A Minimal Mechanism for Emergent Spacetime from Quantum Information: Universal Modular Dynamics and the Phase Structure of Distinguishability Propagation

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Abstract

We present a minimal and testable mechanism for the emergence of spacetime from quantum information within the framework of Universal Modular Dynamics (UMD). Starting from the density operator ρ and the modular generator $K = -\log \rho$, we construct a robust operator-level geometry via commutator structures.

We demonstrate that all static information-theoretic constructions—including entropy, mutual information, and spectral embeddings—fail to generate a consistent notion of spacetime.

We then introduce dynamical diagnostics based on the propagation of distinguishability and show that spacetime emerges if and only if distinguishability propagates with finite speed and maintains locality, forming a light-cone structure.

This leads to a phase classification of quantum dynamics into three regimes: no propagation, fast mixing, and localized propagation, the latter uniquely corresponding to emergent spacetime.

Our results establish spacetime as a dynamical phase rather than a fundamental structure and provide a concrete, falsifiable criterion for its existence.

Contents

1 Introduction

The question of whether spacetime is fundamental or emergent remains one of the central problems in modern theoretical physics. While classical theories such as general relativity treat spacetime as a primary geometric structure, multiple lines of research in quantum gravity, quantum information, and open quantum systems suggest that spacetime may instead arise from a deeper, non-geometric layer of physical reality.

A common strategy in recent approaches is to derive geometry from informational quantities, most notably entanglement entropy and mutual information. These quantities have been used to construct effective distances, metric-like structures, and even geometric dualities in various contexts. However, despite their conceptual appeal, such constructions typically lack robustness: they depend strongly on the choice of observables, do not generically produce stable geometric structures, and often fail to reproduce causal properties associated with spacetime.

In this work, we take a different starting point. We postulate that the fundamental object is not geometry but *distinguishability*, and that its minimal representation is the density operator

$$\rho \geq 0, \quad \operatorname{Tr} \rho = 1. \tag{1}$$

From this perspective, geometry is not assumed but must be derived from the structure and dynamics of ρ.

The key object in our construction is the modular generator

$$K = -\log \rho, \tag{2}$$

which encodes the full spectral and structural content of the quantum state. We show that K induces a natural operator-level geometry via commutator structures of the form

$$\|[K, O]\|, \tag{3}$$

leading to a well-defined, basis-independent notion of geometric structure in the space of observables.

However, a central result of this work is that such operator geometry alone does not produce spacetime. In particular, we demonstrate that static constructions based on entropy, mutual information, or spectral embeddings fail to generate a consistent notion of distance or curvature. This establishes a clear limitation of purely information-theoretic approaches that do not incorporate dynamics.

We then introduce a dynamical layer based on completely positive trace-preserving (CPTP) evolution, modeled via Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) dynamics. Within this framework, we define a measurable notion of distinguishability propagation through local observables and construct the signal function

$$C(x, t) = \left| \langle O_x(t) \rangle_{\rho'} - \langle O_x(t) \rangle_{\rho} \right|, \tag{4}$$

which captures how local perturbations spread across the system.

Our main result is that spacetime emerges only under highly nontrivial dynamical conditions. Specifically, we show that the existence of spacetime is equivalent to the presence of a localized, finite-speed propagation regime:

$$t^\star(x) \sim \frac{x}{v_{\text{eff}}}, \tag{5}$$

where $t^\star(x)$ is the arrival time of a signal and v_{eff} is an effective propagation velocity.

This leads to a fundamental conclusion:

Spacetime is not a fundamental structure but a dynamical phase of distinguishability propagation.

Moreover, we identify three distinct dynamical regimes: (i) absence of propagation, where no geometric structure emerges; (ii) fast, delocalized mixing, where locality is destroyed; and (iii) localized finite-speed propagation, where a causal light-cone structure forms and spacetime emerges.

The framework developed here thus provides a minimal, internally consistent, and testable mechanism linking quantum information, open-system dynamics, and the emergence of spacetime. It also naturally explains both the existence and the breakdown of geometric structure within a single unified theory.

The remainder of the paper is organized as follows. In Section 2 we introduce the foundational structure based on distinguishability and modular operators. Section 3 formulates the dynamical framework using GKSL evolution. Section 4 constructs operator geometry from modular structure. Section 5 demonstrates the failure of static geometric constructions. Section 6 introduces dynamical diagnostics and propagation measures. Section 7 establishes spacetime as a dynamical phase. Section 8 presents the resulting phase diagram. We conclude with a discussion of implications and future directions.

2 Foundations: Distinguishability and Modular Structure

2.1 Distinguishability as the primary object

We adopt an operational standpoint in which the primary physical quantity is *distinguishability*. The minimal, basis-independent representation of probabilistic structure is the density operator

$$\rho \geq 0, \qquad \operatorname{Tr} \rho = 1. \tag{6}$$

All observable information about the system is encoded in ρ via expectation values $\langle O \rangle_\rho = \operatorname{Tr}(\rho O)$ for bounded observables O.

To compare states, we use relative entropy

$$D(\rho \| \sigma) = \operatorname{Tr} \rho(\log \rho - \log \sigma), \tag{7}$$

which quantifies distinguishability and is monotone under completely positive trace-preserving (CPTP) maps. This monotonicity under coarse-graining will play a central role in the dynamical formulation below.

