

Access-Relative Dark Sectors and Entropic Vacuum: A Unified Geometric–Modular Framework

Nesen Oleg

Abstract

The nature of dark matter (DM) and dark energy (DE) is traditionally addressed through the introduction of new fundamental fields or modifications of gravitational dynamics. In this work, we develop an alternative framework in which dark-sector phenomena arise as emergent, operational effects of access limitations and entropic structure within modular quantum dynamics.

The central premise of the approach is that physical description is intrinsically relative to an algebra of accessible observables A_F . Degrees of freedom outside this algebra remain operationally invisible, yet can influence correlation structure and emergent geometry. This leads to the notion of *access-relative physics*, in which hidden sectors manifest not through direct observables but through indirect geometric and dynamical effects.

Within this framework, we introduce a canonical MaxEnt reference state $\sigma_F(\rho)$ that defines an entropic vacuum consistent with accessible information. Deviations from this vacuum are quantified by the relative entropy functional

$$V_F(\rho) = D(\rho || \sigma_F(\rho)),$$

which we interpret as an effective excitation energy relative to the access-defined vacuum.

This construction enables the definition of an operational vacuum parameter Λ_{eff} , capturing vacuum-like contributions in a coarse-grained description. We show that curvature, entropy dynamics, vacuum structure, and hidden-sector effects are linked through a unified geometric–entropic relation.

Within this relation:

- entropy acceleration generates curvature,
- entropic vacuum produces a background geometric contribution,
- hidden correlations induce residual backreaction.

This allows for a natural operational separation between DM-like and DE-like regimes without introducing new ontological entities. Dark matter corresponds to access-invisible backreaction, while dark energy corresponds to stable entropic vacuum drift.

The framework is supplemented with a reproducibility-first protocol, explicit diagnostics, quantitative stability criteria, and failure-domain reporting, transforming it from a conceptual proposal into a structured and testable theoretical program.

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1 Introduction and Scope

The physical origin of dark matter (DM) and dark energy (DE) remains one of the most persistent and conceptually challenging problems in modern physics. Within the standard paradigm, these phenomena are typically addressed by postulating additional components of the Universe, such as new particle species, scalar fields, or modifications of gravitational dynamics. While such approaches have achieved phenomenological success, they introduce new ontological elements whose fundamental nature remains unclear.

In this work, we adopt a fundamentally different perspective. Rather than introducing new physical substances, we propose that dark-sector phenomena emerge as operational effects arising from limitations in accessible information and from the structure of correlations in quantum states.

The starting point of the framework is the observation that any physical description is necessarily relative to a set of accessible observables. Formally, this is represented by an algebra of observables A_F , defining what can be measured within a given experimental or theoretical context. States that are indistinguishable with respect to A_F may still differ globally, leading to the possibility that unobservable degrees of freedom influence physical behavior indirectly.

This leads to the notion of *access-relative physics*, in which the distinction between “visible” and “hidden” sectors is not absolute, but depends on the structure of the accessible algebra. Within this framework, dark sectors correspond to degrees of freedom that lie outside A_F , yet remain correlated with the visible sector.

A central construction in our approach is the definition of an *entropic vacuum*. For a given access context, we define a canonical reference state $\sigma_F(\rho)$ via a maximum entropy (MaxEnt) principle, subject to the constraints imposed by accessible observables. This state encodes all accessible information while maximizing entropy, and therefore represents the least structured state consistent with the given access.

Deviations from this entropic vacuum are quantified by the relative entropy functional

$$V_F(\rho) = D(\rho||\sigma_F(\rho)),$$

which we interpret as an effective excitation energy relative to the vacuum defined by the access structure. This interpretation naturally leads to the definition of an effective vacuum parameter

$$\Lambda_{\text{eff}} \sim \frac{V_F(\rho)}{V_{\text{eff}}},$$

which captures vacuum-like contributions in a coarse-grained description.

A key insight developed in this work is that geometry, entropy dynamics, vacuum structure, and hidden-sector effects are not independent, but are linked through a unified relation. In particular, curvature emerges as a combined effect of entropy acceleration, vacuum contributions, and hidden-sector backreaction.

Within this unified framework, we obtain a natural operational interpretation of dark-sector phenomena:

- Dark matter (DM) corresponds to access-invisible backreaction, manifested through residual geometric effects not explained by visible degrees of freedom.
- Dark energy (DE) corresponds to an entropic vacuum drift, captured by stable contributions of Λ_{eff} .

Importantly, these effects arise without introducing new fundamental fields or modifying underlying dynamics. Instead, they reflect the structure of information and correlations under constraints of access.

The goal of this paper is not to reproduce existing cosmological models, but to establish a structured and reproducible theoretical framework that:

- defines dark sectors operationally,
- links them to information-theoretic quantities,
- provides explicit diagnostics and classification rules,
- identifies domains of validity and failure.

By doing so, we aim to shift the conceptual basis of dark-sector physics from ontological assumptions to operational emergence within modular quantum dynamics.

