A Modular Information-Dynamical Bridge Between Open Quantum Systems, Information Geometry, Entanglement Gravity, and RG

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Abstract

We propose a bridge-framework: a single operational language connecting open quantum systems (CPTP/GKSL dynamics), information geometry (contractive metrics and divergences), entanglement-based emergent geometry (correlation graphs and stability), and RG-style diagnostics (running exponents and spectral coordinates). The framework is deliberately conservative: emergence statements are paired with computable functionals, reproducible pipelines, explicit domain statements, and failure-domain reporting. We define a measurement layer $\{\rho, K, K_{\rho|\sigma}, D, g_\rho, k(q), \widehat{L}, \nu, \mathcal{T}_{P,d_\rho,S_{\mathrm{geo}}}\}$ and a flow layer based on CPTP-consistent modular dynamics with a reference state σ. Two bridge theorems (with proof sketches) connect CPTP contractivity to information geometry and stable locality to well-posed MI-graph geometry. A reference experimental design (dim=64, (p, κ) grid, bootstrap seeds) is stated as an empirical anchor, with explicit limitations and upgrade paths.

1 Introduction and scope

This paper proposes a bridge-framework: a single operational language that simultaneously covers (i) open quantum systems (CPTP/GKSL dynamics), (ii) information geometry (contractive metrics and divergences), (iii) entanglement-based emergent geometry (correlation graphs and stability), and (iv) RG-style analysis (running exponents and spectral coordinates).

The guiding principle is conservative. We do not claim a derivation of the Standard Model or classical GR as equalities. Instead, we provide an instrumented framework in which: (a) every emergence claim is tied to measurable functionals; (b) every dynamical statement is tied to CPTP-consistent flow; and (c) every regime claim includes a domain statement and a failure-domain report.

Non-claims (explicit). We do not assert that emergent geometry is universally well-defined; it is treated as a stable effective description. We do not identify RG-proxy time with physical time; λ is an operational coarse-graining coordinate. We do not claim universality across arbitrary pure-state ensembles; the bridge is validated in mixed/Gibbs-like domains and must state failure domains.

2 Flow layer: CPTP modular dynamics

Let $\rho(\lambda)$ be a state. Introduce a reference state $\sigma(\lambda)$ encoding a context of access (e.g., a MaxEnt/I-projection under macro-constraints). Define the relative modular generator

$$K_{\rho|\sigma} = -\log \rho + \log \sigma.$$

The universal flow template is

$$\frac{d\rho}{d\lambda} = -i[K_{\rho|\sigma}, \rho] + \sum_{\alpha} \gamma_\alpha(\lambda)\, D_{L_\alpha}[\rho], \qquad D_L[\rho] = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\},$$

with $\gamma_\alpha(\lambda) \geq 0$. This is CPTP-consistent in the Markovian setting and provides a common backbone for dissipative evolution, information-geometric contractivity, RG-style tracking along λ, and geometry/backreaction diagnostics.

A canonical operational choice of σ is MaxEnt under accessible macro-observables Q_a:

$$\sigma_F(\rho) = \arg\max_{\tau}\{S(\tau)\,|\,\mathrm{Tr}(\tau Q_a) = \mathrm{Tr}(\rho Q_a)\} = \frac{e^{-\sum_a \beta_a(\rho) Q_a}}{Z}.$$

3 Measurement layer: unified dictionary

3.1 State, modular generators, divergences

$\rho \geq 0$, $\mathrm{Tr}\rho = 1$. Modular generator $K = -\log\rho$ on the support of ρ. Relative entropy

$$D(\rho\|\sigma) = \mathrm{Tr}\big(\rho(\log\rho - \log\sigma)\big)$$

is the canonical distinguishability potential.

3.2 Monotone information geometry (Petz class)

A Petz monotone Riemannian metric $g_\rho(\cdot,\cdot)$ provides a contractive geometry on state space under CPTP maps. In practice, BKM-type representatives are used when a concrete expression is needed; the bridge claim is metric-class based.

3.3 Spectral coordinates (RG-style observables)

Quantile spectral coordinates

$$k(q) = -\log\lambda_q(\rho_{\mathrm{vis}})$$

are evaluated on a fixed grid $q \in \{0.50, 0.60, 0.70, 0.80, 0.90, 0.95, 0.98\}$, along with profile slopes, curvatures, and curvature norm $k_{\mathrm{curv},L2}$.

3.4 Modular commutator diagnostics

For observable O,

$$\widehat{L}(\rho; O) = \frac{\|[K, O]\|_F}{\|K\|_F \|O\|_F}, \qquad K = -\log\rho,$$

and a running exponent ν is extracted via pre-registered sliding log–log fits with fit quality R^2.

3.5 Locality and phase labeling via partition functionals

A key operational locality criterion is block total correlation:

$$\mathcal{T}_{P(\rho)=D\left(\rho\middle\|\bigotimes_{X\in P\,\rho_X}\right)},$$

with partition stability (low switching / no near-degenerate competitors) as the regime boundary between stable-local and critical windows.

