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The cosmological constant as a QCD observable: derivation, nonlinear screening, and falsification

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The cosmological constant as a QCD observable: derivation, nonlinear screening, and falsification

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Abstract

We extend the Einstein–Hilbert action by two terms—an R^2 elastic correction (Starobinsky) and a vacuum–matter coupling $(1 + 2\alpha)$—where the coupling α is derived from QCD sigma terms. Taking the trace of the resulting field equations and imposing a self-consistency condition on the vacuum–matter–metric cycle, with the non-perturbative curvature response estimated via naive dimensional analysis (NDA), yields a closed-form expression for the cosmological constant: $\Lambda_0 = \alpha \cdot f_b \cdot \sigma^6 / m_{Pl}^4$. Every input is measured independently. Using the derived $\alpha = 0.005$, the result is $\Lambda_0 \approx 3.0 \times 10^{-52}$ m^{-2}, compared to the observed 1.1×10^{-52} m^{-2} (ratio 2.8). The nonlinear screening factor entering α is confirmed by particle-mesh N-body simulations at 128^3 resolution presented here, which yield a converged screening of 3.9× via Richardson extrapolation. Five falsifiable predictions are stated. The simulation code is provided in Appendix A.

Keywords: *cosmological constant; vacuum energy; QCD sigma terms; f(R) gravity; Schwinger–DeWitt effective action; naive dimensional analysis; N-body simulations; nonlinear screening*

1. Introduction

The cosmological constant problem has two layers. The Planck-scale estimate exceeds the observed value by 10^{120}. More tractably, the QCD chiral condensate energy ~(250 MeV)^4 exceeds Λ by ~10^{42}. This letter addresses the second layer.

The Zel'dovich identification (1967) [1] equates Λ (a geometric quantity) with ρ_{vac} (a field-theoretic quantity). This was never derived from a variational principle; it was postulated. We adopt the trace-free formulation [2], in which Λ enters as an integration constant separated from the matter sector by construction. This is an interpretive choice; see Refs. [2–6] for independent support.

Our strategy: write an action, derive α from QCD sigma terms, obtain the field equations, take the trace, impose self-consistency with the curvature response estimated via NDA [7], and obtain Λ_0. The derivation contains one O(1) uncertainty (a nonlinear screening factor), which we confirm independently by N-body simulations (Section 3). The result is a parameter-free estimate, not a rigorous derivation; this distinction is maintained throughout.

2. The action

We consider:

$$S = \int d^4x \sqrt{(-g)} \{ (m^2\_Pl / 2) [R + R^2/(6M^2)] + (1 + 2\alpha) \mathscr{L}\_m \} \quad (1)$$

where $m_{Pl} = 2.44 \times 10^{18}$ GeV, $M \approx 3 \times 10^{13}$ GeV (CMB amplitude [8]), α is derived below, and $\mathscr{L}_m$ is the matter Lagrangian. The R^2 term is Starobinsky inflation [8]. The factor $(1 + 2\alpha)$ before $\mathscr{L}_m$ does not modify the gravitational constant G. It reflects the inclusion of vacuum energy $\alpha\rho_m$ as an additional Poisson source: $\nabla^2\Phi = 4\pi G(\rho_m + 2\alpha\rho_m) = 4\pi G(1 + 2\alpha)\rho_m$. In the solar system, ρ_m in the interplanetary medium is $\sim 10^{-20}$ kg/m³; the vacuum source $2\alpha\rho_m \sim 10^{-22}$ kg/m³ produces a gravitational acceleration $\sim 10^{-30}$ times the solar field at 1 AU. Lunar laser ranging [9] operates 25 orders of magnitude above this effect. The 1% cosmological enhancement emerges only at galactic scales, where the integrated vacuum mass becomes comparable to luminous matter. Four limits verify the action: (i) $M \to \infty$ recovers GR; (ii) $\alpha \to 0$ recovers Starobinsky; (iii) both $\to 0$ recovers Einstein 1915; (iv) $T_{\mu\nu} \to 0$ gives vacuum Starobinsky.

3. Derivation of α from nuclear physics

Step 1: Schwinger–DeWitt correction. The heat-kernel expansion gives $\delta\rho_{vac} = c_1 R + O(R^2)$ [10–12]. In FRW, the Weyl tensor vanishes and $R_{\mu\nu} = (R/4)g_{\mu\nu}$, so all Seeley–DeWitt invariants reduce to powers of R.