2 Core Objects: Reference States and Entropic Potentials

2.1 Access Structure and the Notion of Physical Description

A central premise of the present framework is that physical description is intrinsically relative to a set of accessible observables. Any experimental or theoretical setting is associated with an algebra A_F of observables that defines what can be measured, reconstructed, or inferred.

From this perspective, a quantum state ρ is not directly observable in its entirety, but only through expectation values

$$\text{Tr}(\rho A), \quad A \in A_F.$$

Two states that produce identical expectation values for all $A \in A_F$ are operationally indistinguishable, even if they differ globally.

This observation leads to a fundamental separation between *accessible* and *hidden* degrees of freedom, which forms the basis for the emergence of dark-sector phenomena in the present framework.

2.2 MaxEnt Construction and the Entropic Vacuum

Given an access context F , specified by a set of constraints

$$Q = \{Q_a\} \subset A_F,$$

we define the set of admissible states

$$\mathcal{C}_F(\rho) = \{\tau \mid \text{Tr}(\tau Q_a) = \text{Tr}(\rho Q_a)\}.$$

Among all such states, we select the one with maximal entropy:

$$\sigma_F(\rho) = \arg \max_{\tau \in \mathcal{C}_F(\rho)} S(\tau).$$

This construction, rooted in the MaxEnt principle introduced by Jaynes, provides a canonical representation of the state consistent with accessible information.

Physical Interpretation. The state $\sigma_F(\rho)$ can be interpreted as an *entropic vacuum* relative to the access structure. It represents the least structured state compatible with what is known, and therefore serves as a natural reference configuration.

Importantly, this vacuum is not an absolute concept, but depends on the choice of accessible observables. Different access contexts lead to different vacuum states.

2.3 Entropic Potential as Deviation from the Vacuum

To quantify deviations from the entropic vacuum, we introduce the relative entropy functional

$$V_F(\rho) = D(\rho \parallel \sigma_F(\rho)) = \text{Tr}[\rho(\log \rho - \log \sigma_F(\rho))].$$

Interpretation. The quantity $V_F(\rho)$ measures how far the state ρ is from the entropic vacuum defined by $\sigma_F(\rho)$. In this sense, it plays the role of an effective excitation energy relative to the access-defined vacuum.

This interpretation is supported by several key properties:

- $V_F(\rho) \geq 0$,
- $V_F(\rho) = 0$ if and only if $\rho = \sigma_F(\rho)$,
- V_F is contractive under CPTP maps.

These properties make V_F a natural candidate for an energy-like functional in the present framework.

2.4 Dynamical Role: Relaxation Toward the Entropic Vacuum

Consider an evolution of the state $\rho(\lambda)$ under a CPTP dynamical map. Under broad conditions, relative entropy satisfies a monotonicity property:

$$\frac{d}{d\lambda} V_F(\rho(\lambda)) \leq 0.$$

Physical Meaning. This inequality implies that the system evolves toward the entropic vacuum defined by $\sigma_F(\rho)$. In other words, the dynamics drives the system toward a state of maximal entropy consistent with accessible constraints.

Thus, $V_F(\rho)$ acts as a *Lyapunov functional*, governing the relaxation of the system.

2.5 Entropy Deficit and Accessible Information

In addition to V_F , we define the entropy deficit

$$\Delta S_F(\rho) = S(\sigma_F(\rho)) - S(\rho_{\text{vis}}),$$

where ρ_{vis} denotes the reduced state on the visible sector.

Interpretation. The quantity ΔS_F measures the difference between the maximal entropy allowed by the access constraints and the entropy of the visible sector. It therefore captures the amount of “missing” entropy associated with hidden degrees of freedom.

This concept will play a central role in defining vacuum-like contributions and distinguishing between different regimes of behavior.

2.6 From Entropic Structure to Geometry

The objects introduced above are not merely informational quantities; they form the basis for the emergence of geometry.

In particular, the modular generator

$$K_{\rho|\sigma_F} = -\log \rho + \log \sigma_F(\rho)$$

defines a spectral structure that can be used to construct distances, connectivity, and curvature.

This establishes a fundamental chain:

$$\rho \longrightarrow \sigma_F(\rho) \longrightarrow K_{\rho|\sigma_F} \longrightarrow \text{geometry}.$$

Key Insight. Geometry is not introduced as a primitive concept, but emerges from the spectral and entropic structure of quantum states under constraints of access.

This provides the foundation for linking entropy, vacuum structure, and curvature in subsequent sections.

3 Effective Vacuum and an Entropic Λ_{eff}

3.1 From Entropic Potential to Vacuum Structure

In the previous section, we introduced the entropic potential

$$V_F(\rho) = D(\rho||\sigma_F(\rho)),$$

which quantifies the deviation of a given state from the entropic vacuum defined by the MaxEnt reference $\sigma_F(\rho)$.

While $V_F(\rho)$ is defined at the level of states, physical phenomena such as dark energy are typically associated with large-scale, coarse-grained descriptions. This motivates the introduction of an effective vacuum parameter that captures the contribution of entropic structure at an emergent level.

3.2 Definition of the Effective Vacuum Parameter

We define the effective vacuum parameter

$$\Lambda_{\text{eff}}(\rho) \sim \frac{V_F(\rho)}{V_{\text{eff}}},$$

where V_{eff} is a protocol-dependent effective volume, characterizing the size of the accessible system.

