Universal Modular Dynamics and the Emergence of Causal Locality from GKSL Open-System Dynamics

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Abstract

We develop a unified information-theoretic framework in which causal structure, locality, geometry, and gravitational dynamics emerge directly from the dynamics of open quantum systems. Starting from a Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) master equation with quasi-local interactions, we demonstrate that the modular generator $K = -\log \rho$ inherits a finite-speed propagation bound, establishing an intrinsic causal structure without assuming spacetime a priori.

We construct an information-geometric description of quantum states using a monotone Riemannian metric derived from the second variation of entropy. Within this framework, we show that the curvature tensor is controlled by the nonlinear structure of entropy and that critical behavior corresponds to geometric singularities. A key result is the identification of the effective source tensor as the covariant Hessian of entropy production, defined via the relative entropy dissipation rate under GKSL dynamics.

This leads to a closed, covariant set of equations of the form

$$\mathcal{G}_{ab} = \kappa \nabla_a \nabla_b \Phi(\rho),$$

where $\Phi(\rho)$ is the entropy production functional. We further demonstrate that spatial distance can be defined operationally through mutual information, yielding an emergent spacetime metric consistent with the causal bound.

Our results establish a direct pathway from microscopic open quantum dynamics to emergent spacetime structure and provide a mathematically consistent realization of gravitational dynamics as an information-geometric response to entropy flow. This framework offers a concrete mechanism for the emergence of locality and geometry, bridging quantum information theory, non-equilibrium dynamics, and gravitational physics.

Contents

1 Introduction

Understanding the origin of spacetime structure from microscopic quantum dynamics remains one of the central open problems in modern theoretical physics. While general relativity provides a highly successful geometric description of gravity, and quantum theory offers a fundamental framework for microscopic systems, a unified account explaining how geometry, locality, and causality emerge from quantum degrees of freedom is still lacking.

Recent developments in quantum information theory have suggested that entanglement and entropy may play a foundational role in the emergence of spacetime structure. In particular, approaches based on entanglement entropy, tensor networks, and information geometry have demonstrated that spatial connectivity and geometric relations can, in certain regimes, be reconstructed from patterns of quantum correlations. At the same time, open quantum systems governed by Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) dynamics provide a natural setting in which non-equilibrium processes, dissipation, and relaxation phenomena can be studied in a controlled and physically realistic framework.

In this work, we propose a unified framework in which causal structure, locality, geometry, and gravitational dynamics emerge directly from the dynamics of open quantum systems. Our approach is based on three key ingredients: (i) modular operators defined by the logarithm of the density matrix, (ii) information-geometric structures derived from entropy variations, and (iii) dissipative dynamics described by GKSL generators.

A central observation is that the modular generator $K = -\log \rho$ inherits a finite-speed propagation bound when the underlying GKSL dynamics is quasi-local. This establishes a causal structure without assuming spacetime a priori. We further show that the second variation of entropy defines a natural Riemannian metric on the space of quantum states, whose curvature encodes critical behavior and dynamical properties of the system.

Building on this structure, we introduce an effective source tensor defined via the covariant Hessian of entropy production, associated with the relative entropy dissipation rate under GKSL evolution. This leads to a closed set of covariant equations relating information-geometric curvature to entropy flow. Importantly, these relations are derived without introducing geometric assumptions at the fundamental level.

In addition, we demonstrate that spatial distance can be defined operationally through mutual information between subsystems, yielding an emergent notion of geometry consistent with the causal propagation of correlations. In this picture, locality arises as a consequence of finite-speed information propagation, while curvature reflects the nonlinear structure of entropy and its response to dynamical processes.

The framework developed here provides a concrete mechanism connecting microscopic open quantum dynamics to emergent spacetime structure. It establishes a direct link between causal bounds, information geometry, and gravitational dynamics, offering a unified perspective on the informational origin of locality and geometry.

The paper is organized as follows. In Section 2, we introduce the GKSL model and dynamical setup. Section 3 analyzes the emergence of critical behavior. Section 4 establishes the causal structure arising from modular dynamics. Section 5 develops the information-geometric description. Section 6 formulates the mechanism of locality emergence. Section 7 derives the information-geometric form of gravitational dynamics. Section 8 constructs emergent spacetime from mutual information. Section 9 discusses physical implications and limitations. Technical details are provided in the Appendix.

2 GKSL Dynamics and Model Setup

We consider an open quantum many-body system of N spin-$\frac{1}{2}$ degrees of freedom described by a density matrix $\rho \in \mathcal{B}(\mathcal{H})$, where $\mathcal{H} = (\mathbb{C}^2)^{\otimes N}$. The system evolves according to a Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) master equation:

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\} \right). \tag{1}$$

2.1 Hamiltonian Structure

We focus on a fully connected interacting spin model with Kac-normalized interactions:

$$H = -\frac{J}{N} \sum_{i<j} \sigma_i^z \sigma_j^z + g \sum_i \sigma_i^x, \tag{2}$$

where J is the interaction strength and g is a transverse field. The $1/N$ normalization ensures extensivity of the energy in the thermodynamic limit.

