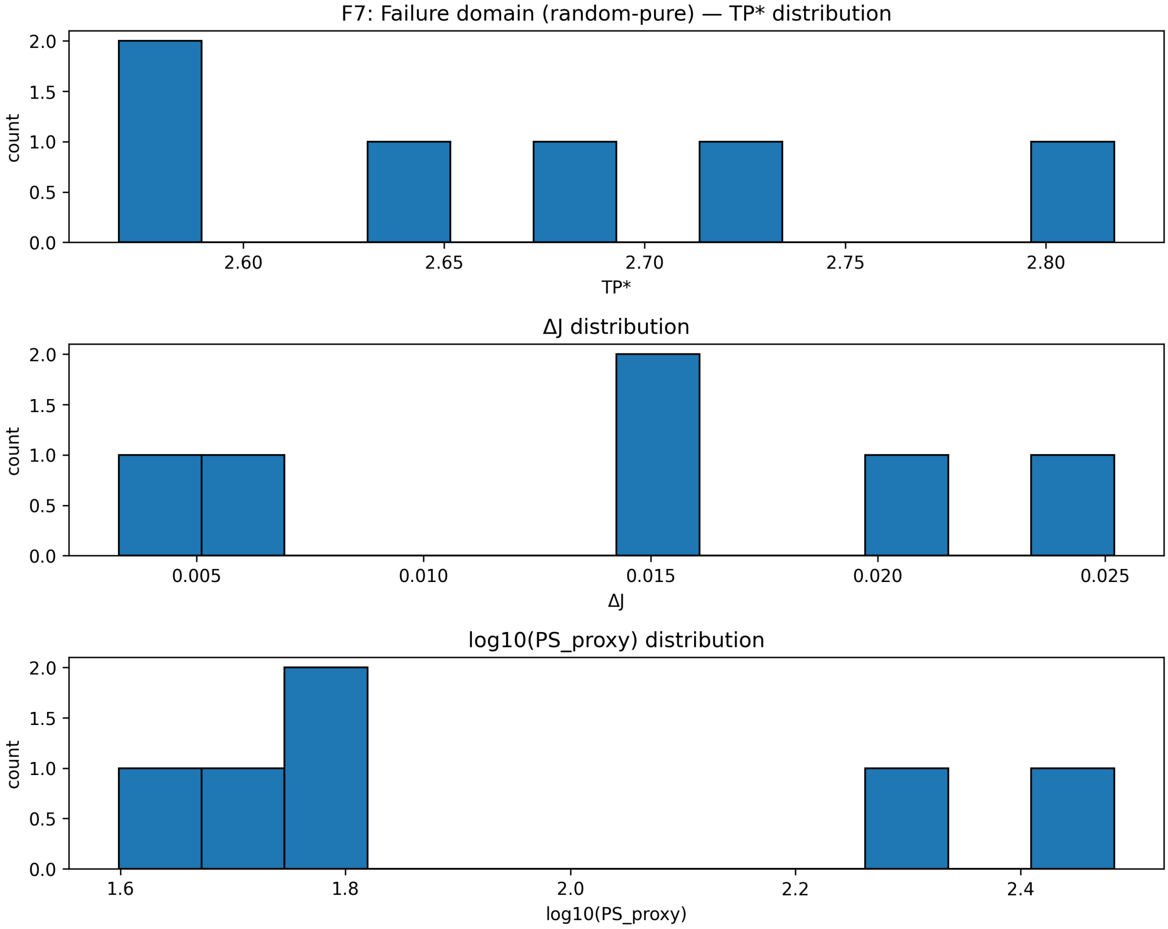

Figure 6: Failure domain (near-pure random): distributions of TP^*, ΔJ, and $\log_{10}(\text{PS}_{\text{proxy}})$.