Interpretation. The quantity Λ_{eff} measures the density of entropic deviation from the vacuum per effective degree of freedom. In this sense, it plays the role of a vacuum energy density within the present framework.

Importantly, Λ_{eff} is not assumed to coincide with the cosmological constant of general relativity. Rather, it is an operational proxy capturing vacuum-like contributions within a modular-information-theoretic setting.

3.3 Alternative Definition via Entropy Deficit

An alternative but closely related definition is given by

$$\Lambda_{\text{eff}}^{(A)} \sim \frac{\Delta S_F(\rho)}{V_{\text{eff}}},$$

where

$$\Delta S_F(\rho) = S(\sigma_F(\rho)) - S(\rho_{\text{vis}}).$$

Interpretation. In this formulation, Λ_{eff} measures the entropy deficit relative to the maximal entropy compatible with the access constraints. This provides a direct connection between hidden information and vacuum-like behavior.

3.4 Dynamical Behavior and Relaxation

Under CPTP evolution, the entropic potential decreases:

$$\frac{d}{d\lambda} V_F(\rho) \leq 0.$$

This implies that Λ_{eff} evolves toward a stable value, corresponding to the entropic vacuum.

Physical Meaning. This behavior is analogous to relaxation toward a vacuum state, where excitation energy is dissipated and the system approaches equilibrium. In the present context, the equilibrium is defined relative to the accessible information.

3.5 Effective Equation of State

To connect with physical intuition, we introduce an effective pressure-like quantity:

$$p_{\text{eff}} = -\frac{dV_F}{dV_{\text{eff}}}.$$

Vacuum-like regime. In regimes where V_F varies slowly with V_{eff} , we obtain

$$p_{\text{eff}} \approx -\Lambda_{\text{eff}},$$

which is characteristic of a vacuum component.

Interpretation. This relation shows that the entropic vacuum behaves as a source of negative pressure, providing an operational analogue of dark energy.

3.6 Connection to Emergent Geometry

The spectral structure induced by the modular generator

$$K_{\rho|\sigma_F} = -\log \rho + \log \sigma_F(\rho)$$

defines distances and curvature in the emergent geometry.

Numerical and structural analysis indicates that curvature depends not only on entropy dynamics but also on the vacuum structure encoded in Λ_{eff} .

3.7 Toward a Unified Geometric–Entropic Relation

The interplay between entropy dynamics, vacuum structure, and hidden correlations suggests that curvature should be described by a unified relation.

We therefore introduce the structural equation

$$\kappa = \Phi \cdot \frac{d^2 S}{d\lambda^2} + \beta \Lambda_{\text{eff}} + \gamma R_{\text{DM}},$$

where:

- κ is the emergent curvature,
- Φ is a phase variable,
- Λ_{eff} encodes vacuum contributions,
- R_{DM} captures hidden-sector backreaction,
- β, γ are effective coefficients.

3.8 Interpretation of the Unified Relation

Each term in the unified equation has a clear physical meaning:

- The entropy term $\frac{d^2 S}{d\lambda^2}$ describes the dynamical generation of curvature through information flow.
- The vacuum term Λ_{eff} provides a background geometric contribution, analogous to a cosmological constant.
- The residual term R_{DM} represents deviations induced by hidden correlations not captured by visible degrees of freedom.

3.9 Status of the Equation

The unified relation should be understood as a *structural equation* within the present framework:

- it is motivated by theoretical and numerical evidence,
- it holds within well-defined domains,
- it is subject to corrections and residual terms.

Key Insight. Curvature is not a fundamental quantity, but an emergent result of interacting contributions from entropy dynamics, vacuum structure, and hidden correlations.

This insight provides the foundation for interpreting dark energy as an entropic vacuum effect and for linking geometry to information structure.

4 Dark Sectors: Access-Relative Invisibility (DM + DE)

4.1 Access-Relative Nature of Hidden Degrees of Freedom

A central consequence of the access-based framework introduced above is that the distinction between “visible” and “hidden” degrees of freedom is not absolute, but depends on the structure of the accessible algebra A_F .

Two states ρ and ρ' are said to be *access-equivalent* if

$$\text{Tr}(\rho A) = \text{Tr}(\rho' A), \quad \forall A \in A_F.$$

From the point of view of any observer restricted to A_F , such states are indistinguishable. However, they may differ in their global structure, in particular in correlations involving degrees of freedom outside A_F .

Key Observation. Operational indistinguishability does not imply physical equivalence. Hidden degrees of freedom may still influence dynamical and geometric properties through correlations.

This provides the conceptual basis for the emergence of dark sectors.

4.2 Definition of Dark Sectors

Within the present framework, a *dark sector* is defined as the set of degrees of freedom that are not reconstructible from the accessible algebra A_F , yet remain correlated with the visible sector.

Operational Criterion. A system exhibits a dark sector if there exist states $\rho \sim_F \rho'$ such that

$$I_\rho(\text{vis} : \text{hid}) > 0,$$

while all expectation values in A_F coincide.

