QUANTUMOGRAPH v14

THE NO-ASYMPTOTIC-CHAOS THEOREM

Ontic chaos is impossible; observable chaos is an artefact of coarse-graining

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Abstract

Within Quantumograph theory—where the universe is modelled as a finite directed quantum graph with unitary evolution—asymptotic chaos is impossible in three rigorous senses:
(1) spectral quasiperiodicity: the Hilbert-space trajectory is almost periodic in the evolution parameter;
(2) Poincaré recurrence: the system returns arbitrarily close to any initial state after finitely many unitary iterations;
(3) saturation of the quantum Lyapunov exponent: the OTOC function is bounded above and saturates at n* ~ ln N / λ_{max} fter which there is no further exponential divergence. We present here is an explicit separation of ontic chaos (a property of the exact microdynamics) from epistemic/coarse-grained chaos (an artefact of incomplete information), together with a rigorous proof that coarse-graining at scale ℓ >> a generates observable stochasticity from deterministic unitary dynamics.

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1. Key Distinction: Ontic vs Epistemic Chaos

Central thesis: Chaos is not an ontological property of the exact microdynamics. It is an emergent property of coarse-grained descriptions.

Standard definitions of chaos (sensitive dependence on initial conditions, topological mixing, positive Lyapunov exponent) apply to a specific model and require specification of the level of description. We introduce a rigorous distinction:

Definition 1.1 (Ontic Chaos). Chaos at the level of the exact microdynamics: positive asymptotic Lyapunov exponent $\lambda_L = \limsup_{t\to\infty} \frac{1}{t} \ln C(t) > 0$ for the exact unitary evolution $\hat{U}^t|\Psi\rangle$. This would imply irreversible exponential divergence of quantum trajectories.

Definition 1.2 (Epistemic / Coarse-grained Chaos). Chaos at the level of the coarse-grained description: positive λ_L^{eff} for the dynamics $\Pi_\ell \circ \hat{U}^t$, where Π_ℓ is the coarse-graining operator to scale ℓ. This is a finite-window, finite-amplitude effect arising from incomplete information.

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The present theorem proves: ontic chaos in Quantumograph is impossible. Epistemic chaos exists and is mathematically derived from coarse-graining (Corollary 6.4 and §7.3). This is not a contradiction - it is a precise delineation of levels.

Three routes to the proof:

- **Via OTOC and Lyapunov:** $\lambda_L = 0$ follows directly from boundedness of C(t) (Lemma 3.3 + Lemma 3.4).
- **Via almost-periodicity:** The trajectory is a trigonometric polynomial - an almost-periodic function (Lemma 3.1). True chaos is incompatible with almost-periodicity.
- **Via coarse-graining:** The operator $\Pi_\ell \circ \hat{U}^t$ generates effective variance growing as $C_3 \varepsilon_{spec}$ as ℓ → a (§7.3).

2. Axiomatics: Quantumograph Structure

In Quantumograph, the evolution parameter n counts the steps of the unitary operator $\hat{U}$, which itself is constructed from the topological charges q_e as described in Quantumograph v14. Thus n is an emergent quantity, not an external coordinate.

(Q1) $N = |V| < \infty$ - finite number of vertices.

(Q2) Each vertex carries a qubit: $H_v \cong \mathbb{C}^2$. Total Hilbert space $H = \bigotimes_v H_v \cong \mathbb{C}^{2^N}$.

(Q3) Evolution operator: $\hat{U} = \hat{U}_{global} \circ \hat{R}_{local}$.

(Q4) Unitarity: $\hat{U}^\dagger \hat{U} = \hat{I}$. Reversible Fredkin–Toffoli gates guarantee unitarity.

(Q5) Noise Hamiltonian: $\|\hat{H}\| \le J_{max}|E| + h_{max} N < \infty$.

3. Definitions of Chaos in Quantum Systems

Definition 3.1 (Quantum Lyapunov Exponent / OTOC). $F(t) = \langle \hat{W}^\dagger(t)\, \hat{V}^\dagger(0)\, \hat{W}(t)\, \hat{V}(0)\rangle_\beta$, $\hat{W}(t) = \hat{U}^t\ \hat{W}\ \hat{U}^{-t}$. Scrambling function: $C(t) = 1 - \Re F(t)$. Ontic Lyapunov exponent: $\lambda_L = \limsup_{t\to\infty} \frac{1}{t} \ln C(t)$.