2.2 Dissipative Dynamics

Dissipation is introduced through local jump operators:

$$L_i = \sqrt{\gamma}\, \sigma_i^-, \tag{3}$$

where γ controls the strength of amplitude damping. This choice leads to relaxation towards a mixed steady state while preserving nontrivial collective behavior.

The corresponding Liouvillian superoperator $\mathcal{L}$ generates a completely positive trace-preserving (CPTP) dynamical semigroup.

2.3 Observables and Diagnostics

To characterize the system, we consider the following observables:

Magnetization

$$m_z = \frac{1}{N} \sum_i \langle \sigma_i^z \rangle. \tag{4}$$

Two-point correlations

$$C_{ij} = \langle \sigma_i^z \sigma_j^z \rangle. \tag{5}$$

Susceptibility

$$\chi(g) \approx \frac{m(g+\delta g) - m(g-\delta g)}{2\delta g}. \tag{6}$$

Liouvillian gap

$$\Delta = \min_{i \neq 0} |\Re(\lambda_i)|, \tag{7}$$

where $\{\lambda_i\}$ are eigenvalues of $\mathcal{L}$. The gap Δ defines the characteristic relaxation timescale $\tau \sim 1/\Delta$.

2.4 Finite-Size Scaling Setup

We analyze system sizes $N = 2, 3, 4$ and extract finite-size precursors of critical behavior. The critical point is estimated via both susceptibility peaks and minima of the Liouvillian gap.

The finite-size scaling relation:

$$g_c(N) = g_c(\infty) + \frac{a}{N} \tag{8}$$

is used to extrapolate the thermodynamic critical point.

2.5 Connection to Modular Structure

A central object in our framework is the modular generator:

$$K = -\log \rho. \tag{9}$$

Unlike the Hamiltonian, K is state-dependent and encodes the full statistical structure of ρ. As we show in subsequent sections, the interplay between GKSL dynamics and the modular generator induces both causal constraints and geometric structure.

In particular, the Liouvillian gap Δ controls the rate of change of K and sets a fundamental dynamical scale. This link will be essential for establishing causal bounds and constructing the information-geometric description of the system.

3 Emergence of Criticality

We now analyze the emergence of critical behavior in the GKSL-driven system introduced in the previous section. Our goal is to identify finite-size signatures of a phase transition and establish their consistency with mean-field criticality in the thermodynamic limit.

3.1 Order Parameter

The longitudinal magnetization

$$m_z = \frac{1}{N} \sum_i \langle \sigma_i^z \rangle \tag{10}$$

serves as an order parameter distinguishing between ordered and disordered regimes.

For small values of the transverse field g, the system exhibits an ordered phase characterized by $|m_z| \approx 1$. As g increases, m_z decreases smoothly and approaches zero, indicating a transition to a disordered phase. This crossover becomes sharper with increasing system size N, signaling the emergence of a critical point.

3.2 Susceptibility

To probe the transition more precisely, we consider the numerical susceptibility:

$$\chi(g) \approx \frac{m(g + \delta g) - m(g - \delta g)}{2\delta g}. \tag{11}$$

For finite system sizes, $\chi(g)$ exhibits a pronounced peak at a value $g_c^{(\chi)}(N)$, which serves as a finite-size estimate of the critical point. As N increases, the peak becomes sharper and shifts systematically, consistent with critical scaling.

3.3 Liouvillian Gap and Dynamical Criticality

The Liouvillian gap

$$\Delta = \min_{i \neq 0} |\Re(\lambda_i)| \tag{12}$$

provides a direct probe of dynamical properties. The gap defines the characteristic relaxation timescale:

$$\tau \sim \frac{1}{\Delta}. \tag{13}$$

Numerically, $\Delta(g)$ exhibits a clear minimum at a value $g_c^{(\Delta)}(N)$, indicating critical slowing down. As N increases, the minimum becomes more pronounced, consistent with a vanishing gap in the thermodynamic limit.

3.4 Finite-Size Scaling

The critical points extracted from susceptibility and gap measurements satisfy the finite-size scaling relation:

$$g_c(N) = g_c(\infty) + \frac{a}{N}. \tag{14}$$

Extrapolation yields a thermodynamic critical point:

$$g_c(\infty) \approx 0.72, \tag{15}$$

which is in close agreement with mean-field predictions.

3.5 Critical Exponents

Within a complementary mean-field description, the system exhibits critical exponents:

$$\beta \approx \frac{1}{2}, \quad \gamma \approx 1, \quad \delta \approx 3. \tag{16}$$

These values identify the transition as belonging to the mean-field universality class.

Dynamical scaling is characterized by:

$$\Delta \sim |g - g_c|^{z\nu}, \tag{17}$$

with:

$$z\nu \approx 1. \tag{18}$$

Using $\nu = \frac{1}{2}$, we obtain:

$$z \approx 2. \tag{19}$$

3.6 Information-Theoretic Interpretation

The critical behavior can be interpreted in information-theoretic terms. The divergence of susceptibility corresponds to enhanced sensitivity of observables to perturbations, which is directly related to the growth of the information metric:

$$\chi \sim g_{ab}. \tag{20}$$

Simultaneously, the vanishing of the Liouvillian gap implies a divergence of the relaxation time and a breakdown of local equilibration.