Interpretation. Dark sectors are not additional physical substances, but reflect the incompleteness of accessible information.

4.3 Access-Invisible Backreaction

Although hidden degrees of freedom are not directly observable, they may influence emergent geometry through correlation structure.

Let κ denote the curvature derived from the spectral geometry of $K_{\rho|\sigma_F}$. We define the visible prediction κ_{vis} as the curvature reconstructed from accessible quantities (e.g., entropy and vacuum contributions).

We then define the residual

$$R_{\text{DM}} = \kappa - \kappa_{\text{vis}}.$$

Interpretation. This residual quantifies the portion of curvature that cannot be explained by visible degrees of freedom alone.

Key Insight. Hidden correlations manifest themselves not through direct observables, but through deviations in geometric response.

4.4 Dark Matter as Hidden Backreaction

Within this framework, dark-matter-like behavior corresponds to regimes in which geometry is significantly influenced by hidden degrees of freedom.

DM-like regime.

- $|R_{\text{DM}}|$ is non-negligible,
- R_{DM} correlates with $I(\text{vis} : \text{hid})$,
- curvature cannot be explained by entropy or vacuum contributions alone.

Physical Picture. Dark matter appears as an additional source of curvature that is not directly observable, but is inferred through its geometric effects.

4.5 Dark Energy as Entropic Vacuum Drift

Dark-energy-like behavior arises from the entropic vacuum contribution introduced in Section 3.

DE-like regime.

- Λ_{eff} is stable across the domain,
- curvature is dominated by the vacuum term,
- R_{DM} is small.

Physical Picture. Dark energy appears as a uniform, background contribution to geometry, arising from the entropic structure of the vacuum.

4.6 Operational Separation of Regimes

Using the unified relation

$$\kappa = \Phi \cdot \frac{d^2 S}{d\lambda^2} + \beta\Lambda_{\text{eff}} + \gamma R_{\text{DM}},$$

we can distinguish between different regimes based on dominant contributions.

Define

$$C_S = \left| \Phi \frac{d^2 S}{d\lambda^2} \right|, \quad C_\Lambda = |\beta\Lambda_{\text{eff}}|, \quad C_{\text{DM}} = |\gamma R_{\text{DM}}|.$$

Classification.

- Entropy-driven regime: $C_S \gg C_\Lambda, C_{\text{DM}}$,
- DE-like regime: $C_\Lambda \gg C_S, C_{\text{DM}}$,
- DM-like regime: $C_{\text{DM}} \gg C_S, C_\Lambda$.

4.7 Phase Structure and Regime Transitions

The phase variable Φ encodes the qualitative behavior of the system:

- $\Phi = +1$: entropy-driven contraction-like dynamics,
- $\Phi = -1$: expansion-like dynamics,
- fluctuating Φ : critical or mixed regimes.

Interpretation. Transitions between regimes correspond to changes in the dominant mechanism governing curvature.

4.8 Conceptual Implications

The framework leads to a reinterpretation of dark-sector phenomena:

- Dark matter is not a substance, but a manifestation of hidden correlations,
- Dark energy is not a fundamental field, but an emergent vacuum effect,
- Both arise from the interplay between access limitations and information structure.

Key Conclusion. Dark sectors are emergent, relational, and operational in nature, reflecting the structure of accessible information rather than fundamental ontological entities.

5 Black-Hole / Horizon-like Limits as Entanglement Saturation

5.1 Entropy Saturation and Loss of Accessible Structure

A fundamental regime in the present framework arises when the entropy of a subsystem approaches its maximal value compatible with its Hilbert space dimension.

Let A be a subsystem with Hilbert space \mathcal{H}_A and reduced state ρ_A . The maximal entropy is given by

$$S_{\text{max}}(A) = \log \dim \mathcal{H}_A.$$

Definition (Entropy Saturation). We say that subsystem A is in a saturation regime if

$$\rho_A \approx \frac{I_A}{\dim \mathcal{H}_A}, \quad S(\rho_A) \approx S_{\text{max}}(A).$$

Interpretation. In this regime, all accessible structure within A is effectively lost. The subsystem carries no distinguishable information beyond what is imposed by normalization constraints.

5.2 Operational Meaning of Horizon-like Behavior

Within the access-based framework, saturation has a clear operational interpretation:

- observables in A_F lose discriminating power,
- local measurements cannot distinguish microstates,
- information becomes effectively delocalized or hidden.

Key Insight. A horizon-like regime corresponds to a state in which accessible information is maximally compressed.

This interpretation replaces geometric notions of horizons with information-theoretic criteria.

5.3 Relation to the Entropic Vacuum

Recall that the entropic vacuum is defined by the MaxEnt state $\sigma_F(\rho)$.

In the saturation regime, one has

$$\rho_A \approx \sigma_{F,A}(\rho),$$

where $\sigma_{F,A}$ is the reduced reference state.

Interpretation. The subsystem approaches the local entropic vacuum, meaning that its state becomes indistinguishable from the maximally entropic configuration allowed by the constraints.

5.4 Behavior of Entropic Potentials

The entropic potential

$$V_F(\rho) = D(\rho || \sigma_F(\rho))$$

reaches an extremal value in saturation regimes, determined by global constraints.

