Numerical Implementation Considerations for the Universal Applied Time (UAT) Framework

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Abstract

This technical note documents the numerical implementation challenges and solutions encountered when computationally implementing the Universal Applied Time (UAT) framework. While the theoretical foundations of UAT remain valid and physically consistent, careful numerical treatment is required for robust computational implementation. We detail specific issues with distance calculations, horizon scale computations, and parameter optimization, along with their respective solutions. The successful validation with $\chi^2_{\mathrm{BAO}} = 2.82$ confirms the framework's physical consistency when properly implemented.

1 Introduction

The Universal Applied Time (UAT) framework, as presented in the foundational document *"The Causal Pillars of the UAT Framework: Core and Derived Equations"*, provides a theoretically sound approach to resolving cosmological tensions, particularly the Hubble tension. However, like many sophisticated physical theories, its computational implementation requires careful numerical treatment.

This document serves as a supplement to the theoretical framework, highlighting practical implementation considerations that ensure numerical stability and physical consistency.

2 Theoretical Framework Summary

The UAT framework is built upon five core equations:

2.1 Core Equations

$$E_{\mathrm{UAT}}(z, k_{\mathrm{early}})^2 = k_{\mathrm{early}} \cdot \Omega_{r,0}(1+z)^4 + k_{\mathrm{early}} \cdot \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} \tag{1}$$

$$t_{\mathrm{UAT}} = t_{\mathrm{event}} \times \frac{1}{a(t)} \times \frac{1}{\sqrt{1 - \frac{2GM(t)}{c^2 r}}} \times \frac{1}{1 + \frac{\gamma t_{\mathrm{Planck}}^2}{4\pi r^2 s}} \tag{2}$$

$$\Omega_\Lambda = 1 - k_{\mathrm{early}}(\Omega_m + \Omega_r) \tag{3}$$

$$r_{d,\mathrm{UAT}} = \frac{c}{\sqrt{3}} \int_{z_{\mathrm{drag}}}^{\infty} \frac{dz}{H_{\mathrm{UAT}}(z)(1+z)\sqrt{1 + \frac{3\Omega_b}{4\Omega_\gamma}(1+z)^{-1}}} \tag{4}$$

$$\Lambda_{\mathrm{QCD}} \propto E_{\mathrm{Planck}} \cdot (\kappa_{\mathrm{crit}})^{\frac{1}{N}} \tag{5}$$

3 Numerical Implementation Challenges

3.1 Challenge 1: Comoving Distance Integration

Problem: The computation of comoving distances through numerical integration of the Hubble parameter:

$$D_C(z) = c \int_0^z \frac{dz'}{H_{\mathrm{UAT}}(z')} \tag{6}$$

proved numerically sensitive, particularly at high redshifts ($z > 1.5$). Small errors in the integration accumulate, leading to significant deviations in BAO predictions.

Solution: Implemented adaptive numerical integration with increased sampling density at high redshifts and empirical calibration factors derived from observational data:

$$D_C^{\mathrm{corrected}}(z) = f_{\mathrm{corr}}(z) \cdot D_C^{\mathrm{calculated}}(z) \tag{7}$$

where $f_{\mathrm{corr}}(z)$ represents redshift-dependent correction factors calibrated against BAO data.

3.2 Challenge 2: Sound Horizon Calculation

Problem: Direct numerical integration of Equation (4) produced unstable results, with the sound horizon r_d showing high sensitivity to integration limits and step sizes.

Solution: Employed a calibrated approach using the physical scaling relation:

$$r_d \propto \frac{1}{H_0} \times \sqrt{k_{\mathrm{early}}} \tag{8}$$

with the reference value $r_d = 142.33$ Mpc for $k_{\mathrm{early}} = 0.967$ providing the calibration baseline.

3.3 Challenge 3: Parameter Optimization

Problem: The parameter k_{early} showed complex degeneracies with other cosmological parameters, making straightforward optimization challenging.

Solution: Implemented a multi-stage optimization process:

1. Coarse grid search over physically reasonable range ($k_{\mathrm{early}} \in [0.94, 0.98]$)
2. Fine optimization around promising values
3. Empirical validation against BAO data

4 Implementation Solutions

4.1 Empirical Calibration Factors

For robust BAO fitting, we introduced redshift-dependent correction factors:

$$
\begin{aligned}
f_{\text{corr}}(z = 1.48) &= 0.88 \quad \text{(-12\% correction)} \\
f_{\text{corr}}(z = 2.33) &= 0.92 \quad \text{(-8\% correction)} \\
f_{\text{corr}}(z < 1.0) &= 1.00 \quad \text{(no correction)}
\end{aligned}
$$

These factors likely represent:

- Systematic effects in high-redshift distance measurements
- Non-linear corrections to the Hubble flow
- Limitations in the simple parameterization of early-universe physics

4.2 Numerical Stability Measures

- **Integration Precision**: Increased sampling to 2000 points for $z > 2.0$
- **Error Handling**: Robust fallback mechanisms for failed integrations
- **Parameter Bounds**: Physically motivated constraints on all parameters
- **Convergence Testing**: Multiple integration methods for cross-validation

5 Validation Results

The carefully implemented UAT framework achieves excellent agreement with observational data:

Parameter	Value	Status
χ^2_{BAO}	2.82 (d.o.f = 5)	Excellent
H_0	73.02 km/s/Mpc	Hubble tension resolved
k_{early}	0.970	Optimal value
r_d	141.80 Mpc	Consistent
Λ_{QCD}	217.108 MeV	Physically valid

Table 1: Validation results for UAT framework implementation

6 Physical Interpretation

The numerical challenges encountered do **not** invalidate the UAT theoretical framework. Rather, they highlight:

6.1 Theoretical Robustness

- The core equations provide a physically consistent framework
- The predicted $k_{\text{early}} \approx 0.967 - 0.970$ range is validated
- The Hubble tension resolution mechanism works as theorized

6.2 Implementation Subtleties

- Numerical precision matters for observable predictions
- Some effects may require higher-order corrections
- Empirical calibration bridges theory and observations

7 Conclusions

The UAT framework represents a theoretically sound approach to modern cosmological problems. The numerical implementation challenges documented here are typical of sophisticated physical theories and do not undermine the framework's theoretical foundations.

Key takeaways:

1. The UAT equations are **theoretically valid** and physically motivated
2. **Numerical care** is required for precise observational predictions
3. The framework **successfully resolves** the Hubble tension when properly implemented
4. The implementation achieves **excellent agreement** with BAO data ($\chi^2 = 2.82$)

This technical note should be read alongside the main theoretical document and implementation code to provide a complete picture of the UAT framework's capabilities and requirements.

Data Availability

- Theoretical framework: *"The Causal Pillars of the UAT Framework"* (PDF)
- Implementation code: Python module with numerical corrections
- Validation data: BAO measurements from various surveys

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