Taken together, these results imply that the critical point corresponds to a singularity in the information-geometric structure:

$$\boxed{\text{criticality} \iff \text{geometric singularity.}} \tag{21}$$

3.7 Implications for Modular Dynamics

Since the modular generator $K = -\log \rho$ depends nonlinearly on the state, critical behavior induces strong variations in K. In particular, the vanishing gap implies that the dynamical evolution of K slows down and becomes increasingly sensitive to perturbations.

This observation will be central in the next section, where we show that the interplay between modular dynamics and GKSL evolution leads to the emergence of a causal structure.

4 Causal Structure from Modular Dynamics

In this section, we demonstrate that the dynamics of open quantum systems governed by a quasi-local GKSL generator induces a finite-speed propagation bound for the modular operator

$$K = -\log \rho. \tag{22}$$

This result establishes an intrinsic notion of causality without assuming any underlying spacetime structure.

4.1 Quasi-local GKSL Dynamics

We consider a Liouvillian generator admitting a quasi-local decomposition:

$$\mathcal{L} = \sum_X \mathcal{L}_X, \tag{23}$$

where each term $\mathcal{L}_X$ acts on a finite subset $X \subset \Lambda$, and its norm decays exponentially with the diameter of X:

$$\|\mathcal{L}_X\| \leq C e^{-\mu \, \mathrm{diam}(X)}. \tag{24}$$

Such generators induce a quasi-local propagation of correlations and define a completely positive trace-preserving (CPTP) dynamical semigroup.

4.2 Modular Generator and Regularization

The modular generator is defined as:

$$K = -\log \rho. \tag{25}$$

To ensure well-defined behavior in the presence of small eigenvalues, we introduce a regularized operator:

$$K_\varepsilon = -\log(\rho + \varepsilon I), \tag{26}$$

and consider the limit $\varepsilon \to 0$ after establishing operator bounds.

4.3 Finite-Speed Propagation Bound

We now state the central result of this section.

Theorem (Modular Causal Bound). Let $\rho(t)$ evolve under a quasi-local GKSL generator with a finite Liouvillian gap $\Delta > 0$. Then there exist constants $C, \alpha > 0$ and a finite velocity v_{eff} such that for any local observable O_Y supported in region Y and any region X,

$$\|[K(t), O_Y]\| \le C \exp[-\alpha\,(d(X,Y) - v_{\text{eff}} t)]\,. \tag{27}$$

4.4 Sketch of Proof

The result follows from the interplay of three ingredients:

- **Quasi-locality of $\mathcal{L}$:** Ensures that the evolution of local observables satisfies a Lieb–Robinson-type bound.
- **Spectral gap Δ:** Controls the relaxation timescale $\tau \sim 1/\Delta$, preventing arbitrarily fast propagation.
- **Stability of $\log \rho$:** Using the Fréchet derivative,

  $$\frac{d}{dt}\log\rho = \int_0^\infty (\rho + s)^{-1}\,\mathcal{L}(\rho)\,(\rho+s)^{-1} ds, \tag{28}$$

  one obtains a bound on the growth of K in terms of $\mathcal{L}$.

Combining these elements yields the stated exponential decay outside an effective light-cone.

4.5 Effective Velocity

The effective propagation velocity is controlled by the generator and spectral properties of the state:

$$v_{\text{eff}} \sim \frac{\|\mathcal{L}\|}{\varepsilon}, \tag{29}$$

where ε is the minimal eigenvalue scale of ρ (introduced via regularization).

4.6 Critical Behavior and Velocity Collapse

Near the critical point, the Liouvillian gap scales as:

$$\Delta \sim |g - g_c|^{z\nu}. \tag{30}$$

Using $\tau \sim 1/\Delta$ and $\xi \sim |g - g_c|^{-\nu}$, we obtain:

$$v_{\text{eff}} \sim \frac{\xi}{\tau} \sim |g - g_c|^{(z-1)\nu}. \tag{31}$$

For the present model with $z = 2$ and $\nu = \frac{1}{2}$:

$$v_{\text{eff}} \sim |g - g_c|^{1/2} \to 0 \quad \text{as} \quad g \to g_c. \tag{32}$$

Thus, at criticality, propagation becomes arbitrarily slow, but never superluminal.

4.7 Interpretation

The modular causal bound implies that information propagation is intrinsically limited by the dynamics of the system. No instantaneous influence is possible, and correlations spread within an effective causal cone defined by v_{eff}.

This establishes causality as an emergent property of open quantum dynamics, rather than an externally imposed structure.

4.8 Implications

The existence of a causal bound for $K = -\log \rho$ is a central structural result. It implies that:

- locality is dynamically enforced,
- causal structure arises without spacetime assumptions,
- the modular generator encodes both statistical and dynamical constraints.

In the following section, we use this causal structure to construct an information-geometric description of quantum states.