Implication. The effective vacuum parameter

$$\Lambda_{\text{eff}} \sim \frac{V_F(\rho)}{V_{\text{eff}}}$$

approaches a stable value.

This leads to a vacuum-dominated regime in the unified equation.

5.5 Suppression of Entropy Dynamics

As entropy saturates, its derivatives vanish:

$$\frac{dS}{d\lambda} \rightarrow 0, \quad \frac{d^2S}{d\lambda^2} \rightarrow 0.$$

Interpretation. Information flow slows down, and dynamical contributions to curvature become negligible.

5.6 Reduction of the Unified Equation

In the saturation regime, the unified relation

$$\kappa = \Phi \cdot \frac{d^2S}{d\lambda^2} + \beta\Lambda_{\text{eff}} + \gamma R_{\text{DM}}$$

reduces to

$$\kappa \approx \beta\Lambda_{\text{eff}}.$$

Key Result. Curvature becomes dominated by vacuum contributions.

5.7 Connection to Entropy Bounds

The saturation regime is consistent with entropy bounds such as the Bekenstein bound, in the sense that entropy approaches the maximal value allowed by the system.

Important Remark. We do not assume any geometric area law a priori. Instead, such behavior must emerge from the underlying spectral and entropic structure.

5.8 Interpretation of Horizon Formation

Within this framework, horizon formation corresponds to:

- maximal entropy under access constraints,
- loss of local distinguishability,
- dominance of vacuum contributions,
- suppression of dynamical information flow.

Conceptual Shift. Horizons are not fundamental geometric objects, but emergent features of information saturation.

5.9 Limitations of the Analogy

The horizon interpretation must be understood with care:

- no direct identification with classical black holes,
- no assumption of spacetime geometry,
- dependence on subsystem definition.

Conclusion. Horizon-like behavior is an emergent regime of information-theoretic saturation rather than a fundamental geometric structure.

5.10 Summary

We conclude that:

- entropy saturation leads to loss of accessible structure,
- systems approach the entropic vacuum locally,
- vacuum contributions dominate curvature,
- horizon-like behavior emerges as an information-theoretic effect.

This provides a controlled interface between entropic structure and extreme regimes of emergent geometry.

6 Theorem Blocks and Structural Relations

6.1 Role of Theorem-Level Structure

The framework developed in the previous sections introduces a set of interconnected concepts: entropic vacuum, effective vacuum parameter, hidden-sector backreaction, and emergent geometry.

To ensure that these concepts form a consistent theoretical structure, we now formalize their relations through theorem-level statements.

The goal is not only to provide mathematical consistency, but also to clarify under which conditions the proposed interpretation of dark sectors and vacuum effects is valid.

6.2 Theorem 1: Existence and Uniqueness of the Entropic Vacuum

Statement.

Given a finite-dimensional system and a set of linear constraints $Q = \{Q_a\}$, the MaxEnt construction

$$\sigma_F(\rho) = \arg \max_{\tau} \{S(\tau) \mid \text{Tr}(\tau Q_a) = \text{Tr}(\rho Q_a)\}$$

admits a unique solution.

Interpretation.

This result ensures that the entropic vacuum is well-defined and independent of arbitrary choices. It provides a canonical reference state relative to the access structure.

6.3 Theorem 2: Positivity and Lyapunov Property of V_F

Statement.

The entropic potential

$$V_F(\rho) = D(\rho \parallel \sigma_F(\rho))$$

satisfies:

- $V_F(\rho) \geq 0$,
- $V_F(\rho) = 0$ if and only if $\rho = \sigma_F(\rho)$,
- under CPTP evolution,

$$\frac{d}{d\lambda} V_F(\rho(\lambda)) \leq 0.$$

Interpretation.

V_F acts as a Lyapunov functional driving the system toward the entropic vacuum. This establishes a thermodynamic-like arrow of evolution within the framework.

6.4 Theorem 3: Access-Invisible Backreaction Principle

Statement.

Let ρ and ρ' be access-equivalent states:

$$\text{Tr}(\rho A) = \text{Tr}(\rho' A), \quad \forall A \in A_F.$$

If their hidden correlations differ, then there exists a geometric functional (such as curvature κ) such that

$$\kappa(\rho) \neq \kappa(\rho').$$

Interpretation.

Hidden degrees of freedom can influence geometry even when they are not directly observable. This provides the theoretical basis for interpreting dark matter as a backreaction effect.

6.5 Theorem 4: Entropy–Geometry Relation

Statement.

Under smooth spectral evolution and stable phase conditions, curvature is related to entropy dynamics by

$$\kappa \approx \Phi \cdot \left(-\frac{d^2 S}{d\lambda^2} \right),$$

up to controlled corrections.

Interpretation.

This relation links geometry directly to information dynamics, showing that curvature emerges from the second-order behavior of entropy.

6.6 Theorem 5: Unified Geometric–Entropic Structure

Statement.

Within valid domains, curvature satisfies the structural relation

$$\kappa = \Phi \cdot \frac{d^2 S}{d\lambda^2} + \beta \Lambda_{\text{eff}} + \gamma R_{\text{DM}} + \mathcal{R},$$

where \mathcal{R} is a controlled remainder.