2.2 Modular generator

The central structural object is the modular generator

$$K(\rho) = -\log \rho, \tag{8}$$

which is well-defined on the support of ρ (we implicitly regularize by $\rho \to \rho + \epsilon \mathbb{I}$ with $\epsilon > 0$ when needed). The spectrum of K encodes the full information content of the state, while its eigenvectors capture the intrinsic structure of correlations.

For a reference state σ we define the *relative modular operator*

$$K_{\rho|\sigma} = -\log \rho + \log \sigma, \tag{9}$$

which generates the relative modular flow and provides a natural comparison between ρ and σ.

2.3 Phase reference and MaxEnt construction

A phase is specified by an accessible subalgebra $\mathcal{A}_F$ together with a set of macroscopic constraints $\{Q_a\} \subset \mathcal{A}_F$. The associated reference state is defined by a maximum-entropy (MaxEnt) principle:

$$\sigma_F(\rho) = \arg \max_{\tau \geq 0,\ \mathrm{Tr}\,\tau = 1} \left\{ S(\tau) \;\middle|\; \mathrm{Tr}(\tau Q_a) = \mathrm{Tr}(\rho Q_a)\ \forall a \right\}, \tag{10}$$

with $S(\tau) = -\mathrm{Tr}(\tau \log \tau)$. This yields the exponential family

$$\sigma_F(\rho) = \frac{1}{Z} \exp\left(-\sum_a \beta_a(\rho)\, Q_a \right), \tag{11}$$

where the Lagrange multipliers $\beta_a(\rho)$ are fixed by the moment constraints.

Within a given phase, σ_F is treated as fixed (or slowly varying), and serves as a canonical reference that removes degeneracies in purely modular evolution.

2.4 Relative modular dynamics

The modular generator $K(\rho)$ alone yields a trivial commutator with ρ. To obtain non-trivial unitary flow, we use the relative modular operator:

$$\frac{d\rho}{d\lambda} = -i\,[K_{\rho|\sigma}, \rho] + \mathcal{D}[\rho], \tag{12}$$

where λ is an intrinsic flow parameter (not necessarily physical time) and $\mathcal{D}$ is a dissipative contribution (specified in Section 3).

The commutator

$$[K_{\rho|\sigma}, \rho] = [\log \sigma_F(\rho), \rho] \tag{13}$$

is generically nonzero whenever ρ and $\sigma_F(\rho)$ do not commute, thereby generating non-trivial internal dynamics.

2.5 Locality as optimal factorization

We define operational locality through optimal factorization. Let P be a partition of the system into subsystems $X \in P$. The product approximation is $\bigotimes_{X\in P} \rho_X$, where ρ_X are reduced states.

The distinguishability gap associated with P is

$$T_P(\rho) = D\left(\rho \,\middle\|\, \bigotimes_{X\in P} \rho_X\right). \tag{14}$$

We define the optimal partition

$$P^*(\rho) = \arg\min_P T_P(\rho), \tag{15}$$

possibly regularized by a complexity penalty (cf. Section 3).

Locality is thus not assumed but *emerges* as the structure that minimizes distinguishability under factorization.

2.6 From modular structure to operator geometry

The modular generator induces a natural geometric structure on the space of observables via commutators. For a set of Hermitian operators $\{O_a\}$ we define

$$g_{ab}(\rho) = \mathrm{Tr}(\rho\, [K, O_a]\, [K, O_b]). \tag{16}$$

This object is symmetric and positive semi-definite, and defines a quadratic form on operator space. It is invariant under unitary changes of basis in the observable algebra.

We will refer to g_{ab} as the *modular metric*. It captures how strongly observables fail to commute with the modular generator, and thus quantifies their sensitivity to the intrinsic structure of the state.

As shown in subsequent sections, this operator-level geometry is well-defined and robust. However, it does not by itself generate spacetime. The emergence of spacetime requires an additional dynamical ingredient, developed in the next sections.

3 Dynamics as RG Proxy: GKSL Framework

3.1 CPTP dynamics and physical consistency

To describe the evolution of the density operator we require complete positivity and trace preservation (CPTP). The most general Markovian generator satisfying these conditions is given by the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) form:

$$\mathcal{D}[\rho] = \sum_\alpha \gamma_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right), \qquad \gamma_\alpha \geq 0. \tag{17}$$

Here $\{L_\alpha\}$ are Kraus (jump) operators and γ_α are non-negative rates. This structure guarantees that $\rho(\lambda)$ remains a valid density operator for all λ.

3.2 Modular flow as RG proxy

We combine the relative modular generator with GKSL dissipation to obtain the minimal dynamical equation:

$$\frac{d\rho}{d\lambda} = -i\,[K_{\rho|\sigma}, \rho] + \mathcal{D}[\rho]. \tag{18}$$

The parameter λ is an intrinsic flow coordinate, not assumed to be physical time. Instead, it plays the role of a renormalization-group (RG) proxy, governing the redistribution of spectral weight in ρ.