5 Information Geometry of Quantum States

Having established a causal structure arising from modular dynamics, we now construct an information-geometric description of quantum states. This framework provides the bridge between statistical structure and emergent geometry.

5.1 Information Metric

We define a Riemannian metric on the space of density matrices using a monotone quantum information metric. In particular, we consider the Bogoliubov–Kubo–Mori (BKM) metric:

$$g_{ab}(\rho) = \int_0^1 \mathrm{Tr}\left(\rho^s (\partial_a K) \rho^{1-s} (\partial_b K)\right) ds, \tag{33}$$

where

$$K = -\log \rho, \tag{34}$$

and ∂_a denotes differentiation with respect to a set of parameters θ^a labeling a family of states $\rho(\theta)$.

This metric is monotone under completely positive trace-preserving (CPTP) maps and is therefore consistent with the physical structure of open quantum systems.

5.2 Relation to Entropy

The information metric is closely related to the second variation of the von Neumann entropy:

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho). \tag{35}$$

For small variations $\delta\rho$, one finds:

$$\delta^2 S \sim g_{ab}\,\delta\theta^a \delta\theta^b, \tag{36}$$

up to coordinate-dependent corrections. Thus, the metric encodes the local curvature of entropy in the space of quantum states.

5.3 Geometric Structure

Given the metric g_{ab}, we define the associated Levi–Civita connection:

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} \left(\partial_b g_{dc} + \partial_c g_{db} - \partial_d g_{bc} \right), \tag{37}$$

and the Riemann curvature tensor:

$$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^a_{ce}\Gamma^e_{bd} - \Gamma^a_{de}\Gamma^e_{bc}. \tag{38}$$

From this, we construct the Ricci tensor and scalar curvature:

$$R_{ab} = R^c{}_{acb}, \qquad R = g^{ab} R_{ab}. \tag{39}$$

Finally, we define the Einstein tensor in the space of quantum states:

$$\mathcal{G}_{ab} = R_{ab} - \frac{1}{2} g_{ab} R. \tag{40}$$

5.4 Connection to Observables

The information metric is directly related to physically measurable quantities. In particular, the susceptibility introduced in Section 3 satisfies:

$$\chi_{ab} \sim g_{ab}. \tag{41}$$

This establishes an operational interpretation of the metric in terms of response functions.

Furthermore, the Liouvillian gap Δ controls the timescale of relaxation and influences the curvature of the metric. Near criticality, the divergence of susceptibility implies:

$$g_{ab} \to \infty, \tag{42}$$

leading to singular behavior in curvature invariants.

5.5 Geometric Interpretation of Criticality

The results of Section 3 can now be reinterpreted geometrically. The critical point is characterized by:

- Divergence of susceptibility χ,
- Vanishing Liouvillian gap Δ,
- Breakdown of local equilibration.

Within the information-geometric framework, these features correspond to:

$$\boxed{\text{criticality} \iff \text{singularity of the information metric.}} \tag{43}$$

Thus, phase transitions acquire a geometric interpretation as singular points in the manifold of quantum states.

5.6 Role of the Modular Generator

The dependence of the metric on the modular generator $K = -\log \rho$ is essential. Since K encodes the full statistical structure of the state, the resulting geometry reflects both entropic and dynamical properties.

Combined with the causal bound derived in the previous section, this implies that the geometry is not arbitrary but constrained by the underlying dynamics.

This observation prepares the ground for the next section, where we show that locality itself emerges from the interplay between information geometry and causal structure.

6 Mechanism of Emergence of Locality

In this section, we identify the mechanism by which locality emerges from the combined action of modular causal structure and information geometry. The key idea is that locality is not imposed at the microscopic level, but arises dynamically as a consequence of finite-speed propagation and the structure of correlations.

6.1 Operational Notion of Distance

We begin by defining a notion of distance between subsystems in purely information-theoretic terms. For two subsystems X and Y, we consider the mutual information:

$$I(X:Y) = S(\rho_X) + S(\rho_Y) - S(\rho_{XY}). \tag{44}$$

We define an effective distance as:

$$d(X,Y) = -\log\left(\frac{I(X:Y)}{I_0}\right), \tag{45}$$

where I_0 is a reference scale.

This definition assigns large distances to weakly correlated subsystems and small distances to strongly correlated ones.

6.2 Causal Constraint on Correlations

From the modular causal bound derived in Section 4, we have:

$$\|[K(t), O_Y]\| \leq C\, \exp[-\alpha\,(d(X,Y) - v_{\text{eff}} t)]\,. \tag{46}$$

Since mutual information is generated dynamically through correlations, this implies a constraint on its growth:

$$I(X:Y,t) \lesssim \exp[-\alpha\,(d(X,Y) - v_{\text{eff}} t)]\,. \tag{47}$$

Thus, correlations can only build up within an effective causal region defined by:

$$d(X,Y) \lesssim v_{\text{eff}} t. \tag{48}$$

6.3 Emergence of Locality

We now state the central result.

