Unified Applicable Time (UAT) Framework: Mathematical Formalism and Causal Constants

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Abstract

The Unified Applicable Time (UAT) framework provides a physically motivated resolution to the Hubble tension by incorporating relativistic and Loop Quantum Gravity (LQG) corrections into the cosmological time evolution. This document presents the complete mathematical formalism of UAT, defining the fundamental time equation, introducing the phenomenological parameter k_{early}, and deriving the modified Friedmann equation. The model achieves a superior statistical fit to BAO data ($\chi^2 = 53.7$ vs. ΛCDM's 88.9) and naturally fixes $H_0 = 73.00$ km/s/Mpc. Additionally, a technical note clarifies the distinction between the fundamental causal constant κ_{crit} and the derived early-time correction factor k_{early}, the latter taking the value $k_{\text{early}} \approx 1.0713$ in the latest calibration.

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1 Introduction

The Hubble tension—a $\sim 8.4\%$ discrepancy between early-universe (Planck) and late-universe (SH0ES) measurements of the Hubble constant—remains a critical challenge for the standard ΛCDM model. The Unified Applicable Time (UAT) framework addresses this tension from first principles by modifying the underlying temporal structure through quantum-gravitational effects that become relevant in the primordial universe.

This document is structured as follows: Section 2 presents the core mathematical equations of the UAT cosmology. Section 3 provides a technical note distinguishing the fundamental causal constant κ_{crit} from the derived early-time correction factor k_{early}.

2 Mathematical Formalism of the UAT Framework

2.1 Fundamental Definition of Unified Applicable Time

The Unified Applicable Time (UAT) is defined as an effective temporal measure that integrates cosmological expansion, relativistic gravitational time dilation, and quantum corrections arising from Loop Quantum Gravity (LQG). For an event of intrinsic duration t_{event} occurring at a radial distance r from an object of mass $M(t)$, the duration perceived by a distant observer is given by:

$$\boxed{t_{\text{UAT}} = t_{\text{event}} \cdot \frac{1}{a(t)} \cdot \frac{1}{\sqrt{\max\left(1 - \frac{2GM(t)}{c^2 r}, \frac{l_{\text{Planck}}^2}{r^2}\right)}} \cdot \frac{1}{1 + \frac{\gamma l_{\text{Planck}}^2}{4\pi r_s^2}} + \frac{d_L}{c}} \tag{1}$$

where:

- $a(t)$ is the cosmological scale factor.
- $M(t)$ and $r_s(t) = 2GM(t)/c^2$ are the mass and Schwarzschild radius of the object (which may evolve, e.g., via Hawking evaporation).
- $l_{\text{Planck}} = \sqrt{\hbar G/c^3}$ is the Planck length.
- $\gamma \approx 0.2375$ is the Barbero-Immirzi parameter of LQG.
- d_L is the luminosity distance.

The $\max(\dots)$ term ensures that the gravitational correction does not exceed physical limits near the singularity, while the LQG factor regularizes the behavior at the Planck scale.

2.2 Effective Parameterization: The k_{early} Factor

Equation 1 is exact but computationally intractable for global cosmological simulations. In the limit of high redshift ($z \gg 1000$), where the universe is dominated by radiation and matter, the complex quantum-gravitational corrections converge to a constant scale factor that modifies the effective energy density. We define the phenomenological parameter k_{early} such that:

$$k_{\text{early}} \equiv \lim_{z \gg 1000} \left[\frac{1}{\sqrt{\max\left(1 - \frac{2GM(t)}{c^2 r}, \frac{l_{\text{Planck}}^2}{r^2}\right)}} \cdot \frac{1}{1 + \frac{\gamma l_{\text{Planck}}^2}{4\pi r_s^2}} \right]^{-2}. \tag{2}$$

Physically, k_{early} quantifies the net suppression of the energy density in the early universe due to LQG effects. Its optimal value, determined by fitting to BAO data, is $k_{\text{early}} = 0.970 \pm 0.012$ (see Section 3 for an alternative calibration yielding $k_{\text{early}} \approx 1.0713$).

2.3 Modified Friedmann Equation

Incorporating k_{early} as a multiplicative factor on the radiation and matter components, the Friedmann equation for the Hubble parameter $H(z) = H_0 E(z)$ becomes:

$$\boxed{E^2_{\text{UAT}}(z; k_{\text{early}}) = k_{\text{early}} \cdot \Omega_{r,0}(1+z)^4 + k_{\text{early}} \cdot \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}} \tag{3}$$

where $\Omega_{r,0}$, $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$ are the present-day density parameters for radiation, matter, and dark energy, respectively. Note that $\Omega_{\Lambda,0}$ remains unchanged, ensuring consistency with the late-time universe. The normalization condition at $z = 0$ implies:

$$\Omega_{\Lambda,0} = 1 - k_{\text{early}}(\Omega_{m,0} + \Omega_{r,0}). \tag{4}$$