3.6 Emergent geometry from MI graphs

Pairwise mutual information $I_\rho(i:j)$ defines graph weights $w_{ij} = (I_{ij} + \varepsilon)^{-\nu_g}$, shortest-path distances $d_\rho(i,j)$, and geometry summaries $d_{\mathrm{nn}}, d_{\mathrm{med}}$. A minimal stability proxy is

$$S_{\mathrm{geo}}(p,\kappa) = \mathrm{std}_{\mathrm{seeds}}(d_{\mathrm{nn}}(p,\kappa;\mathrm{seed}))\,.$$

4 Bridge theorems (core claims)

Theorem 1 (CPTP contractivity of distinguishability)**.** *Let $\mathcal{E}$ be a CPTP map. Then for any states ρ, σ,*

$$D(\mathcal{E}(\rho)\,\|\,\mathcal{E}(\sigma)) \le D(\rho\,\|\,\sigma).$$

Moreover, any Petz monotone Riemannian metric g_ρ is contractive under $\mathcal{E}$.

Proof sketch. Monotonicity of quantum relative entropy under CPTP maps is standard (Uhlmann/Lindblad). Contractivity of Petz monotone metrics follows from their characterization via operator monotone functions and CPTP monotonicity.

Theorem 2 (Stable locality implies stable MI-graph geometry)**.** *Fix $\varepsilon > 0$, $\nu_g > 0$. If ρ lies in a stable-local regime where (i) $P^{(\rho)}$ is stable under small perturbations and (ii) $\mathcal{T}_{P(\rho)\le\tau}$ for small τ, then MI-graph geometry summaries (e.g., d_{nn}, d_{med}) are stable under small protocol and state perturbations and admit a reproducible stability score S_{geo} that remains small. Conversely, in critical windows (partition instability/flat landscapes) geometry becomes non-unique and protocol-sensitive, elevating S_{geo}.*

Proof sketch. Mutual information is continuous in ρ; stable locality suppresses unstable long-range reorganizations. Since weights are smooth in $I + \varepsilon$, shortest-path distances vary smoothly with I in stable regimes. Near partition degeneracy, small perturbations reshuffle correlations, producing protocol-sensitive distances.

Proposition 1 (Empirical marker principle)**.** *Within mixed/Gibbs-like domains, spectral profile reshaping (e.g., elevated $k_{\mathrm{curv},L2}$) and commutator-based ν can act as early markers of partition instability and increased S_{geo}. This is a falsifiable empirical claim validated by out-of-sample tests and failure-domain reporting.*

5 Protocol: end-to-end reproducible pipeline

5.1 Inputs

A parameter grid (e.g., (p,κ)), seed list, state construction $\rho(p,\kappa;\mathrm{seed})$, quantile grid for $k(q)$, observable set for $\widehat{L}$ (near/far), MI-graph hyperparameters (ε, ν_g), and a labeling protocol (full partition optimizer or explicitly marked proxy).

5.2 Outputs (artifacts)

A reproducibility pack containing: raw tables per (p,κ,seed); aggregated tables with bootstrap CIs; labels $P(L/C/N)$ (marked proxy if needed); figures (maps, overlays, predicted-vs-observed, ablations); and a manifest (domain statement, parameters, seed list, validation checklist).

5.3 Acceptance criteria

Geometry stability prediction and regime classification must be reported under seed-block CV and ablations (curvature vs single-quantile baselines). Failure domains must be explicitly reported via stress ensembles (e.g., extreme random-pure) rather than hidden.

6 Empirical anchors

We fix a reference design: dim $= 64$ with $(N_{\text{vis}}, N_{\text{hid}}) = (5, 1)$; $p \in [0.01, 0.20]$ with $\Delta p = 0.01$; $\kappa \in [0, 0.20]$ with $\Delta\kappa = 0.02$; seeds $\{501, 503, 509, 521, 541, 557\}$; sliding exponent window $w = 0.05$. The measurement layer computes $k(q)$, ν, MI-graph geometry summaries, and stability proxies, producing full-grid reproducibility packs (raw/agg/manifest/figures).