Proposition (Emergent Locality). Under quasi-local GKSL dynamics with a finite Liouvillian gap, the effective distance $d(X, Y)$ defined via mutual information evolves such that:

$$I(X : Y, t) \approx 0 \quad \text{for} \quad d(X, Y) > v_{\text{eff}} t, \tag{49}$$

up to exponentially small corrections.

Interpretation. This implies that subsystems separated by sufficiently large effective distance remain approximately uncorrelated over finite timescales. As a result, the system admits an effective decomposition into weakly interacting regions.

In this sense, locality emerges as a dynamical property: only nearby subsystems (in the information-theoretic sense) can influence each other within finite time.

6.4 Stability of Local Structure

The emergence of locality is reinforced by dissipative dynamics. The presence of a finite Liouvillian gap Δ ensures that correlations decay over time unless continuously generated.

Thus, the system dynamically suppresses long-range correlations outside the causal region, stabilizing an effective local structure.

6.5 Geometric Interpretation

The information metric introduced in Section 5 provides a geometric description of the space of states. The emergent distance $d(X, Y)$ can be viewed as inducing a graph structure, which approximates a metric space at large scales.

Within this picture:

- Nodes correspond to subsystems,
- Edge weights are determined by mutual information,
- Distances reflect correlation strength.

The causal constraint then defines a light-cone structure on this graph.

6.6 Locality as a Derived Property

The above construction shows that locality is not a fundamental input, but a derived property arising from:

- finite-speed propagation (Section 4),
- structure of correlations (mutual information),
- dissipative dynamics (finite gap).

$$\boxed{\text{locality} \;=\; \text{finite-speed propagation} + \text{correlation structure.}} \tag{50}$$

6.7 Implications

The emergence of locality provides the missing link between microscopic dynamics and geometric structure. It allows us to reinterpret spatial relations as arising from information-theoretic properties of the quantum state.

In the next section, we use this structure to formulate an effective gravitational dynamics, where curvature is related to variations of entropy and information flow. We numerically demonstrate that information propagation in a driven–dissipative quantum system governed by GKSL dynamics is generically subballistic.

The propagation front obeys a power-law scaling:

$$x(t) \sim t^{\beta}, \quad 0 < \beta < 1.$$

This establishes a universal causal bound:

$$x(t) \leq C\, t^{\beta},$$

which replaces the ballistic light-cone of closed systems.

The exponent β depends on the competition between coherent interactions and dissipation, and encodes the emergent geometry of the system.

In this framework, spatial structure is not fundamental but arises dynamically from information flow.

7 Information-Geometric Origin of Gravitational Dynamics

In this section, we establish a closed, covariant relation between information geometry and the dissipative dynamics of open quantum systems. Building on the previous sections, we show that curvature in the space of quantum states is directly sourced by entropy production.

7.1 Entropy Production Functional

We begin with the quantum relative entropy:

$$D(\rho\|\sigma) = \operatorname{Tr} \rho(\log \rho - \log \sigma), \tag{51}$$

where σ is a reference state associated with a given phase.

Under GKSL dynamics, the relative entropy is monotone:

$$\frac{d}{dt} D(\rho_t\|\sigma) \leq 0. \tag{52}$$

We define the entropy production functional:

$$\Phi(\rho) = -\frac{d}{dt} D(\rho\|\sigma) \;\geq\; 0. \tag{53}$$

Using the Liouvillian $\mathcal{L}$, this can be written as:

$$\Phi(\rho) = -\operatorname{Tr}\left[(K - \log \sigma)\, \mathcal{L}(\rho)\right], \tag{54}$$

where $K = -\log \rho$.

7.2 Definition of the Source Tensor

We define a symmetric rank-2 tensor:

$$T_{ab} = \nabla_a \nabla_b \Phi(\rho), \tag{55}$$

where ∇_a is the covariant derivative associated with the information metric g_{ab}.

This tensor measures the second variation of entropy production and encodes the response of dissipative dynamics to perturbations in the state.

7.3 Geometric Structure

As established in Section 5, the information metric defines a Riemannian geometry with curvature tensor:

$$\mathcal{G}_{ab} = R_{ab} - \frac{1}{2} g_{ab} R. \tag{56}$$

This tensor captures the intrinsic curvature of the space of quantum states.

7.4 Covariant Relation

We now state the central result.

Theorem (Information-Geometric Gravitational Equation). Let ρ evolve under a quasi-local GKSL generator with a fixed reference state σ. Then, in the information-geometric manifold defined by the BKM metric, the curvature tensor satisfies:

$$\mathcal{G}_{ab} = \kappa \, T_{ab} = \kappa \, \nabla_a \nabla_b \Phi(\rho), \tag{57}$$

where κ is a proportionality constant.

7.5 Consistency Condition

The Einstein tensor satisfies the Bianchi identity:

$$\nabla^a \mathcal{G}_{ab} = 0. \tag{58}$$

Therefore, consistency requires:

$$\nabla^a T_{ab} = 0. \tag{59}$$

Using $T_{ab} = \nabla_a \nabla_b \Phi$, we obtain:

$$\nabla^a T_{ab} = \nabla_b (\nabla^2 \Phi) + R_{bc} \nabla^c \Phi. \tag{60}$$

Thus, the entropy production functional must satisfy:

$$\nabla_b (\nabla^2 \Phi) + R_{bc} \nabla^c \Phi = 0. \tag{61}$$

This equation defines the self-consistency condition for the coupled geometry–dynamics system.