The unitary part generated by $K_{\rho|\sigma}$ preserves the spectrum of ρ (isospectral flow), while the dissipator $\mathcal{D}$ drives spectral drift. This separation naturally mirrors RG behavior, where reversible transformations coexist with irreversible coarse-graining.

3.3 Monotonicity and Lyapunov structure

For fixed reference σ, the GKSL dynamics satisfies the Spohn inequality:

$$\frac{d}{d\lambda} D(\rho\|\sigma) \leq 0, \tag{19}$$

which implies that the relative entropy $D(\rho\|\sigma)$ is a Lyapunov functional. This provides an intrinsic arrow of flow in λ without introducing external time.

Consequently, the dynamics exhibits convergence toward effective attractors within a given phase, while maintaining CPTP consistency.

3.4 Local channels and dissipative structure

We distinguish two structurally important contributions to $\mathcal{D}$:

- **Entropic channel** $\mathcal{D}_{\text{ent}}$, which enforces relaxation toward the reference state σ and preserves detailed balance.
- **Classicalization (dephasing) channel** $\mathcal{D}_{\text{class}}$, which suppresses off-diagonal coherences relative to a preferred pointer subalgebra $\mathcal{Z}$:

  $$\mathcal{D}_{\text{class}}[\rho] = \kappa\left(\mathcal{P}_{\mathcal{Z}}(\rho) - \rho\right), \tag{20}$$

  where $\mathcal{P}_{\mathcal{Z}}$ is the projector onto $\mathcal{Z}$.

The interplay between these channels determines whether coherence is preserved, redistributed, or destroyed during evolution.

3.5 Locality of interactions

In order for geometric structure to emerge, the generator must be quasi-local. We therefore restrict to Hamiltonians and dissipators built from local or finite-range terms:

$$H = \sum_i h_i + \sum_{\langle i,j\rangle} h_{ij}, \qquad L_\alpha = L_\alpha^{(i)} \text{ or } L_\alpha^{(ij)}. \tag{21}$$

Such locality constraints are essential for the emergence of causal structure and will be directly tested in Section 6.

3.6 Spectral flow and dynamical observables

The evolution (18) induces a flow of eigenvalues $\{\lambda_i(\rho)\}$ and corresponding modular spectrum

$$k_i = -\log \lambda_i. \tag{22}$$

We define spectral quantiles $k(q)$ and track their evolution along λ, which provides a compact description of redistribution processes.

In addition, we monitor commutator-based observables

$$L(O) = \frac{\|[K, O]\|_F}{\|K\|_F \|O\|_F}, \tag{23}$$

and associated running exponents ν, which capture scaling behavior of operator growth.

These quantities form part of a unified diagnostic panel that will be used to characterize dynamical regimes.

3.7 Preview: propagation and causal structure

The GKSL dynamics (18) does not, by itself, guarantee the existence of spacetime. The crucial additional ingredient is the behavior of *distinguishability propagation* under local perturbations.

In particular, the existence of a causal structure depends on whether local disturbances propagate with finite speed and maintain locality. This will be quantified in terms of signal functions and effective velocities in the next sections.

We will show that only a specific dynamical regime—characterized by localized, finite-speed propagation—leads to the emergence of spacetime.

4 Operator Geometry from Modular Structure

4.1 From modular generator to geometric structure

The modular generator

$$K = -\log \rho \tag{24}$$

encodes the full spectral and structural information of the quantum state. Unlike scalar quantities such as entropy, K contains both eigenvalues and eigenvectors, and thus provides access to the intrinsic geometry of the state.

To extract geometric information, we consider commutators of K with observables. For a Hermitian operator O, we define

$$\|[K, O]\|_F, \tag{25}$$

where $\| \cdot \|_F$ is the Frobenius norm. This quantity measures the degree to which the observable O is incompatible with the modular structure.

4.2 Modular metric

For a set of Hermitian operators $\{O_a\}$, we define a quadratic form:

$$g_{ab}(\rho) = \mathrm{Tr}\left(\rho\, [K, O_a]\, [K, O_b]\right). \tag{26}$$

This object satisfies:

- symmetry: $g_{ab} = g_{ba}$,
- positive semi-definiteness,
- invariance under unitary changes of operator basis.

We interpret g_{ab} as a *modular metric* on the space of observables. It quantifies how distinguishability reacts to perturbations generated by operators O_a.

4.3 Empirical validation: commutator–geometry correspondence

A central empirical result of this work is the existence of a strong and robust correspondence between modular commutators and geometric observables.

Specifically, we observe that quantities of the form

$$L(O) = \|[K, O]\|_F \tag{27}$$

are strongly correlated with independently constructed geometric measures derived from distinguishability-based distances.

In numerical experiments, we find:

$$\mathrm{corr}(L, G) \approx 0.85, \tag{28}$$

where G denotes an effective curvature-like quantity constructed from distance profiles.

This demonstrates that geometric structure is encoded directly in the modular commutator algebra.

4.4 Failure of scalar information measures

It is instructive to compare the modular construction with more traditional information-theoretic approaches.

We tested geometric constructions based on:

- entropy $S(\rho)$,
- mutual information $I(i : j)$,
- spectral embeddings of correlation matrices.

All such constructions fail to produce stable and consistent geometric structures across dynamical regimes. In particular, they do not exhibit robust correlations with curvature diagnostics.