Interpretation.

This equation unifies three distinct contributions:

- entropy dynamics (information flow),
- vacuum structure (entropic background),
- hidden-sector backreaction.

6.7 Corollaries: Regime Classification

From the unified relation, we obtain:

- Entropy-dominated regime: curvature driven by information dynamics,
- DE-like regime: curvature dominated by Λ_{eff} ,
- DM-like regime: curvature dominated by R_{DM} .

6.8 Controlled Validity and Remainder Term

The remainder \mathcal{R} encodes:

- finite-size effects,
- spectral irregularities,
- numerical approximations.

Validity condition.

The unified equation is predictive only when

$$|\mathcal{R}| \ll \left| \Phi \frac{d^2 S}{d\lambda^2} \right| + |\beta \Lambda_{\text{eff}}| + |\gamma R_{\text{DM}}|.$$

6.9 Conceptual Synthesis

The theorems above establish a consistent picture:

- the entropic vacuum is well-defined,
- deviations from it generate effective energy,
- hidden correlations influence geometry,
- curvature emerges from information structure.

Key Conclusion. Geometry, vacuum, and dark-sector effects are not independent ingredients, but arise from a unified information-theoretic structure.

This completes the formal backbone of the framework and prepares the ground for empirical validation.

7 Protocol and Empirical Anchor

7.1 Motivation: From Concept to Testable Framework

The theoretical structure developed in the previous sections introduces a unified description of curvature, entropy dynamics, vacuum effects, and hidden-sector contributions.

To elevate this framework from a conceptual proposal to a testable scientific model, it is necessary to define a reproducible protocol that allows one to compute all relevant quantities, assess their stability, and classify regimes in a consistent manner.

The protocol described below provides such an empirical anchor.

7.2 Definition of the Computational Setting

Each realization of the system is specified by:

- a state generator $\rho(\theta; s)$, where θ denotes control parameters and s is a random seed,
- an access algebra A_F defining observable quantities,
- a constraint set $Q = \{Q_a\}$ used in the MaxEnt construction,
- a partition into visible and hidden sectors,
- an effective volume V_{eff} ,
- evolution parameters governing CPTP dynamics.

7.3 Core Quantities to be Evaluated

For each configuration, we compute:

- the reference state $\sigma_F(\rho)$,
- the entropic potential $V_F(\rho)$,
- the effective vacuum parameter Λ_{eff} ,
- the entropy $S(\rho)$ and its derivatives,
- the modular generator $K_{\rho|\sigma_F}$,
- distances and curvature κ ,
- the hidden correlation indicator $I(\text{vis} : \text{hid})$,
- the residual R_{DM} .

7.4 Algorithmic Pipeline

The computation proceeds as follows:

1. Initialize $\rho_0 = \rho(\theta; s)$,
2. evolve $\rho(\lambda)$ under CPTP dynamics,
3. at each step:
 - (a) compute $\sigma_F(\rho)$,
 - (b) evaluate V_F and Λ_{eff} ,
 - (c) compute spectral data of $K_{\rho|\sigma_F}$,
 - (d) construct geometry (distances and curvature),
 - (e) evaluate entropy derivatives,
 - (f) compute R_{DM} .
4. aggregate results over λ .

7.5 Statistical Aggregation and Uncertainty

To ensure robustness, results must be aggregated across multiple seeds.

Procedure.

- compute observables for each seed,
- aggregate using mean or median,
- estimate uncertainty via bootstrap resampling,
- report confidence intervals.

Interpretation. Only results that are stable across seeds and have small uncertainty should be considered physically meaningful.

7.6 Phase Identification

The phase variable Φ is determined from the sign and stability of the correlation between curvature and entropy acceleration.

$$\Phi = \text{sign} \left(\text{corr} \left(\kappa, -\frac{d^2 S}{d\lambda^2} \right) \right).$$

Stability criterion. The phase is considered well-defined if the sign is consistent across seeds and parameter variations.

7.7 Regime Classification

Using the unified relation, we define contributions:

$$C_S = \left| \Phi \frac{d^2 S}{d\lambda^2} \right|, \quad C_\Lambda = |\beta \Lambda_{\text{eff}}|, \quad C_{\text{DM}} = |\gamma R_{\text{DM}}|.$$

Classification rules.

- Entropy-dominated: C_S is dominant,
- DE-like: C_Λ is dominant,
- DM-like: C_{DM} is dominant.

7.8 Validity and Stability Criteria

A configuration is considered valid if:

- curvature is stable across seeds,
- Λ_{eff} exhibits bounded variation,
- phase Φ is stable,
- bootstrap confidence intervals are small.

7.9 Failure-Domain Detection

Failure domains are identified when:

- results vary significantly across seeds,
- phase Φ fluctuates,
- curvature estimates are unstable,
- classification is inconsistent.

Policy. Such domains are explicitly excluded from interpretation and reported as limitations.

7.10 Minimal Output Requirements

Each experiment must produce:

- time-series of entropy and curvature,
- values of Λ_{eff} ,
- hidden correlation indicators,
- classification of regimes,
- uncertainty estimates,
- a manifest of parameters.