Step 2: Einstein trace. For dust: $R = \rho_m/m^2_{Pl}$. Hence $\delta\rho_{vac} \propto R \propto \rho_m$.

Step 3: QCD sigma terms. $\sigma_{\pi N} \approx 50$ MeV [13] and $\sigma_s \approx 40$ MeV [14] give $\sigma/m_N = 0.096$.

Step 4: Baryon fraction and screening. Weighted by $f_b = 0.156$ [15] and a screening factor ~3:

$$\alpha = f\_b \times \sigma / (3 \times m\_N) = 0.005 \quad \text{(observed: 0.003, ratio 1.67)} \qquad (2)$$

The screening factor is not a free parameter: we confirm it independently by particle-mesh N-body simulations. Using the standard PM algorithm [16] with CIC assignment, FFT Poisson solve, and leapfrog integration in ln(a), we simulate a 256 Mpc/h box with Planck 2018 parameters (Ω_m = 0.315, h = 0.674, $\sigma_8 = 0.811$), Zel'dovich initial conditions at z = 49, and the Eisenstein–Hu transfer function [17]. The source modification $S = \delta[1 + 2\alpha\Theta(\delta)]$ implements Eq. (1) on the mesh. At 128^3 resolution (2.1×10^6 particles), the σ_8 ratio for α = +0.03 is 1.086 ± 0.002 (10-seed variance), while linear theory predicts 1.352—a screening of 4.1×. Richardson extrapolation across three resolutions ($32^3 \rightarrow 64^3 \rightarrow 128^3$) gives $R_\infty = 1.091 \pm 0.005$, converged screening factor 3.9×. The screening is robust to threshold choice: replacing the Heaviside Θ(δ) with a smooth tanh transition changes the result by <15%. The physical origin is geometric: the void volume fraction evolves from $f_v \approx 0.50$ at z = 49 to $f_v \approx 0.72$ at z = 0, progressively confining the gravitational modification to the shrinking overdense fraction. A predictive formula based on the growth-weighted overdense fraction gives $R_{pred} = 1.088$, matching the measured 1.086. The simulation code is provided in Appendix A.

4. Field equations

Varying $\delta S = 0$ in the trace-free formulation [2]:

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} + H_{\mu\nu} = (1 + 2\alpha)/m^2_{Pl} \cdot T_{\mu\nu} \quad (3)$$

where $H_{\mu\nu} = (1/(3M^2))[RR_{\mu\nu} - (R^2/4)g_{\mu\nu} + g_{\mu\nu}\Box R - \nabla_\mu\nabla_\nu R]$. Λ_0 arises as a constant of integration, not a free parameter; its value is determined below.

5. Derivation of Λ_0

5.1. Trace. Contracting Eq. (3), at late times ($H \approx 0$, $T = -\rho_m$):

$$R + 4\Lambda_0 = (1 + 2\alpha)/m^2_{Pl} \cdot \rho_m \quad (4)$$

5.2. The QCD condensate as the source of Λ_0. The coupling α produces a running vacuum component $\delta\rho_{vac} = \alpha f_b \rho_m$ that tracks the matter density. However, Λ_0 is not this running component; it is the ground-state constant determined by the fundamental QCD energy scale. The chiral condensate energy density is $\sim\sigma^4$, where $\sigma = 90$ MeV. Only the fraction αf_b escapes confinement:

$$\rho_{eff} = \alpha \cdot f_b \cdot \sigma^4 \quad (5)$$

This is not a substitution of ρ_m by σ^4. The running component ($\propto \rho_m$) and the ground-state scale ($\propto \sigma^4$) are physically distinct: both follow from α, but one is cosmologically contingent, the other is a QCD constant.

5.3. Non-perturbative curvature response. The curvature $R = \rho_{eff}/m^2_{Pl}$ feeds back through the Schwinger–DeWitt mechanism. The coefficient c_1 determines the cycling suppression. For perturbative fields, $c_1 \sim m^2/(16\pi^2)$; the $1/(16\pi^2)$ loop suppression would spoil the estimate. However, the chiral condensate is non-perturbative. In naive dimensional analysis (NDA) [7]—

the standard power-counting of chiral perturbation theory—non-perturbative hadronic coefficients enter at $O(f_\pi^2)$ without loop suppression, because strong-coupling dynamics fill in the missing factors. We note that this applies NDA beyond its original domain (low-energy constants in the chiral Lagrangian) to the gravitational curvature response of the condensate—a novel extrapolation, though one that follows the same dimensional logic. Since $f_\pi = 93$ MeV $\approx \sigma = 90$ MeV (both set by chiral symmetry breaking, related through the Gell-Mann–Oakes–Renner relation [18]):

$$c_1 \sim f\pi^2 \approx \sigma^2 \quad \text{(NDA estimate, O(1) accuracy)} \quad (6)$$

This is a falsifiable claim. Measurements of the proton's gravitational form factor via deeply virtual Compton scattering [19] demonstrate that the stress-energy structure of hadrons is experimentally accessible; lattice extensions to the pion sector in curved backgrounds would directly test whether $c_1 = \sigma^2$ holds to better than NDA accuracy.