Definition 3.2 (Asymptotic Ontic Chaos). The system exhibits ontic asymptotic chaos if $\lambda_L > 0$, i.e., C(t) grows exponentially without bound.

Definition 3.3 (Effective / Observable Epistemic Chaos). An observer with resolution ℓ sees chaos if $\lambda_L^{eff}(\ell) = \limsup \frac{1}{t} \ln C_\ell(t) > 0$ for $C_\ell(t) = C(\Pi_\ell \circ \hat{U}^t)$. This is a property of the observer, not the system.

4. Auxiliary Lemmas

Lemma 4.1 - Discrete Spectrum on the Unit Circle

Let $\hat{U}$ be unitary, dim $H = 2^N < \infty$. Then:

(i) $\sigma(\hat{U}) = \{e^{i\omega_k}\}$ - exactly 2^N eigenvalues, $\omega_k \in [0,2\pi)$.

(ii) $|\Psi(t)\rangle = \Sigma_k c_k e^{i\omega_k t}$ |φ_k⟩ - a trigonometric polynomial, hence almost-periodic (Bohr 1925).

(iii) $\langle\hat{O}\rangle(t) = \Sigma_{k,l} c_k c_l \langle\varphi_k|\hat{O}|\varphi_l\rangle e^{i(\omega_l - \omega_k)t}$ - finite sum.

Proof. *A unitary operator on a finite-dimensional space is normal, hence diagonalizable by the spectral theorem. From* $\hat{U}\dagger\hat{U} = \hat{I}$: $|\lambda_k|^2 = 1$. *Parts (ii)–(iii) by direct computation.* □

Lemma 4.2 - Quantum Poincaré Recurrence

Here t denotes the evolution parameter of the exact unitary dynamics; in Quantumograph it is interpreted as an emergent parameter at the effective level.

Under the conditions of Lemma 4.1, for any $|\Psi(0)\rangle \in \mathcal{H}$ and any $\epsilon > 0$, there exists a positive integer finite return step $T = T(\epsilon, |\Psi(0)\rangle)$ such that $[\| |\Psi(T)\rangle - |\Psi(0)\rangle | < \epsilon.]$

Return-step estimate: $[T \le \frac{2\pi}{\delta_\epsilon}, \qquad \delta_\epsilon = \frac{\epsilon^2}{4\cdot 2^N \max\limits_k |c_k|^2}]$.

Caveat. For physically very large N, the return step exceeds any accessible observation window. In practice, the system may appear chaotic over all experimentally relevant ranges of the evolution parameter.

Proof. $\||\Psi(t)\rangle - |\Psi(0)\rangle\|^2 = 4\Sigma_k |c_k|^2 sin^2(\omega_k t/2)$. *Apply the multidimensional Kronecker theorem on Diophantine approximation.* □

Lemma 4.3 - Boundedness of the OTOC

Under Q1–Q5: (i) $|F(t)| \le 1$ for all $t \ge 0$. (ii) $C(t) \in [0,2]$ for all t. (iii) $\sup\limits_t C(t) \le 2$.

Proof. *By Q4,* $\hat{W}(t)$ *is unitary,* $\|\hat{W}(t)\| = 1$. *Hölder's inequality for normal states:* $|F(t)| \le 1$, *hence* $C(t) \in [0,2]$. □

Lemma 4.4 - OTOC Saturation at n*

(i) Growth: for $0 \le n \le n* = \frac{\ln(2^N)}{\lambda_{max}}$: $[C(n) \le C_0 e^{\lambda_L n}, \qquad \lambda_L \le \frac{2\pi k_B T}{\hbar}]$ (MSS bound).

(ii) Saturation: for $n >$ n*: $[C(n) \le 2 + \delta_{\text{rec}}, \qquad \delta_{\text{rec}} \to 0$ as $n \to \infty.]$

(iii) Ontic Lyapunov exponent: $[\lambda_L = lim\,sup_{n\to\infty} \frac{1}{n} \ln C(n) = 0.]$

Proof. *(i) MSS bound from analyticity of OTOC in strip* $0 \le \Im n \le \beta\hbar/2$ *and the Schwarz lemma. (ii) F(n) is almost-periodic (Lemma 4.1), so* $|C(n)| \le 2$ *always; after* $n^* \sim N \ln 2/\lambda_L$ *the function oscillates around the Page value. (iii) From* $C(n) \le 2 < \infty$: $\lambda_L = \lim\sup \ln C(n)/n \le \lim\sup \ln 2/n = 0$; *and* $C(t) \ge 0$ *gives* $\lambda_L \ge 0$; *hence* $\lambda_L = 0$. □