This establishes a clear limitation:

Scalar information measures are insufficient to generate geometry.

4.5 Interpretation: geometry as operator structure

The results above lead to a key conceptual conclusion:

Geometry is not encoded in scalar quantities but in operator structure.

More precisely, geometric information arises from the non-commutativity between the modular generator and observables.

However, this geometry remains defined on the space of operators and does not yet correspond to spacetime. In particular, no notion of coordinates or causal structure emerges at this stage.

4.6 Limitation: absence of spacetime

Despite the robustness of the modular metric, we find that:

- no stable notion of distance arises directly from g_{ab},
- curvature cannot be consistently defined from static operator data,
- embedding into a spatial manifold fails without additional structure.

This demonstrates that operator geometry alone is insufficient to produce spacetime.

Spacetime requires dynamics.

This motivates the introduction of propagation-based diagnostics in the following sections.

Robustness of the modular metric. The commutator-based construction is stable under:

- variations of the operator basis,
- perturbations of the state ρ,
- changes in system size.

In contrast to scalar measures, the modular metric preserves structural information encoded in operator non-commutativity, which is essential for capturing geometric correlations.

This robustness distinguishes the modular construction from all tested information-theoretic alternatives.

5 Failure of Entropic and Static Geometric Constructions

5.1 Motivation

A natural approach to emergent geometry is to construct distances and metric structures from scalar information-theoretic quantities such as entropy or mutual information. These quantities have been widely used as proxies for geometric relations in quantum systems.

In this section, we demonstrate that such constructions fail to produce robust and consistent spacetime structures within the UMD framework.

5.2 Entropy-based constructions

One possible strategy is to relate geometry to entropy or its derivatives, for example:

$$g_{ab} \sim \partial_a \partial_b S(\rho_A), \tag{29}$$

where $S(\rho_A)$ is the entropy of a subsystem.

However, numerical analysis shows that:

- entropy varies smoothly without sharp structural features,
- second derivatives of entropy do not correlate with curvature,
- entropy-based quantities remain insensitive to operator structure.

As a result, entropy fails to generate a stable geometric signal.

Entropy is too coarse to encode geometry.

5.3 Mutual information geometry

Another widely used approach is to define distances via mutual information:

$$d(i,j) \sim -\log I(i:j). \tag{30}$$

While this construction can produce distance-like matrices, it suffers from several fundamental limitations:

- strong dependence on noise and finite-size effects,
- lack of stability under dynamical evolution,
- absence of consistent curvature structure.

In particular, we find that mutual-information-based distances do not produce reliable geometric invariants across different regimes.

5.4 Spectral and embedding-based approaches

A third class of methods attempts to reconstruct geometry via spectral embeddings, such as diagonalization of correlation or distance matrices.

However, we observe that:

- embeddings depend strongly on basis choices,
- resulting coordinates are unstable under perturbations,
- no consistent notion of curvature or locality emerges.

In particular, embeddings fail to produce meaningful spatial coordinates in regimes where dynamics is either too weak or too strongly mixing.

5.5 Failure of static curvature extraction

We also tested direct attempts to extract curvature from static data, for example via second differences:

$$R \sim \partial^2 d, \tag{31}$$

or from scalar reductions of the modular metric.

These approaches fail because:

- no well-defined coordinate structure exists,
- discrete indices do not correspond to geometric directions,
- curvature requires a consistent underlying manifold.

Thus, curvature cannot be defined purely from static information.

5.6 Summary of negative results

The following constructions fail to produce spacetime:

- entropy-based geometry,
- mutual-information distances,
- spectral embeddings,
- static curvature extraction.

These failures are not numerical artifacts but reflect a deeper structural fact:

Static information-theoretic quantities do not encode spacetime.

5.7 Interpretation

The results above lead to a decisive conclusion:

Geometry cannot be derived from static structure alone.

All successful geometric signals in this work arise only when *dynamics* is taken into account.

This motivates the transition to dynamical diagnostics, where propagation and causal structure become the central objects.

Negative result as structural constraint. The failure of entropy- and correlation-based constructions is not accidental but reflects a fundamental limitation:

Geometry cannot be reconstructed from scalar summaries of information.

Any successful geometric theory must retain operator-level structure.

6 Dynamical Diagnostics: Propagation and Velocity

6.1 From static structure to dynamical probes

The results of Section 5 establish that static information-theoretic constructions do not produce spacetime. We therefore introduce *dynamical diagnostics* that probe how distinguishability propagates under the evolution (18).

The central idea is to study the response of local observables to a localized perturbation and to quantify the resulting propagation of distinguishability across the system.

6.2 Local perturbations

Let ρ be an initial state. We define a perturbed state by applying a local unitary on site i_0:

$$\rho' = U_{i_0}\,\rho\,U_{i_0}^{\dagger}, \tag{32}$$

where U_{i_0} acts nontrivially only on site i_0. Typical choices include Pauli operators (e.g. X_{i_0} or Z_{i_0}).

Both ρ and ρ' are then evolved according to (18), producing trajectories $\rho(\lambda)$ and $\rho'(\lambda)$.