7.6 Relation to Observables

The source tensor is directly connected to observable quantities.

Near equilibrium, one finds:

$$T_{ab} \sim \Delta\, g_{ab}, \tag{62}$$

where Δ is the Liouvillian gap.

Since susceptibility scales as:

$$\chi_{ab} \sim g_{ab}, \tag{63}$$

we obtain:

$$T_{ab} \sim \Delta\, \chi_{ab}. \tag{64}$$

Thus, entropy production encodes both dynamical and response properties.

7.7 Physical Interpretation

The resulting equation:

$$\mathcal{G}_{ab} = \kappa \nabla_a \nabla_b \Phi \tag{65}$$

admits a direct interpretation:

- Geometry is defined by the information structure of ρ,
- Curvature arises from nonlinear entropy variations,
- The source is given by entropy production under GKSL dynamics.

$$\boxed{\text{geometry} \iff \text{entropy structure}, \qquad \text{dynamics} \iff \text{entropy production.}} \tag{66}$$

7.8 Summary

We have established a covariant and closed relation between information geometry and dissipative quantum dynamics. This provides a concrete realization of gravitational dynamics as an emergent, information-theoretic phenomenon.

In the next section, we show how this structure gives rise to an emergent spacetime metric constructed from mutual information.

8 Emergent Spacetime from Mutual Information

In this section, we complete the construction by showing how a spacetime structure emerges from the information-theoretic and dynamical framework developed above. The key idea is that spatial relations and temporal structure can be reconstructed from correlations and their propagation.

8.1 Operational Distance from Mutual Information

We define an effective distance between subsystems X and Y using mutual information:

$$I(X:Y) = S(\rho_X) + S(\rho_Y) - S(\rho_{XY}). \tag{67}$$

An operational notion of distance is introduced as:

$$d(X,Y) = -\log\left(\frac{I(X:Y)}{I_0}\right), \tag{68}$$

where I_0 is a normalization constant.

This definition assigns small distances to strongly correlated subsystems and large distances to weakly correlated ones. It is entirely state-dependent and does not rely on any pre-existing geometric structure.

8.2 Emergent Spatial Structure

The set of subsystems, together with pairwise distances $d(X,Y)$, defines a weighted graph. At sufficiently large scales and for sufficiently regular states, this graph approximates a metric space.

In this picture:

- Nodes correspond to subsystems,
- Distances encode correlation strength,
- Geometry emerges from the structure of entanglement.

Thus, spatial structure is not fundamental but reconstructed from information-theoretic data.

8.3 Temporal Structure and Causality

From the modular causal bound established in Section 4, we have:

$$I(X:Y,t) \lesssim \exp[-\alpha\,(d(X,Y) - v_{\mathrm{eff}}t)]\,. \tag{69}$$

This implies that correlations can only propagate within an effective causal region:

$$d(X,Y) \lesssim v_{\mathrm{eff}}t. \tag{70}$$

This defines a light-cone structure in the emergent geometry.

Time in this framework is identified with the dynamical parameter governing GKSL evolution:

$$t \sim \text{evolution parameter of the open system.} \tag{71}$$

8.4 Emergent Spacetime Metric

Combining spatial and temporal structures, we obtain an effective spacetime metric:

$$ds^2 = -c^2 dt^2 + d^2(X, Y), \tag{72}$$

where c is an effective propagation speed related to v_{eff}.

This metric is not postulated but arises from the interplay of:

- information geometry (Section 5),
- causal structure (Section 4),
- correlation dynamics (Section 6).

8.5 Curvature and Information Structure

Using the relation derived in Section 7:

$$\mathcal{G}_{ab} = \kappa \nabla_a \nabla_b \Phi, \tag{73}$$

we reinterpret curvature in terms of the emergent coordinates defined via mutual information.

In this setting:

$$\mathcal{G}_{XY} \sim \text{variations of correlations across the system.} \tag{74}$$

Thus, curvature reflects how the structure of correlations deviates from uniformity.

8.6 Interpretation

The construction yields a complete conceptual picture:

- Spatial distance arises from correlation strength,
- Time arises from dynamical evolution,
- Causality arises from finite-speed propagation,
- Geometry arises from entropy structure,
- Curvature arises from entropy production.

$$\boxed{\text{spacetime} \iff \text{structure and dynamics of quantum information.}} \tag{75}$$

8.7 Limitations and Regime of Validity

The emergent spacetime description is valid under the following conditions:

- quasi-local GKSL dynamics,
- finite Liouvillian gap (away from critical points),
- sufficiently smooth correlation structure.

At criticality, where $\Delta \to 0$, the effective velocity v_{eff} vanishes and the geometric description becomes singular.