5. Main Theorem

Theorem 5.1 - Absence of Ontic Asymptotic Chaos.
Let the universe be described by a quantum graph G satisfying Q1–Q5. Then:

- **(I) Spectral Quasiperiodicity.** $|\Psi(t)\rangle$ is almost-periodic in t; $\sup_t \| |\Psi(t)\rangle \| = 1$.
- **(II) Poincaré Recurrence.** For any $\varepsilon > 0$ $\exists\, T_\varepsilon < \infty : \| |\Psi(T_\varepsilon)\rangle - |\Psi(0)\rangle \| < \varepsilon$.
- **(III) Zero Ontic Lyapunov Exponent.** $\lambda_L = \limsup_{t \to +\infty} \frac{1}{t} \ln C(t). = 0$.
- **(IV) Bounded Scrambling.** $\exists$ finite number of steps n^* : $C(n) \leq 2 < \infty$ for all $n > n^*$.
- **(V) MSS Bound as Corollary.** $\lambda_L^{(eff)}(n) \leq \frac{2\pi k_B T}{\hbar}$, $n^* \sim N \ln \frac{2}{\lambda_L^{(eff)}}$.

In the language of ontic vs epistemic chaos:

Ontic chaos ($\lambda_L > 0$ for $\hat{U}^t$): impossible.

Epistemic chaos ($\lambda_L^{eff} > 0$ for $\Pi_\ell \circ \hat{U}^t$): possible and mathematically derived from coarse-graining (§7.3).

Proof. (I)–(IV) follow from Lemmas 4.1–4.4. (I) Almost-periodicity from Bohr's theorem (1925) and Lemma 4.1. (II) Lemma 4.2. (III) Lemma 4.4(iii). (IV) Lemma 4.3. (V) MSS from boundedness of $\hat{H}$ (Q5). □

6. Corollaries

Corollary 6.1 - Absence of Strange Attractors

In a system with Q1–Q5, there exists no invariant set $A \subset H$ with non-integer Hausdorff dimension to which trajectories converge irreversibly. Unitary evolution is an isometry of H, preserving the Haar measure on $U(2^N)$. The dissipation required for a strange attractor is incompatible with $\hat{U}^\dagger \hat{U} = \hat{I}$.

Corollary 6.2 - Information Conservation

For any two initial states $|\Psi_1(0)\rangle, |\Psi_2(0)\rangle$: $\| |\Psi_1(t)\rangle - |\Psi_2(t)\rangle \| = \| |\Psi_1(0)\rangle - |\Psi_2(0)\rangle \|$ for all t. Information (defined via distance in H) is strictly conserved.

Corollary 6.3 - MSS Bound (consistency condition)

The quantum Lyapunov exponent in Quantumograph satisfies $\lambda_L = \frac{2\pi k_B T}{\hbar}$. As $T \to 0$: $\lambda_L \to 0$. Equality $\lambda_L = \frac{2\pi k_B T}{\hbar}$ is unreachable (it belongs to black holes with $N \to \infty$; in Quantumograph $N < \infty$).

Corollary 6.4 - Apparent Randomness as Coarse-Graining

Any observable randomness in macroscopic predictions arises from coarse-graining ($\Pi_\ell : a \to \ell$), not from fundamental stochasticity. For any observable $\hat{O}$ on scale $\ell >> a$: $Var_\ell(\hat{O}) \leq C_3\, \varepsilon_{spec}$. Variance is nonzero not due to stochasticity but due to imprecise knowledge of the initial state or the choice of a coarse observable. As shown in the companion [13] Reducibility Theorem, coarse-grained observables can be predicted polynomially, explaining why macroscopic randomness does not require fundamental stochasticity.

7. Limitations

Theorem 5.1 rigorously proves the absence of ontic asymptotic chaos. It does not assert:

- **On observation window 0 < n ≤ n*, C(n) may grow exponentially - real, but finite.**
- **Absence of scrambling:** Poincaré return step ~ $(2\pi/\varepsilon)^{2^N}$ - astronomically large for physical N.
- **Predictability:** measurement outcomes remain open; the theorem applies to unitary evolution between measurements.
- **Determinism of measurements:** in the thermodynamic limit return step → ∞ and chaos becomes effectively real. Chaos in the limit N → ∞.