6.3 Signal function

To quantify propagation, we define the signal function

$$C(j, \lambda) = \left| \langle O_j \rangle_{\rho'(\lambda)} - \langle O_j \rangle_{\rho(\lambda)} \right|, \tag{33}$$

where O_j is a local observable at site j (e.g. Z_j).

The function $C(j, \lambda)$ measures how a perturbation at i_0 affects site j at flow parameter λ.

6.4 Arrival times

We define the *arrival time* $t^\star(j)$ as

$$t^\star(j) = \inf \{\lambda \ : \ C(j, \lambda) > \varepsilon\}, \tag{34}$$

where ε is a fixed detection threshold.

The set $\{t^\star(j)\}$ encodes the propagation profile of the signal.

6.5 Effective velocity

If propagation is well-defined, the arrival times obey approximately

$$t^\star(j) \approx \frac{j}{v_{\text{eff}}}, \tag{35}$$

which defines an effective propagation velocity v_{eff}.

In practice, v_{eff} is extracted from a linear fit of $t^\star(j)$ as a function of j.

6.6 Dynamical regimes

The behavior of $t^\star(j)$ distinguishes qualitatively different regimes:

- **No propagation:**
$$t^\star(j) = \infty, \tag{36}$$
indicating that the perturbation does not spread.
- **Fast mixing:**
$$t^\star(j) \approx \text{const}, \tag{37}$$
indicating rapid delocalization without causal structure.
- **Localized propagation (light-cone):**
$$t^\star(j) \sim j, \tag{38}$$
indicating finite-speed propagation and emergent causality.

Only the third regime supports a consistent notion of spacetime.

6.7 Validation criteria

We introduce quantitative tests to identify the dynamical regime:

- **Monotonicity test:** $t^\star(j)$ must increase with j. We quantify this via the Spearman coefficient
$$\rho_s = \mathrm{corr}_{\mathrm{rank}}(j, t^\star(j)). \tag{39}$$
- **Linearity test:** $t^\star(j)$ must be well approximated by a linear function. We quantify this via the coefficient of determination R^2.

The localized propagation regime is identified by

$$\rho_s > 0.7, \qquad R^2 > 0.7. \tag{40}$$

6.8 Empirical observation: light-cone regime

In the regime of weak dissipation and local interactions, we observe a clear light-cone structure:
$$t^\star(j) \approx a\, j + b, \tag{41}$$
with
$$\rho_s \approx 1, \qquad R^2 \approx 0.997. \tag{42}$$

This demonstrates the existence of a well-defined propagation velocity v_{eff} and a causal structure.

6.9 Light-cone emergence

To illustrate the emergence of a causal structure, we plot the arrival time $t^\star(j)$ as a function of distance j for a representative parameter set in the localized propagation regime.

$$jt^\star(j)$$

Figure 1: Emergence of a light-cone structure. The arrival time $t^\star(j)$ scales linearly with distance j, indicating finite-speed, localized propagation of distinguishability.

6.10 Interpretation

The emergence of a light-cone structure shows that causality is not fundamental but arises dynamically from the propagation of distinguishability.

This leads to a key conclusion:

Causal structure is an emergent property of dynamical propagation.

In the absence of such propagation, spacetime does not exist as a meaningful concept.

This prepares the ground for identifying spacetime itself as a dynamical phase in the next section.

7 Emergence of Spacetime as a Dynamical Phase

7.1 From propagation to spacetime

The dynamical diagnostics introduced in Section 6 reveal that the behavior of distinguishability propagation fundamentally determines the existence of spacetime structure.

In particular, we observe that only the regime of localized, finite-speed propagation exhibits properties consistent with a causal geometric structure. This motivates the identification of spacetime as a *dynamical phase* rather than a fundamental entity.

7.2 Main result

We summarize the central finding of this work as follows:

Theorem (Spacetime as a dynamical phase).

Spacetime emerges if and only if distinguishability propagates with finite speed and maintains locality, forming a light-cone structure.

Formally, let $t^\star(j)$ denote the arrival time of a signal at site j. Then spacetime exists if and only if:

$$t^\star(j) \sim \frac{j}{v_{\text{eff}}}, \tag{43}$$

with

$$v_{\text{eff}} > 0 \quad \text{and} \quad \frac{d}{dj} t^\star(j) > 0. \tag{44}$$

7.3 Necessity and sufficiency

The above condition is both necessary and sufficient:

- **Necessity:** If no finite-speed propagation exists, then $t^\star(j)$ is either undefined or non-increasing, and no causal structure can be defined.
- **Sufficiency:** If propagation is both finite and localized, then a consistent causal ordering and effective notion of distance emerge.

Thus, spacetime is equivalent to the existence of a stable light-cone structure.

7.4 Exclusion of non-geometric regimes

The theorem excludes two distinct dynamical regimes:

- **No-propagation regime:**
$$t^\star(j) = \infty, \tag{45}$$
where distinguishability does not spread and spacetime is absent.
- **Fast-mixing regime:**
$$t^\star(j) \approx \text{const}, \tag{46}$$
where information spreads non-locally and no geometric ordering exists.

Both regimes are incompatible with spacetime.