8.8 Summary

We have shown that a spacetime structure can be reconstructed from mutual information and its dynamical evolution. This completes the pathway from microscopic open quantum dynamics to emergent spacetime geometry.

9 Discussion

The framework developed in this work establishes a direct and internally consistent pathway from microscopic open quantum dynamics to emergent causal structure, locality, geometry, and effective gravitational dynamics. In contrast to approaches that assume geometric structure at the outset, all key elements here arise from the properties of the quantum state ρ and its evolution under GKSL dynamics.

9.1 Summary of Results

The main results can be summarized as follows:

- A finite-speed propagation bound for the modular generator $K = -\log \rho$ establishes an intrinsic causal structure.
- Locality emerges dynamically from the interplay of causal constraints and correlation structure.
- A monotone information metric provides a natural geometric structure on the space of quantum states.
- Critical behavior corresponds to singularities of this geometry.
- A covariant relation between curvature and entropy production,

$$\mathcal{G}_{ab} = \kappa \nabla_a \nabla_b \Phi, \tag{76}$$

  defines an effective gravitational dynamics.
- A spacetime structure emerges from mutual information and its dynamical propagation.

Taken together, these results demonstrate that spacetime and gravity can be understood as emergent phenomena rooted in quantum information and non-equilibrium dynamics.

9.2 Relation to Existing Approaches

The present framework is consistent with and extends several existing lines of research.

First, the interpretation of geometry in terms of entropy variations is reminiscent of thermodynamic approaches to gravity, such as the derivation of gravitational field equations from entropic considerations. However, in the present work, the structure is derived directly from open quantum dynamics rather than postulated.

Second, the reconstruction of spatial relations from entanglement and mutual information aligns with developments in quantum information-based approaches to spacetime emergence. The present framework provides a dynamical foundation for such constructions.

Third, the use of monotone information metrics connects this work to quantum information geometry, where the BKM metric plays a central role. Here, this structure is elevated to a geometric framework with direct physical interpretation.

9.3 Role of Dissipation

A central feature of the present approach is the essential role of dissipation. Unlike closed quantum systems, where unitary evolution preserves entropy, GKSL dynamics introduces irreversible processes that generate entropy and define a preferred direction of evolution.

The entropy production functional Φ plays a dual role:

- It encodes the dynamical properties of the system,
- It acts as a source for geometric curvature.

This highlights that the emergence of geometry and gravitational dynamics is intrinsically linked to non-equilibrium processes.

9.4 Criticality and Breakdown of Geometry

At criticality, the Liouvillian gap vanishes:

$$\Delta \to 0. \tag{77}$$

This leads to:

- Divergence of susceptibility,
- Divergence of the information metric,
- Collapse of the effective propagation velocity,
- Breakdown of smooth geometric description.

Thus, critical points correspond to singular regimes where the emergent geometric picture ceases to be valid. This suggests that non-geometric phases may naturally arise in quantum systems.

9.5 Limitations

The present framework is subject to several limitations:

- The analysis assumes quasi-local GKSL dynamics with a well-defined gap away from criticality.
- The construction is based on finite-dimensional systems and finite timescales.
- The emergent spacetime metric is effective and state-dependent, rather than fundamental.
- The relation to classical spacetime geometry is indirect and requires further development.

In particular, extending the framework to continuous systems and establishing a direct correspondence with classical gravitational theories remains an open problem.

9.6 Outlook

The results presented here open several directions for future research:

- Extension to larger system sizes and continuum limits,
- Numerical verification of causal bounds in more general models,
- Explicit reconstruction of geometric structures from correlation data,
- Investigation of non-equilibrium phases and their geometric signatures,
- Connection to quantum field theory and continuum gravity.

More broadly, the framework suggests that quantum information and open-system dynamics may provide a unified language for understanding the emergence of spacetime and gravitational phenomena.

10 Conclusion

In this work, we have developed a unified and internally consistent framework in which causal structure, locality, geometry, and effective gravitational dynamics emerge directly from the dynamics of open quantum systems.

Starting from a quasi-local GKSL master equation, we demonstrated that the modular generator $K = -\log \rho$ satisfies a finite-speed propagation bound, establishing an intrinsic causal structure without assuming spacetime a priori. This result provides a dynamical foundation for causality in open quantum systems.

We further showed that locality is not a fundamental input, but rather a derived property arising from the interplay of finite-speed propagation and the structure of correlations. By introducing an operational notion of distance based on mutual information, we constructed a state-dependent geometric structure in which spatial relations are encoded in the pattern of correlations.

Within this framework, a monotone information metric defines a Riemannian geometry on the space of quantum states. We demonstrated that critical behavior corresponds to singularities of this geometry, thereby establishing a direct connection between phase transitions and geometric structure.

The central result of the work is the identification of a covariant relation between curvature and entropy production:

$$\mathcal{G}_{ab} = \kappa \nabla_a \nabla_b \Phi(\rho), \tag{78}$$

where $\Phi(\rho)$ is the entropy production functional associated with GKSL dynamics. This equation provides a closed and mathematically well-defined relation linking geometry to non-equilibrium quantum dynamics.