7.3 Coarse-Graining Generates Epistemic Chaos (key section)

Let Π_ℓ : H → $\mathcal{H}_\ell$ be a coarse-graining operator to scale ℓ >> a, dim $\mathcal{H}_\ell$ = M = $\left(\frac{L}{\ell}\right)^4$ << 2^N. Effective dynamics:

$$\widehat{U_{\text{eff}}}(t) = \Pi_1 \circ U^t \circ \Pi_1 \colon \mathcal{H}_\ell \to \mathcal{H}_\ell$$

Lemma 7.1 (Irreversibility of coarse-grained dynamics). The operator U_{eff}(t) = $\Pi_\ell \hat{U}^t \Pi_\ell$ is generically non-unitary: $U_{eff}^{\dagger} U_{eff} \neq I_\ell$. The unitarity defect is bounded by spectral leakage:

$$\left\| U_{eff}^{\dagger} U_{eff} - I_\ell \right\| \leq C_1 \left(\frac{\ell}{L}\right)^2 \cdot \frac{\|\hat{H}\|}{\hbar} =: \varepsilon_{unit} \ , \ \widehat{U} = e^{-i\widehat{H}\delta t}.$$

Proof. Π_ℓ is not a partial isometry w.r.t. the inner product of $\widehat{H}$: spectral modes with k > $\frac{\pi}{\ell}$ (UV modes) project non-trivially into $\widehat{H_\ell}$. The defect norm is estimated via the commutator $[\Pi_\ell, \hat{U}]$ using the Lieb–Robinson bound. □

Theorem 7.2 (Coarse-graining generates positive effective Lyapunov exponent).
For the coarse-grained dynamics $\Pi_\ell \circ \hat{U}^t$ with $\varepsilon_{unit} > 0$:

$$\lambda_L^{\text{eff}}(\ell) \geq \frac{1}{t_*} \ln\left(\varepsilon_{\text{unit}}^{-1}\right) > 0$$

An observer with resolution ℓ sees a positive Lyapunov exponent, even though the exact microdynamics has $\lambda_L = 0$.

Corollary: chaos is a property of the observation scale, not the system. As ℓ → a (exact observation): $\lambda_L^{eff} \to 0$ and the ontic result is recovered.

This explains why macroscopic systems ($N \sim 10^{23}$) appear chaotic: coarse-graining to macroscopic degrees of freedom inevitably introduces $\varepsilon_{\text{unit}} > 0$. The observed irreversibility is therefore an emergent property of the effective description rather than a property of the exact microdynamics.

Why coarse-graining does not rescue Wolfram's cellular automata: in cellular automata, the microdynamics is itself irreversible (W2). Coarse-graining does not introduce irreversibility - it is already present at the micro level. In Quantumograph: no irreversibility at the micro level; coarse-graining creates it. This is the fundamental distinction between the quantum and classical cases.

7.4 Remark.

A useful conceptual analogy can be drawn with geometric flows and the Poincaré program. In Ricci flow with surgery, singularities are removed to maintain a global control; here, coarse-graining removes microscopic degrees of freedom, preventing the appearance of strange attractors at the ontic level. In both cases, the apparent complexity belongs to the effective description rather than to the fundamental dynamics. Coarse-graining plays a role analogous to surgery: it regularizes the observed behavior while leaving the underlying reversible structure intact. Thus, the observed chaotic regime should be viewed as an artifact of coarse-graining, not as a property of the microscopic system itself.

8. Comparison Table

Property	**Classical Chaos**	**Quantumograph (ontic level)**	**Quantumograph (epistemic level)**
Lyapunov exponent	$\lambda > 0$ forever	$\lambda_L = 0$	$\lambda_L^{eff} > 0$ for $n \leq n^*$
Strange attractors	Yes (fractal)	No (Cor. 6.1)	Effectively possible
Information conservation	No (dissipation)	Yes (Cor. 6.2)	No ($\varepsilon_{unit}>0$)
Recurrence	No	Yes (Theorem 5.1 II)	Not practically
Scrambling	N/A	Yes, $C(t) \leq 2$	Yes, effective
Sensitivity to init. cond.	Exponential forever	Exponential to n*, then oscillations	Exponential
Source of randomness	Microdynamics	None (deterministic)	Coarse-graining

Table 1. Classical chaos vs Quantumograph at the ontic and epistemic levels.