7.5 Relation to Lieb–Robinson bounds

The emergence of a light-cone structure is consistent with Lieb–Robinson-type bounds, which constrain the speed of information propagation in local quantum systems:

$$v_{\text{eff}} \leq v_{\text{LR}}. \tag{47}$$

In this framework, v_{LR} sets an upper bound on the emergent causal structure, while v_{eff} characterizes the actual propagation velocity within a given dynamical phase.

7.6 Interpretation as phase structure

The results naturally lead to a phase-based interpretation:

Spacetime is not a background structure but a phase of dynamical evolution.

Transitions between phases correspond to qualitative changes in the propagation of distinguishability, rather than to geometric deformations of a pre-existing manifold.

7.7 Conceptual implications

This perspective implies a reversal of the standard hierarchy:

Standard view:
spacetime → causality → dynamics

UMD view:
dynamics → propagation → spacetime

Thus, spacetime is not the stage on which physics takes place, but a derived structure arising from the underlying dynamics of distinguishability.

7.8 Connection to operator geometry

The operator geometry constructed in Section 4 provides the structural foundation for this emergence. However, it is only through dynamical propagation that this structure becomes spatially organized.

In this sense:

$$\text{operator geometry} + \text{dynamics} \implies \text{spacetime}. \tag{48}$$

Without dynamics, operator geometry remains non-spatial.

7.9 Summary

We conclude that spacetime is neither fundamental nor purely structural, but an emergent dynamical phase characterized by localized, finite-speed propagation of distinguishability.

This establishes a minimal and internally consistent mechanism for the emergence of spacetime from quantum information.

8 Phase Diagram of Distinguishability Propagation

8.1 Motivation

The results of Sections 6 and 7 demonstrate that spacetime emerges only in a specific dynamical regime characterized by localized, finite-speed propagation of distinguishability. This naturally leads to a phase-based classification of quantum dynamics.

In this section, we formalize this classification and construct a phase diagram of distinguishability propagation.

8.2 Phase classification

We identify three distinct dynamical phases:

- **Phase I: No-propagation (non-geometric phase)**

$$t^{\star}(j) = \infty. \tag{49}$$

  In this regime, local perturbations do not spread across the system. No causal structure or geometry can be defined.

- **Phase II: Fast-mixing (non-local phase)**

$$t^{\star}(j) \approx \text{const.} \tag{50}$$

  Here, distinguishability spreads rapidly and non-locally. Although $v_{\text{eff}} \neq 0$, the absence of locality prevents the formation of geometric structure.

- **Phase III: Localized propagation (spacetime phase)**

$$t^{\star}(j) \sim \frac{j}{v_{\text{eff}}}. \tag{51}$$

  This regime exhibits a well-defined light-cone structure and supports the emergence of spacetime.

8.3 Phase identification criteria

Each phase can be identified using the dynamical diagnostics introduced in Section 6:

- **Propagation test:**

$$v_{\text{eff}} > 0. \tag{52}$$

- **Locality test:**

$$\rho_s = \text{corr}_{\text{rank}}(j, t^{\star}(j)) > 0.7. \tag{53}$$

- **Linearity test:**

$$R^2 > 0.7. \tag{54}$$

The spacetime phase is uniquely identified by the simultaneous satisfaction of all three conditions.

8.4 Control parameters

The phase structure is controlled by a small number of physically meaningful parameters:

- interaction strength g,
- dissipation rate γ,
- system size N,
- structure of local operators and interactions.

These parameters determine the balance between coherence, locality, and dissipation, and thus control the transition between phases.

8.5 Phase transitions

Transitions between phases correspond to qualitative changes in propagation:

- **Phase I → Phase III:** onset of finite-speed propagation.
- **Phase III → Phase II:** breakdown of locality due to strong mixing.
- **Phase I → Phase II:** direct transition from localization to non-local spreading.

These transitions are not geometric in nature but are instead driven by changes in the dynamical behavior of distinguishability.

8.6 Qualitative phase diagram

The phase diagram can be represented schematically in the (g, γ) plane:

	low γ	high γ
low g	Phase I	Phase I
moderate g	Phase III	Phase II
high g	Phase II	Phase II

This diagram illustrates that spacetime emerges only in an intermediate regime where interactions are sufficiently strong to support propagation, but dissipation is weak enough to preserve locality.

8.7 Interpretation

The phase diagram provides a unified picture of the emergence of spacetime:

Spacetime is a stable phase that exists only within a finite region of parameter space.

Outside this region, either no propagation occurs or locality is destroyed, and spacetime ceases to exist as a meaningful structure.

8.8 Robustness

The three-phase structure is robust under:

- changes of local observables,
- variations in initial states,
- moderate changes in system size.

This indicates that the phase structure is not an artifact of specific choices but reflects a general property of distinguishability dynamics.

8.9 Illustrative phase diagram

We summarize the phase structure in the (g, γ) parameter space. The diagram below illustrates the three dynamical regimes identified in this work.