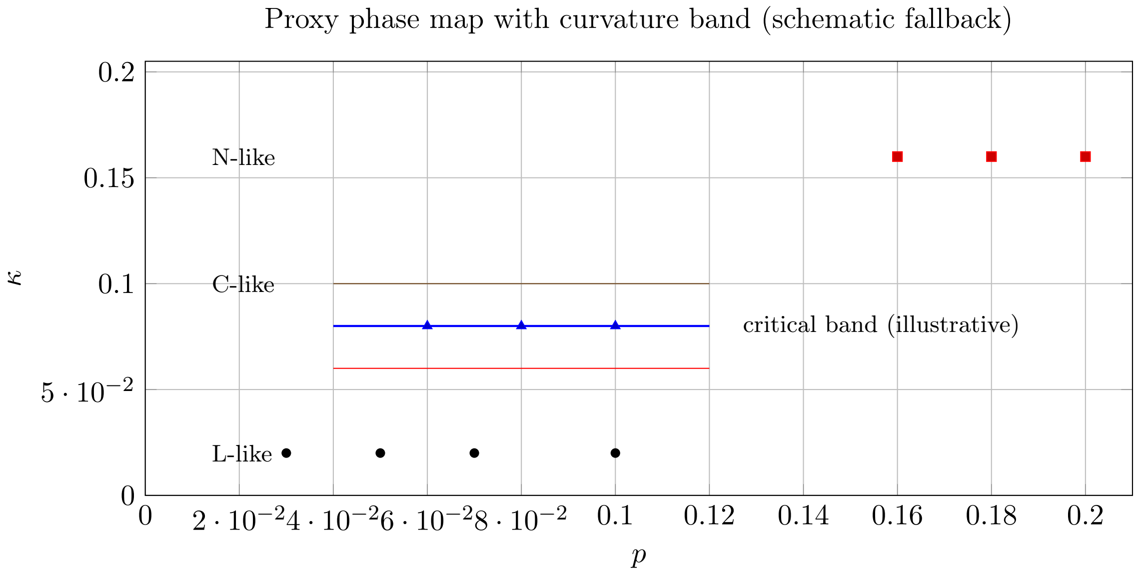

Figure 1: Phase-style map over (p, κ): proxy critical band / curvature contours (PNG if present, otherwise schematic fallback).

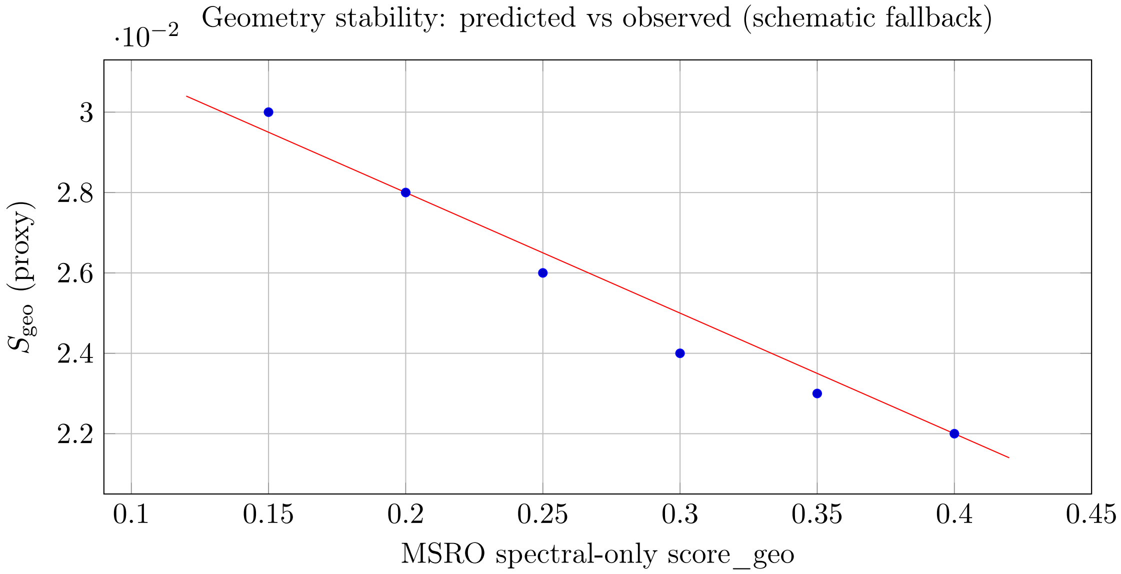

Figure 2: Geometry stability: spectral-only predictor vs observed S_{geo} (PNG if present, otherwise schematic fallback).

7 Limitations and failure domains

The bridge is domain-aware. Known failure drivers include extreme random-pure ensembles where spectral closure and portability degrade; preliminary proxy regime labels (to be replaced by full partition optimization); MI-geometry protocol sensitivity near critical windows; finite-grid and windowing effects in running-exponent extraction; and computational scaling beyond modest visible sizes. Each claim must include (i) domain statement, (ii) reproducibility statement, (iii) failure-domain statement, and (iv) mitigation path.

8 Conclusion

Scientific value

This paper formulates a bridge-framework: a single operational language connecting open quantum systems, information geometry, entanglement-based emergent geometry, and RG-style diagnostics via a CPTP modular flow layer and a unified measurement layer of computable functionals.

Degree of development (depth)

We provide explicit scope and non-claims, two bridge theorem blocks with proof sketches, a fully specified reproducibility protocol, a reference experimental anchor, and first-class limitations and failure-domain reporting.

Applied value and future directions

The bridge provides a reusable instrument panel for regime mapping and geometry-stability screening under realistic open-system dynamics. Next steps are: replace proxy labels by full partition-based $P(L/C/N)$, complete closure-quality modeling Q_{close}, extend geometry stability to protocol sensitivity (ε, ν_g), and scale via a two-stage pipeline (spectral screening then targeted full labeling/geometry).

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