9. Summary

Main result.

$\boldsymbol{\lambda_L = 0}$ ***zero ontic asymptotic Lyapunov exponent***

$\boldsymbol{C(n) \leq 2}$ ***for all n (OTOC is bounded)***

For every $\epsilon > 0$, there exists a finite return step T_ϵ

Effective chaotic behavior may appear over a finite observation window, with $n^* \sim \frac{N \ln 2}{\lambda_L^{(\mathrm{eff})}}$, but it remains finite in both amplitude and extent.

Ontic chaos (in exact unitary microdynamics): impossible.

Epistemic chaos (in coarse-grained description): Effective chaotic behavior may appear and mathematically derived from Theorem 7.2.

10. Implications and Practical Outlook.

The main consequence is not a universal cryptanalytic breakthrough, but a precise separation between principled and operational predictability. Exact microstate prediction remains exponentially costly, whereas coarse-grained observables admit polynomial forecasting. This makes the framework relevant for model reduction, spectral inference, and device-level analysis, while the cryptographic threat landscape remains governed by dedicated quantum algorithms and by the ongoing transition to post-quantum cryptography.

11. Notation

In Quantumograph, "time" is understood as an emergent evolution parameter labeling unitary steps, not as a fundamental external background variable:

- **time t → n** (evolution parameter);
- **timescale → observation window**;
- **finite time → finitely many evolution steps.**

12. Reference:

1. T. Konopka, F. Markopoulou, and L. Smolin, "Quantum Graphity," *arXiv preprint hep-th/0611197*, 2006. DOI: 10.48550/arXiv.hep-th/0611197
2. T. Konopka and F. Markopoulou, "Constrained Mechanics and Noiseless Subsystems," *arXiv preprint gr-qc/0601028*, 2006. DOI: 10.48550/arXiv.gr-qc/0601028
3. J. Maldacena, S. H. Shenker, and D. Stanford, "A bound on chaos," *J. High Energy Phys.*, vol. 2016, no. 8, p. 106, 2016. DOI: 10.1007/JHEP08(2016)106
4. N. Tsuji, T. Shitara, and M. Ueda, "Bound on the exponential growth rate of out-of-time-ordered correlators," *Phys. Rev. E*, vol. 98, no. 1, p. 012216, 2018. DOI: 10.1103/PhysRevE.98.012216
5. A. Dymarsky and M. Smolkin, "Krylov complexity in conformal field theory," *Phys. Rev. D*, vol. 104, no. 8, p. 086025, 2021. DOI: 10.1103/PhysRevD.104.086025
6. D. Pazó and M. Cencini, "Universal scaling of Lyapunov-exponent fluctuations in space-time chaos," *Phys. Rev. E*, vol. 87, no. 6, p. 062909, 2013. DOI: 10.1103/PhysRevE.87.062909
7. N. Israeli and N. Goldenfeld, "On computational irreducibility and the predictability of complex physical systems," *arXiv preprint nlin/0309047*, 2003. DOI: 10.48550/arXiv.nlin/0309047
8. I. García-Mata, M. Saraceno, and M. E. Spina, "Classical decays in decoherent quantum maps," *Phys. Rev. E*, vol. 67, no. 4, p. 046215, 2003. DOI: 10.1103/PhysRevE.67.
9. X. Huang and B. Zhang, "Growth of a renormalized operator as a probe of chaos," *arXiv preprint arXiv:2110.15306*, 2021.
10. S. Wolfram, *A New Kind of Science*. Champaign, IL: Wolfram Media, 2002.
11. H. Zwirn and J.-P. Delahaye, "Unpredictability and computational irreducibility," *arXiv preprint arXiv:1111.4121*, 2011. DOI: 10.48550/arXiv.1111.4121
12. G. J. Martinez, H. Zenil, and S. V. Chvykov, "Wolfram's Classification and Computation in Cellular Automata Classes III and IV," in *Irreducibility and Computational Equivalence*, Springer, 2012. DOI: 10.1007/978-3-642-35482-2_6
13. S. Materov, “Computational reducibility theorem”, 2026, DOI: 10.5281/zenodo.19307648
14. S. Materov, “Quantumograph: A Testable Quantum Graph Theory of Spacetime” 2026, DOI: 10.5281/zenodo.18410313