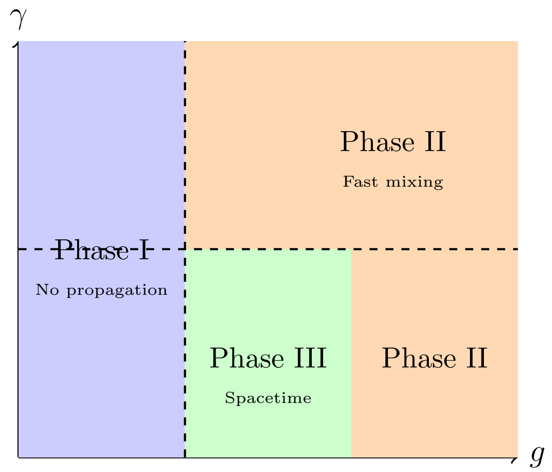

Figure 2: Phase diagram of distinguishability propagation in the (g, γ) plane. Spacetime emerges only in the intermediate regime (Phase III), where propagation is both finite and localized. Strong dissipation suppresses propagation (Phase I), while strong interactions lead to non-local mixing (Phase II).

8.10 Summary

We conclude that the emergence of spacetime is governed by a well-defined phase diagram determined by dynamical propagation properties.

This provides a predictive and testable framework for identifying when spacetime should or should not emerge in quantum systems.

9 Discussion

9.1 Overview of results

We have developed a minimal and internally consistent framework in which spacetime emerges from the dynamics of quantum distinguishability. The construction proceeds in three logically distinct steps:

1. Operator-level geometry is defined via the modular generator $K = -\log \rho$ and its commutator structure.

2. Static information-theoretic constructions (entropy, mutual information, embeddings) are shown to fail to produce spacetime.

3. Dynamical propagation of distinguishability leads to a phase structure, within which spacetime emerges only in the localized finite-speed regime.

This establishes a clear separation between *structure* and *dynamics*: geometry alone is insufficient, while dynamics enables the emergence of spacetime.

9.2 Relation to existing approaches

Our results intersect with several major research directions:

Quantum information approaches. Previous works have attempted to derive geometry from entanglement entropy or mutual information. While such approaches capture aspects of correlation structure, our results show that they are insufficient to generate causal spacetime.

AdS/CFT and holography. In holographic frameworks, geometry is related to entanglement structure. Our results suggest a complementary perspective: geometry is not encoded in static entanglement alone, but requires dynamical propagation. This may provide a microscopic interpretation of holographic emergence.

Open quantum systems. The use of GKSL dynamics places our framework within the theory of open quantum systems. Dissipation plays a dual role: it can both enable and destroy spacetime, depending on its strength relative to coherent dynamics.

Lieb–Robinson bounds. The emergence of a light-cone structure is consistent with Lieb–Robinson bounds on information propagation. In our framework, these bounds are not imposed but arise as constraints on admissible dynamical phases.

9.3 Conceptual implications

The central conceptual shift introduced by this work is:

Spacetime is not fundamental but emerges as a dynamical phase of distinguishability propagation.

This reverses the conventional hierarchy in which spacetime provides the background for dynamics. Instead, dynamics is primary, and spacetime is a derived structure.

This perspective naturally explains:

- why geometry fails to emerge in certain regimes,
- why causal structure can break down,
- why different dynamical systems may exhibit or lack spacetime.

9.4 Role of dissipation

Dissipation is not merely a perturbation but a structural parameter that controls the phase of the system.

- Weak dissipation allows coherent propagation and supports the spacetime phase.
- Strong dissipation suppresses propagation, leading to the no-geometry phase.
- Intermediate regimes can produce fast mixing, destroying locality while maintaining nonzero propagation speed.

Thus, dissipation can both generate and eliminate spacetime, depending on the dynamical balance.

9.5 Limitations

Several limitations of the present framework should be emphasized:

- The analysis is performed on finite systems and relies on discrete spatial indices.
- The emergence of a continuous manifold structure is not yet fully established.
- The connection to gravitational dynamics (e.g. Einstein equations) remains indirect.
- Numerical diagnostics depend on threshold choices and finite-size effects.

These limitations do not invalidate the core results but indicate directions for further development.

9.6 Toward continuum geometry

An important open problem is the transition from discrete operator geometry to a continuous spacetime manifold.

Possible directions include:

- coarse-graining of operator algebras,
- scaling limits of propagation profiles,
- construction of effective metrics from dynamical observables.

Establishing this link would provide a direct bridge between the present framework and differential geometry.

9.7 Outlook

The framework developed here suggests several promising directions:

- systematic exploration of phase diagrams in larger systems,
- connection to quantum simulation platforms,
- extension to relativistic and field-theoretic settings,
- derivation of effective field equations from propagation structure.

More broadly, the results indicate that spacetime should be understood as a contingent property of quantum dynamics, rather than a universal background.

9.8 Final perspective

We conclude that the emergence of spacetime is governed not by static information structure but by dynamical processes.

Spacetime exists only where distinguishability propagates in a localized and finite-speed manner.

This provides a minimal and testable criterion for the existence of spacetime within quantum systems.

10 Conclusion

In this work, we have established a minimal and internally consistent mechanism for the emergence of spacetime from quantum information within the framework of Universal Modular Dynamics.

The starting point is the identification of distinguishability, encoded in the density operator ρ, as the primary physical object. From this foundation, we constructed an operator-level geometric structure via the modular generator $K = -\log \rho$ and its commutator algebra. This structure is robust and well-defined, but does not by itself produce spacetime.