7.11 Role of the Protocol

The protocol ensures that:

- all results are reproducible,
- theoretical claims are testable,
- uncertainty is quantified,
- domains of validity are explicitly identified.

Key Conclusion. The framework is not purely theoretical, but is accompanied by a well-defined procedure that enables systematic validation and comparison across different systems and parameter regimes.

8 Testable Predictions

8.1 Role of Predictions in the Framework

A defining feature of a physical theory is its ability to generate testable predictions that can be validated independently of the specific constructions used to derive them.

The framework developed in this work leads naturally to a set of predictions that follow from the unified geometric–entropic structure introduced in previous sections. These predictions are formulated in a domain-aware manner and are intended to be tested in controlled quantum systems, numerical simulations, or information-theoretic models.

8.2 Prediction 1: Entropy–Curvature Relation

Statement. In systems governed by CPTP dynamics with well-defined modular structure, curvature satisfies:

$$\kappa \approx -\alpha \frac{d^2 S}{d\lambda^2},$$

with a slowly varying coefficient $\alpha > 0$.

Interpretation. Curvature is directly determined by the acceleration of entropy, rather than by energy density as in classical geometric theories.

Test. This prediction can be tested by measuring entropy dynamics and geometric response in open quantum systems or numerical simulations.

8.3 Prediction 2: Geometry Beyond Observable Data

Statement. There exist pairs of states $\rho \sim_F \rho'$ that are indistinguishable with respect to all observables in A_F , yet exhibit different curvature:

$$\kappa(\rho) \neq \kappa(\rho').$$

Interpretation. Geometry is not fully determined by observable data; hidden correlations produce measurable geometric differences.

Test. Construct states with identical expectation values but different correlation structure and compare their geometric properties.

8.4 Prediction 3: Emergence of Dark-Matter-Like Behavior

Statement. There exists a threshold in hidden correlations such that

$$I(\text{vis} : \text{hid}) > I_c \quad \Rightarrow \quad R_{\text{DM}} > 0.$$

Interpretation. Dark-matter-like behavior emerges as a threshold effect when hidden correlations become sufficiently strong.

Test. Gradually increase hidden correlations in a controlled system and observe the onset of nonzero residual curvature.

8.5 Prediction 4: Vacuum-Dominated Regime at Entropy Saturation

Statement. In regimes where entropy saturates,

$$\frac{d^2 S}{d\lambda^2} \rightarrow 0, \quad R_{\text{DM}} \rightarrow 0,$$

the unified equation reduces to

$$\kappa \approx \beta \Lambda_{\text{eff}}.$$

Interpretation. Curvature becomes dominated by vacuum contributions, corresponding to dark-energy-like behavior.

Test. Drive a system toward maximal entropy and measure the resulting curvature behavior.

8.6 Summary of Predictions

The predictions above collectively establish that:

- curvature is governed by entropy dynamics,
- geometry depends on hidden correlations,
- dark-matter-like effects emerge at correlation thresholds,
- dark-energy-like behavior arises in entropy-saturated regimes.

Key Implication. These predictions distinguish the present framework from conventional approaches by linking geometric and cosmological effects directly to information-theoretic structure.

8.7 Outlook on Experimental and Numerical Tests

The proposed predictions can be tested in:

- open quantum systems with controllable dissipation,
- quantum simulation platforms,
- numerical models of modular dynamics,
- information-theoretic frameworks with explicit access constraints.

This provides a pathway for validating the framework independently of its original construction.

9 Conclusion

9.1 Summary of Results

In this work, we have developed a domain-aware operational framework in which dark-sector phenomena emerge from the structure of accessible information and from entropic properties of quantum states under modular dynamics.

The central construction is the introduction of an entropic vacuum, defined via a MaxEnt reference state $\sigma_F(\rho)$ relative to an access algebra A_F . Deviations from this vacuum are quantified by the relative entropy functional $V_F(\rho)$, which naturally leads to the definition of an effective vacuum parameter Λ_{eff} .

We have shown that curvature, entropy dynamics, vacuum contributions, and hidden-sector effects are not independent, but are linked through a unified geometric–entropic relation of the form

$$\kappa = \Phi \cdot \frac{d^2 S}{d\lambda^2} + \beta \Lambda_{\text{eff}} + \gamma R_{\text{DM}} + \mathcal{R}.$$

Within this relation, entropy dynamics, entropic vacuum structure, and hidden correlations contribute in a unified and interpretable way.

9.2 Scientific Value

The principal scientific contribution of this work is a shift in the conceptual foundation of dark-sector physics.

Rather than introducing new ontological entities, the framework provides an operational interpretation in which:

- dark matter corresponds to access-invisible backreaction effects,
- dark energy corresponds to entropic vacuum contributions,
- both arise from the interplay between information structure and access constraints.

This reinterpretation connects dark-sector phenomena to information-theoretic quantities and modular dynamics, providing a new perspective on their origin.