5.5 Geometry diagnostics (secondary layer)

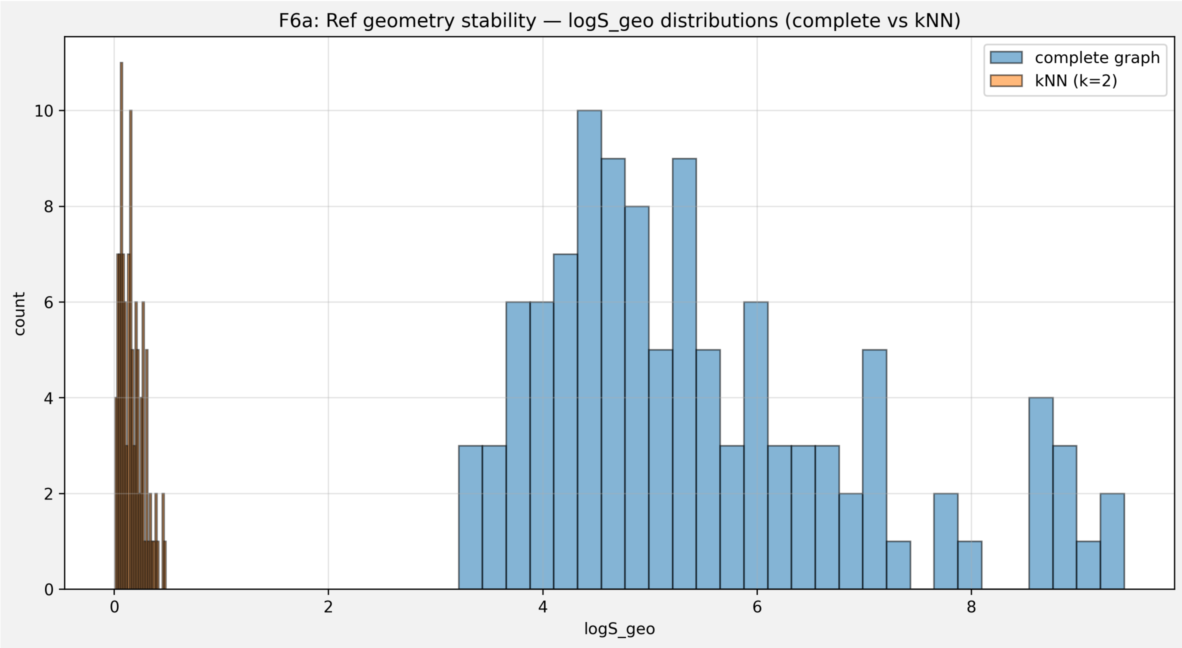

Figure 7: Ref geometry stability: $\log S_{\text{geo}}$ distributions (complete vs kNN).

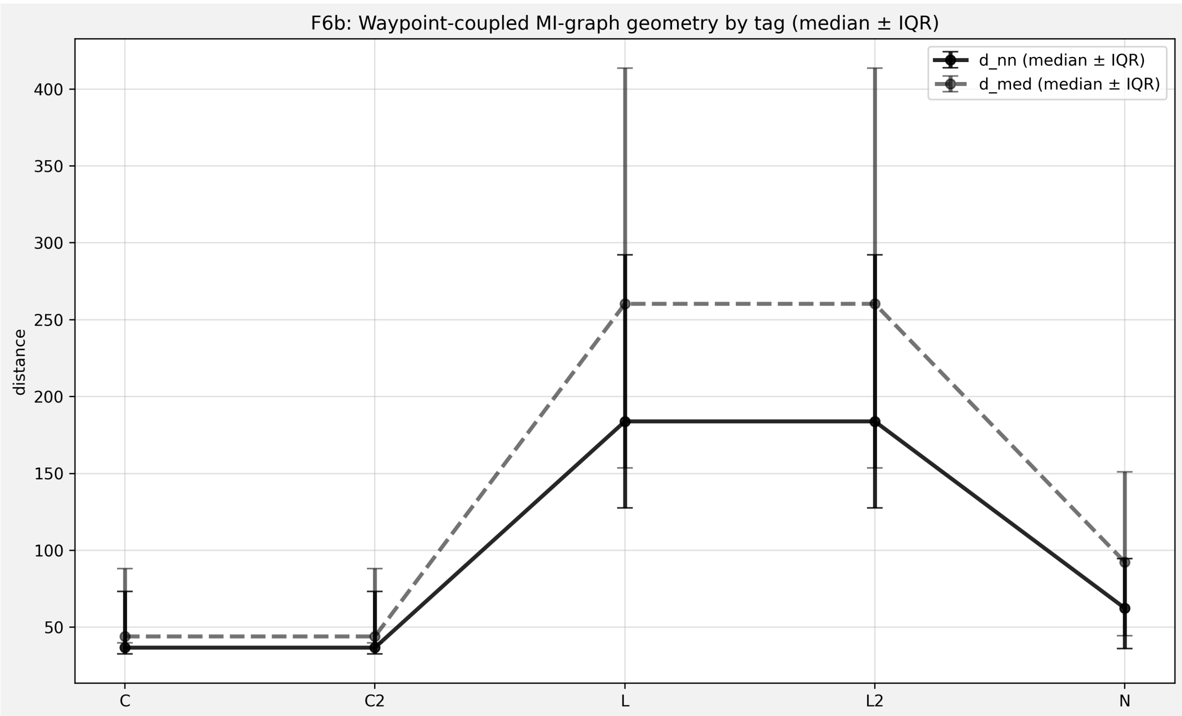

Figure 8: Waypoint-coupled MI-graph geometry by tag (median ± IQR).

6 Discussion and limitations

Operational locality is defined as small TP^* together with a stable protocol-defined optimal partition P^*. Criticality is diagnosed by partition competition (flatness ΔJ and/or switching along trajectories) at small/moderate TP^*. Thresholds are calibrated on D_{ref} and transferred to D_{hold} without recalibration. Near-pure random families constitute an explicit boundary of applicability. Geometry diagnostics are treated as secondary and ablation-dependent; no universal "criticality implies increased geometry variance" requirement is imposed.

7 Conclusion

This work establishes a concrete operational standard for diagnosing locality without imposing a geometric postulate.

Scientific value. The criterion turns locality/criticality into a reproducible, auditable diagnostic pipeline with explicit domain statements.

Depth of investigation. The work combines information-theoretic functionals with protocol stability and transfer testing, enforcing reproducibility via artifacts.

Potential discoveries enabled. Partition-competition diagnostics can reveal critical windows and regime boundaries that are not captured by correlation-length narratives.

References

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