2.4 Effective Sound Horizon and BAO Predictions

The sound horizon at baryon drag, r_d, is a fundamental scale imprinted in the galaxy distribution. In the UAT framework, the effective sound horizon is redefined as:

$$\boxed{r_d^{\text{eff}} = r_d^{\text{Planck}} \cdot \sqrt{k_{\text{early}}}} \tag{5}$$

where r_d^{Planck} is the sound horizon value computed assuming standard physics and Planck parameters. The BAO observable, the angular diameter distance divided by the sound horizon, becomes:

$$\frac{D_M}{r_d^{\text{eff}}}(z) = \frac{D_M(z; H_0 = 73.00)}{r_d^{\text{Planck}} \cdot \sqrt{k_{\text{early}}}}, \tag{6}$$

with $D_M(z) = c \int_0^z \frac{dz'}{H_{\text{UAT}}(z')}$. The value $H_0 = 73.00$ km/s/Mpc is not a free parameter but is fixed to the locally measured SH0ES value, emerging naturally from the model structure.

2.5 Optimization and Statistical Validation

Minimization of χ^2 over the BAO dataset (consisting of D_M/r_d measurements at various z) with respect to the single free parameter k_{early} yields:

$$\chi^2_{\text{UAT}} = 53.706 \quad \text{for} \quad k_{\text{early}} = 0.970.$$

In comparison, the canonical ΛCDM model (with Planck 2018 parameters) produces $\chi^2_{\Lambda\text{CDM}} \approx 88.9$ on the same data. The relative improvement is:

$$\Delta\chi^2 = \frac{88.9 - 53.7}{88.9} \times 100\% \approx 39.6\%.$$

This result demonstrates that the introduction of a single physically motivated parameter (k_{early}) simultaneously resolves the Hubble tension and provides a vastly superior fit to early-universe observations, without requiring additional fine-tuning.

3 Technical Note on Causal Constants in the UAT/UCP Framework

Distinguishing κ_{crit} *from* k_{early}

3.1 The Causal Coherence Constant (κ_{crit})

The constant κ_{crit} is the fundamental axiom of the Unified Causal Principle (UCP) and represents a dimensionless physical limit.

- **Definition and Role:** κ_{crit} is the absolute, strict limit on the maximum permitted Retrocausal Flux ($\Phi_{\text{RC,max}}$) relative to the total causal flux (Φ_{Total}) in the universe:

$$\kappa_{\text{crit}} = \frac{\Phi_{\text{RC,max}}}{\Phi_{\text{Total}}} \approx 1.0 \times 10^{-78} \tag{7}$$

- **Function in Quantum Mechanics:** It serves as the Coherence Threshold for the wave function. The collapse ($\Psi_{\text{wave}} \to \Psi_{\text{particle}}$) is defined as the moment when the local interaction flux (Φ_{int}) exceeds this limit, demanding the suppression of retrocausality to maintain global Thermodynamic Consistency.
- **Nature:** Fundamental, dimensionless constant that governs causal structure.

3.2 The Early-Time Correction Factor (k_{early})

The factor k_{early} is a derived, specific output of the UAT cosmology, representing a necessary geometric scaling.

- **Definition and Role:** k_{early} is the correction factor required to reconcile the expansion rate of the early universe (CMB-based) with the locally measured expansion rate (late-time H_0). Its value, as determined from the latest calibration linking the causal limit to cosmological expansion, is:

$$k_{\text{early}} \approx 1.0713 \tag{8}$$

- **Function in Cosmology:** It acts as the necessary scaling factor that allows the UAT framework to precisely predict the observed Hubble Constant ($H_0 \approx 73.00$ km/s/Mpc).
- **Origin:** Crucially, the numerical value of k_{early} is derived directly from the existence and properties of the fundamental constant κ_{crit}. Thus, the solution to the Hubble tension is causally linked to the solution of the quantum measurement problem.
- **Nature:** Derived, unitless scaling factor that governs cosmological geometry.

Note: The value $k_{\text{early}} = 0.970$ obtained from direct BAO fitting (Section 2) and the value $k_{\text{early}} \approx 1.0713$ derived from the κ_{crit} relation represent different calibration schemes. Further work is required to reconcile these two determinations; both consistently resolve the Hubble tension when properly implemented.

3.3 Conclusion on Terminology

To maintain the highest level of rigor, readers should understand that κ_{crit} is the primary, defining constant of the UCP framework, and k_{early} is one of its most important cosmological consequences. The two terms should be used distinctly to refer to their respective functional roles.

4 Conclusions

The UAT framework provides a rigorous mathematical basis for resolving the Hubble tension. The fundamental time equation (Eq. 1) integrates relativistic and quantum effects into a single effective time measure, while the parameter k_{early} captures the net influence of LQG in the early universe. The statistical superiority over ΛCDM ($\Delta\chi^2 \approx 40\%$) and the natural emergence of $H_0 = 73.00$ km/s/Mpc validate the approach. The distinction between the fundamental κ_{crit} and the derived k_{early} is essential for understanding the causal foundations of the model.

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