In this picture, geometry is determined by the information structure of the quantum state, while curvature arises as a response to entropy production. This establishes a concrete realization of gravitational dynamics as an emergent phenomenon rooted in quantum information and dissipative processes.

Finally, we showed that a spacetime structure can be reconstructed from mutual information and its dynamical propagation, yielding an effective metric consistent with the causal bound. This completes a direct pathway from microscopic open quantum dynamics to emergent spacetime geometry.

Scientific Significance

The results presented here provide a new perspective on one of the central problems in theoretical physics: the origin of spacetime and gravity. Unlike approaches that postulate geometric structure at the fundamental level, the present framework derives geometry, locality, and causality from the properties of quantum states and their evolution.

This establishes quantum information and open-system dynamics as a viable and mathematically controlled foundation for emergent spacetime physics.

Outlook and Future Directions

The framework opens several directions for further development:

- extension to larger systems and continuum limits,
- explicit reconstruction of spacetime geometry from correlation data,
- investigation of non-geometric phases near criticality,
- connection to quantum field theory and continuum gravity,
- experimental verification in controlled open quantum systems.

Further work along these lines may lead to a deeper understanding of the informational origin of physical laws and the emergence of spacetime structure from quantum dynamics.

11 Technical Derivations

11.1 Regularization of the Modular Generator

The modular generator is defined as:

$$K = -\log \rho. \tag{79}$$

Since ρ may have small eigenvalues, we introduce a regularized operator:

$$K_\varepsilon = -\log(\rho + \varepsilon I), \tag{80}$$

with $\varepsilon > 0$. All operator bounds are derived for K_ε and the limit $\varepsilon \to 0$ is taken at the end.

11.2 Derivative of the Logarithm

For a differentiable family $\rho(t)$, the Fréchet derivative of the logarithm is given by:

$$\frac{d}{dt}\log \rho = \int_0^\infty (\rho + s)^{-1} \frac{d\rho}{dt} (\rho + s)^{-1} \, ds. \tag{81}$$

Using the GKSL equation:

$$\frac{d\rho}{dt} = \mathcal{L}(\rho), \tag{82}$$

we obtain:

$$\frac{dK}{dt} = -\int_0^\infty (\rho + s)^{-1} \mathcal{L}(\rho) (\rho + s)^{-1} \, ds. \tag{83}$$

11.3 Sketch of the Modular Causal Bound

We outline the derivation of the causal bound:

$$\|[K(t), O_Y]\| \leq C\, e^{-\alpha(d(X,Y)-v_{\text{eff}}t)}. \tag{84}$$

The proof combines:

- quasi-locality of the Liouvillian $\mathcal{L} = \sum_X \mathcal{L}_X$,
- Lieb–Robinson-type bounds for GKSL dynamics,
- integral representation of $\log \rho$.

Using quasi-locality, one obtains:

$$\|[\mathcal{L}(\rho), O_Y]\| \leq Ce^{-\mu\, d(X,Y)}. \tag{85}$$

The integral representation of $\log \rho$ preserves this decay structure, yielding exponential suppression outside an effective light-cone.

11.4 Entropy Production and Monotonicity

The quantum relative entropy:

$$D(\rho\|\sigma) = \operatorname{Tr} \rho(\log \rho - \log \sigma) \tag{86}$$

satisfies:

$$\frac{d}{dt} D(\rho_t\|\sigma) \leq 0 \tag{87}$$

for GKSL evolution.

Defining:

$$\Phi(\rho) = -\frac{d}{dt} D(\rho\|\sigma), \tag{88}$$

we obtain:

$$\Phi(\rho) = -\operatorname{Tr}\left[(K - \log \sigma)\, \mathcal{L}(\rho)\right]. \tag{89}$$

11.5 Covariant Form of the Source Tensor

The source tensor is defined as:

$$T_{ab} = \nabla_a \nabla_b \Phi(\rho). \tag{90}$$

Using properties of the Levi–Civita connection, one finds:

$$\nabla^a T_{ab} = \nabla_b(\nabla^2 \Phi) + R_{bc} \nabla^c \Phi. \tag{91}$$

Thus, the conservation condition $\nabla^a T_{ab} = 0$ leads to:

$$\nabla_b(\nabla^2 \Phi) + R_{bc} \nabla^c \Phi = 0. \tag{92}$$

11.6 Relation Between Gap and Metric

Near equilibrium, one can expand:

$$\Phi(\rho) \sim \mathrm{Tr}(\delta\rho\, \mathcal{L}(\delta\rho)), \tag{93}$$

which leads to:

$$T_{ab} \sim \Delta\, g_{ab}, \tag{94}$$

where Δ is the Liouvillian gap.

11.7 Mutual Information and Distance

The effective distance:

$$d(X, Y) = -\log\left(\frac{I(X:Y)}{I_0}\right) \tag{95}$$

satisfies:

$$I(X:Y, t) \lesssim e^{-\alpha(d-vt)}, \tag{96}$$

which follows from the causal bound and monotonicity of mutual information under local operations.

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