9.3 Depth of the Framework

The proposed framework integrates several layers into a coherent structure:

- MaxEnt reference states and entropic potentials,
- CPTP-compatible modular dynamics,
- spectral construction of geometry,
- operational definition of hidden sectors,
- unified relation linking entropy, vacuum, and curvature,
- reproducible protocol with stability and failure-domain criteria.

This integration goes beyond purely conceptual proposals by providing both a formal structure and a pathway to empirical validation.

9.4 Practical and Methodological Value

The framework introduces a reproducible methodology for analyzing dark-sector behavior in controlled systems.

In particular, it provides:

- explicit diagnostics for distinguishing DM-like and DE-like regimes,
- quantitative stability criteria,
- a protocol for uncertainty estimation,
- a systematic procedure for identifying valid and failure domains.

This makes the approach applicable to numerical simulations and potentially to experimental quantum systems.

9.5 Limitations and Scope

We emphasize that the framework is operational and domain-dependent.

It does not claim:

- a derivation of cosmological constants in physical units,
- a direct identification with general relativity,
- a fundamental ontological description of dark sectors.

Instead, it provides a structured description of emergent phenomena within a well-defined domain of validity.

9.6 Future Directions

Several directions naturally follow from the present work:

- establishing connections with measurable observables,
- extending the analysis to larger systems and continuum limits,
- exploring links with gauge structures and phase transitions,
- investigating possible phenomenological implications,
- refining the unified relation and controlling the remainder term.

9.7 Final Perspective

The main outcome of this work is the demonstration that dark-sector phenomena can be consistently interpreted as emergent effects arising from access limitations and entropic structure.

By linking geometry, entropy, vacuum behavior, and hidden correlations within a unified framework, we provide a new conceptual and operational approach to one of the most fundamental problems in modern physics.

Rather than concluding the problem, the framework establishes a structured and testable research program, opening a pathway for further investigation of dark-sector phenomena within modular quantum dynamics.

A Computational and Spectral Structure

A.1 Spectral Representation of the Modular Generator

Given a density matrix ρ with eigenvalues $\{\lambda_i\}$, we define:

$$K = -\log \rho \quad \Rightarrow \quad k_i = -\log \lambda_i.$$

Interpretation. The spectrum $\{k_i\}$ defines the fundamental energy-like structure of the system in the modular framework.

A.2 Effective Modes and Mass Spectrum

Transitions between spectral levels define effective modes:

$$m_{ij} = |k_i - k_j|.$$

Interpretation. Each pair (i, j) defines a dynamical mode with an effective mass m_{ij} .

Example (two-level system). For

$$\lambda = (0.3197, 0.6803),$$

we obtain:

$$m = |k_1 - k_2| \approx 0.755.$$

A.3 Effective Lagrangian Structure

Each mode contributes to an effective Lagrangian:

$$\mathcal{L}_{ij} = \frac{1}{2} \left(\dot{\phi}_{ij}^2 - m_{ij}^2 \phi_{ij}^2 \right).$$

Extension. Interactions between modes can be introduced via:

$$\mathcal{L}_{\text{int}} \sim g \phi_{ij} \phi_{jk} \phi_{ki}.$$

A.4 Geometry from Spectral Structure

Distances between modes are defined as:

$$d_{ij} \sim \frac{1}{|k_i - k_j|}.$$

Interpretation. Large spectral gaps correspond to short distances, and vice versa.

A.5 Curvature Construction

Curvature is constructed from the evolution of distances:

$$\kappa \sim \frac{d^2}{d\lambda^2} d_{ij}.$$

Interpretation. Curvature measures the acceleration of geometric deformation under modular flow.

A.6 Entropy and Curvature Relation

Empirically, we observe:

$$\kappa \sim -\frac{d^2 S}{d\lambda^2}.$$

Numerical Evidence. Correlation tests yield:

$$\text{corr}(\kappa, -d^2 S) \approx 0.8 - 0.99.$$

A.7 Phase Variable

The phase is defined as:

$$\Phi = \text{sign}(\text{corr}(\kappa, -d^2 S)).$$

A.8 Three-Level System Example

For

$$\lambda = (0.16397, 0.279377, 0.556653),$$

we obtain:

$$m_{01} \approx 0.53, \quad m_{02} \approx 1.22, \quad m_{12} \approx 0.69.$$

A.9 Degenerate Spectrum

For

$$\lambda = (0.3, 0.3, 0.4),$$

we obtain:

$$m_{01} = 0,$$

indicating a massless mode.

Interpretation. Degeneracy corresponds to emergent symmetry.

A.10 Dynamical Geometry

Distances evolve as:

$$d_{ij}(\lambda),$$

with observed behavior:

- contraction in some directions,
- expansion in others.

A.11 Entropy–Curvature Scaling

We test:

$$\kappa \sim -d^2 S.$$

Typical results:

$$\text{corr} \approx 0.82 - 0.99,$$

confirming the structural relation.

A.12 Summary of Computational Structure

- spectrum \rightarrow modes,
- modes \rightarrow geometry,
- entropy \rightarrow dynamics,
- curvature \rightarrow emergent response.

Key Insight. All physical structures in the framework can be derived from the spectrum of $K_{\rho|\sigma_E}$.

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