5.4. The cycling suppression. The second-order vacuum correction is $\delta\rho_{vac}^{(2)} = c_1 \cdot R = \sigma^2 \times \alpha f_b \sigma^4/m^2_{Pl} = \alpha f_b \sigma^6/m^2_{Pl}$. The dimensionless cycling factor is $(\sigma/m_{Pl})^2 \approx 1.4 \times 10^{-39}$—the QCD-to-Planck scale ratio squared. Further iterations are negligible.

5.5. Result. Converting to geometric units:

$$\Lambda_0 = \alpha \cdot f\_b \cdot \sigma^6 / m^4\_Pl \quad (7)$$

The two suppression factors—confinement ($\alpha f_b \sim 5 \times 10^{-4}$) and cycling ($(\sigma/m_{Pl})^2 \sim 10^{-39}$)—account for 42 orders of magnitude. The Zel'dovich estimate (σ^4/m^2_{Pl}) is missing both.

6. Numerical evaluation

Inputs: $\sigma = 90$ MeV [13,14]; $f_b = 0.156$ [15]; $m_{Pl} = 2.44 \times 10^{18}$ GeV $= 2.44 \times 10^{21}$ MeV.

With α = 0.005 (derived): $\Lambda_0 \approx 3.0 \times 10^{-52}$ m^{-2}. Ratio to observed (1.1×10^{-52}): 2.8.

With α = 0.003 (observed from fσ_8): $\Lambda_0 \approx 1.8 \times 10^{-52}$ m^{-2}. Ratio: 1.65.

The factor ~1.7 in α propagates linearly into Λ_0. Its source is the nonlinear screening (factor ~3 in Eq. 2). The N-body simulations (Section 3) measure a converged screening of 3.9× at α = 0.03; if applicable at α = 0.005 (a milder modification), the screening factor may be ~5, giving α = 0.003 and ratio ~1. The NDA estimate $c_1 = \sigma^2$ adds an independent O(1) uncertainty. Overall accuracy is O(1)—an NDA-level prediction resolving 42 of 43 orders.

7. Consequences of the action

(i) $n_s = 0.964$ (observed 0.965 [15]); r = 0.004 (LiteBIRD).

(ii) Gravitational wave speed c (GW170817 [20]).

(iii) Effective Poisson source $(1 + 2\alpha)\rho_m$ at galactic scales. The additional vacuum source produces a 1% enhancement in the growth rate of structure, compatible with the S_8 tension.

(iv) Running vacuum $\delta\rho_{vac} = \alpha f_b \rho_m \propto (1+z)^3$, distinct from the constant Λ_0. At z = 12 it was ~2200× present value, enhancing early structure formation (JWST). Observable signatures (modified BAO, ISW) are developed elsewhere.

8. Falsification

The model is falsified by: (1) lattice QCD yielding c_1 inconsistent with σ^2 beyond NDA accuracy; (2) α excluded from 0.001–0.01 at >5σ; (3) r outside 0.002–0.01; (4) $c_{grav} \neq c$ at $>10^{-15}$; (5) a rigorous derivation of $\Lambda = 8\pi G\rho_{vac}$ from a symmetry principle.

9. Discussion

The central result is Eq. (7). The ingredients are standard: Schwinger–DeWitt [10–12], Einstein trace, QCD sigma terms [13,14], Starobinsky [8], and NDA [7]. The novelty is the connection: using the NDA-estimated curvature response of the chiral condensate as the cycling coefficient.

Two O(1) uncertainties are identified: the screening factor (~3, confirmed at ~3.9 by the N-body simulations of Section 3) and the NDA coefficient ($c_1 \sim \sigma^2$). Both are improvable by lattice QCD. Compared to the Zel'dovich estimate (off by 10^{42}), this estimate is off by ~3 using derived α or ~1.7 using observed α. The broader consequences of the action (1)—including galactic rotation curves, the Bullet Cluster morphology, and the Hubble tension—will be developed in companion papers.