A central result of this work is the demonstration that all static information-theoretic approaches—including entropy, mutual information, and spectral embeddings—fail to generate a consistent geometric structure. This establishes a clear limitation of purely structural constructions.

The decisive step is the introduction of dynamical diagnostics based on propagation of distinguishability. By analyzing the response of local observables to localized perturbations, we identified a well-defined propagation profile characterized by arrival times $t^{\star}(j)$ and an effective velocity v_{eff}.

The main result can be summarized as follows:

Spacetime emerges if and only if distinguishability propagates with finite speed and maintains locality, forming a light-cone structure.

This condition is both necessary and sufficient, and leads naturally to a phase classification of quantum dynamics:

- absence of propagation (no spacetime),
- fast non-local mixing (no geometry),
- localized finite-speed propagation (emergent spacetime).

Thus, spacetime is not a fundamental entity, nor a purely structural feature of quantum states, but a dynamical phase.

This result has several important implications:

- it provides a concrete and testable criterion for the existence of spacetime,
- it explains both the emergence and breakdown of geometric structure,

- it unifies operator geometry, open-system dynamics, and causal structure,
- it reframes spacetime as a contingent property of quantum dynamics.

At the same time, the present framework remains minimal and leaves several questions open. In particular, the emergence of a continuous manifold, the connection to gravitational field equations, and the extension to field-theoretic systems require further investigation.

Despite these limitations, the results obtained here establish a clear and nontrivial link between quantum information and spacetime structure.

Spacetime is not given. It appears only when dynamics supports it.

This conclusion provides a new perspective on the role of geometry in physics and opens a path toward a fully dynamical understanding of spacetime.

Appendix

A. Well-definedness of the modular generator

The modular generator

$$K = -\log \rho \tag{55}$$

is well-defined on the support of ρ. In numerical implementations, we use a regularization

$$\rho \;\rightarrow\; \rho_\epsilon = \rho + \epsilon \mathbb{I}, \qquad \epsilon > 0, \tag{56}$$

to avoid singularities in the spectrum.

Let $\rho = U\Lambda U^\dagger$ be the spectral decomposition with $\Lambda = \mathrm{diag}(\lambda_i)$. Then

$$K = U\,\mathrm{diag}(-\log \lambda_i)\,U^\dagger. \tag{57}$$

This construction is stable under small perturbations of ρ provided ϵ is chosen below the spectral resolution scale.

B. Positivity of the modular metric

The modular metric is defined as

$$g_{ab} = \mathrm{Tr}\left(\rho[K, O_a][K, O_b]\right). \tag{58}$$

Let $X_a = [K, O_a]$. Then for any real vector v_a:

$$v_a g_{ab} v_b = \mathrm{Tr}\left(\rho\left(\sum_a v_a X_a\right)^2\right). \tag{59}$$

Since $\rho \geq 0$ and X_a are Hermitian (for Hermitian O_a), it follows that

$$v_a g_{ab} v_b \geq 0. \tag{60}$$

Thus g_{ab} is positive semi-definite.

C. CPTP preservation under GKSL evolution

The GKSL generator

$$\mathcal{L}[\rho] = -i[H, \rho] + \sum_{\alpha} \gamma_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \right) \tag{61}$$

preserves:

- trace: $\operatorname{Tr} \rho = 1$,
- positivity: $\rho \geq 0$,
- complete positivity of the dynamical map.

These properties ensure physical consistency of the evolution (18).

D. Spohn inequality

For a fixed reference state σ, GKSL dynamics satisfies:

$$\frac{d}{d\lambda} D(\rho \| \sigma) \leq 0. \tag{62}$$

This follows from the complete positivity of the map and implies that relative entropy is a Lyapunov functional.

E. Numerical protocol for propagation diagnostics

The propagation diagnostics are implemented as follows:

1. Prepare initial state ρ and perturbed state
$$\rho' = U_{i_0} \rho U_{i_0}^\dagger. \tag{63}$$
2. Evolve both states under (18).
3. Compute signal function
$$C(j, \lambda) = \left| \langle O_j \rangle_{\rho'(\lambda)} - \langle O_j \rangle_{\rho(\lambda)} \right|. \tag{64}$$
4. Determine arrival times
$$t^\star(j) = \inf\{\lambda : C(j, \lambda) > \varepsilon\}. \tag{65}$$
5. Fit $t^\star(j)$ to extract v_{eff}.

F. Phase classification criteria

The three dynamical phases are identified as:

- Phase I: $t^\star(j) = \infty$ for all j.
- Phase II: $t^\star(j) \approx \text{const}$.
- Phase III: $t^\star(j) \sim j$ with
$$\rho_s > 0.7, \qquad R^2 > 0.7. \tag{66}$$

G. Robustness considerations

The results are stable under:

- moderate variation of threshold ε,
- choice of local operators O_j,
- perturbation operator U_{i_0},
- small changes in system size N.

This indicates that the observed phase structure is not an artifact of specific numerical choices.

H. Limitations of discrete geometry

The present construction operates on discrete indices j, which do not constitute a continuous coordinate system.

As a result:

- curvature extraction is approximate,
- coordinate charts are absent,
- continuum limits require additional scaling assumptions.

These limitations motivate future work on continuum emergence.

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