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Appendix A. Particle-mesh simulation code

The following Python code implements the particle-mesh N-body algorithm used in Section 3 to measure the nonlinear screening factor. The PM algorithm follows the standard procedure [16]: CIC assignment, source modification $S = \delta[1 + 2\alpha\Theta(\delta)]$, FFT Poisson solve, central-difference forces, CIC interpolation, leapfrog in ln(a). Requirements: NumPy, Numba. Each 128^3 run takes ~96 s on a single CPU. The complete code (pm_sim.py) will be deposited on Zenodo upon acceptance.

```
"""pm_sim.py - PM N-body for gravitating vacuum model."""
import numpy as np; from numba import njit

# Cosmology (Planck 2018)
Om, Ob, h0, sig8 = 0.315, 0.0493, 0.674, 0.811
L, N, Nsteps, z_i = 256.0, 128, 150, 49.0

def transfer_EH(k):
    """Eisenstein-Hu (1998) zero-baryon transfer function."""
    Omh2, Obh2 = Om*h0**2, Ob*h0**2
```

```
    th = 2.725/2.7; zeq = 2.5e4*Omh2*th**(-4)
    keq = 7.46e-2*Omh2*th**(-2)
    b1 = 0.313*Omh2**(-0.419)*(1+0.607*Omh2**0.674)
    b2 = 0.238*Omh2**0.223
    zd = 1291*Omh2**0.251/(1+0.659*Omh2**0.828)
    zd *= (1+b1*Obh2**b2)
    s = 2/(3*keq)*np.sqrt(6/(31.5e3*Obh2*th**(-4)
        *(1e3/zeq)))
    q = k/(13.41*keq)
    ac = (46.9*Omh2)**0.670*(1+(32.1*Omh2)**(-0.532))
    C = 14.2/ac + 386/(1+69.9*q**1.08)
    return np.log(np.e+1.8*q)/(np.log(np.e+1.8*q)
        +C*q**2)


@njit
def cic_deposit(pos, Ng, Lbox):
    """Cloud-in-Cell mass assignment."""
    rho = np.zeros((Ng,Ng,Ng)); inv = Ng/Lbox
    for p in range(pos.shape[0]):
        x,y,z = pos[p,0]*inv, pos[p,1]*inv, pos[p,2]*inv
        i,j,k = int(x)%Ng, int(y)%Ng, int(z)%Ng
        dx,dy,dz = x-int(x), y-int(y), z-int(z)
        for di in range(2):
            for dj in range(2):
                for dk in range(2):
                    w = ((1-dx,dx)[di] * (1-dy,dy)[dj]
                        * (1-dz,dz)[dk])
                    rho[(i+di)%Ng,(j+dj)%Ng,
                        (k+dk)%Ng] += w
    return rho


def solve_poisson(rho, alpha, a):
    """FFT Poisson with source modification."""
    delta = rho/rho.mean() - 1.0
    src = delta.copy()
    src[delta>0] *= (1 + 2*alpha)  # Eq.(1)
    kf = 2*np.pi/L
    kx = np.fft.fftfreq(N,d=1.0)*N*kf
    ky, kz = kx.copy(), np.fft.rfftfreq(N,d=1.0)*N*kf
    KX,KY,KZ = np.meshgrid(kx,ky,kz,indexing="ij")
    K2 = KX**2+KY**2+KZ**2; K2[0,0,0] = 1.0
    sk = np.fft.rfftn(src); pk = -sk/K2; pk[0,0,0]=0
    pf = 1.5*Om*(100*h0/3e5)**2/a
    fx = np.fft.irfftn(-1j*KX*pk*pf, s=(N,N,N))
    fy = np.fft.irfftn(-1j*KY*pk*pf, s=(N,N,N))
    fz = np.fft.irfftn(-1j*KZ*pk*pf, s=(N,N,N))
    return np.stack([fx,fy,fz],axis=-1), delta


def run(alpha=0.0, seed=42):
    """Leapfrog in ln(a), z=49 to z=0."""
    # [IC generation and time integration; see
    #  full pm_sim.py at Zenodo deposit]
    # Returns: sigma8, void_fraction_history
    pass
```

Note: The cic_interp function (CIC interpolation of forces to particles) mirrors cic_deposit with accumulation replaced by weighted sampling. The full code including Zel'dovich IC generation, σ_8 computation, and multi-seed variance analysis is available at the Zenodo DOI to be assigned upon acceptance.

Declaration of interests

☒The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: