Universal Modular Dynamics: From Spectral Distributions to Physical Reality

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Abstract

We develop a structural framework in which physical reality is derived from the spectral properties of quantum states. Within the approach of Universal Modular Dynamics (UMD), the density operator ρ is taken as the fundamental object, and its modular generator $K = -\log \rho$ provides a representation of informational content.

We show that the distribution of modular energies, defined by the spectrum of K, serves as a primary structure from which higher-level physical organization emerges. In particular, we demonstrate how spectral distributions give rise to stable structures, induce locality through optimal partitioning, and generate geometric relations via correlation-based distances.

Dynamics is introduced as an intrinsic ordering of states under modular evolution, while renormalization appears as a spectral phenomenon governed by statistical properties of the modular spectrum. This establishes a connection between informational structure and physical scale.

The resulting construction defines a sequential chain of emergence,

$$\rho \;\rightarrow\; p(k) \;\rightarrow\; \text{structure} \;\rightarrow\; \text{locality} \;\rightarrow\; \text{geometry} \;\rightarrow\; \text{dynamics} \;\rightarrow\; \text{scale},$$

in which each level arises from properties of the previous one without introducing external assumptions.

This work provides a unified perspective in which physical structure is understood as an emergent consequence of spectral information encoded in quantum states.

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1 Introduction

The origin of physical structure remains one of the central questions in theoretical physics. Conventional approaches typically assume the existence of fundamental entities such as spacetime, fields, or interactions, from which observable phenomena are derived.

However, this strategy leaves open a deeper problem: whether such structures are truly fundamental, or whether they themselves can be understood as emergent.

In this work, we pursue the latter perspective within the framework of Universal Modular Dynamics (UMD). In this approach, the density operator ρ is taken as the primary object, encoding the full informational content of a system. No prior assumptions about spacetime, geometry, or dynamical laws are imposed at the outset.

The key idea of the present work is that the spectral properties of the modular generator

$$K = -\log \rho \tag{1}$$

provide a sufficient basis for the emergence of higher-level physical structures.

Rather than treating individual spectral values, we focus on the distribution of modular energies. This distribution defines a global structural object that captures the informational organization of the state.

We demonstrate that a sequence of increasingly complex structures can be derived from this spectral distribution. In particular:

- stable spectral modes define internal structure,
- optimal partitions give rise to locality,
- correlation patterns induce geometric relations,
- modular evolution introduces an intrinsic notion of dynamics,
- spectral statistics determine characteristic scales through renormalization.

These elements combine into a coherent chain of emergence, in which each level arises from properties of the preceding one.

An important feature of this construction is that no external structures are introduced at any stage. Geometry, dynamics, and scale all emerge as consequences of the informational content encoded in ρ.

The resulting framework provides a unified perspective in which physical reality is understood as a manifestation of spectral organization.

The structure of the paper is as follows. In Section 1 we introduce the informational starting point and define the spectral representation. Section 2 develops the distributional description of information. Section 3 shows how structure emerges from spectral modes. Sections 4 and 5 establish the emergence of locality and geometry. Section 6 introduces dynamics as modular evolution. Section 7 formulates renormalization as a spectral law. Sections 8 and 9 extend the framework to matter-like structures and large-scale organization. Section 10 presents a synthesis of the full chain of emergence.

2 Starting Point: Informational State

2.1 Minimal Ontological Structure

We begin from the assumption that the fundamental description of a physical system is given by a density operator ρ acting on a Hilbert space $\mathcal{H}$:

$$\rho \geq 0, \qquad \mathrm{Tr}(\rho) = 1. \tag{2}$$

No additional structure, such as spacetime, geometry, or fields, is assumed at this level. The operator ρ encodes all physically relevant information about the system.

This choice should be understood as a minimal informational starting point, rather than as an a priori commitment to the standard formulation of quantum mechanics.

2.2 Modular Generator

Given a full-rank density operator ρ, we define the modular generator

$$K = -\log \rho. \tag{3}$$

The operator K provides a representation of the informational content of the state. Its spectral properties reflect the internal organization of the system.

In particular, if

$$\rho = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i|, \tag{4}$$

then

$$K = \sum_i k_i |\psi_i\rangle\langle\psi_i|, \qquad k_i = -\log \lambda_i. \tag{5}$$

2.3 From Spectrum to Distribution

The set of modular eigenvalues $\{k_i\}$ defines a spectral structure associated with the state.

Rather than treating these values individually, we interpret them collectively as defining a distribution:

$$p(k) = \frac{1}{N} \sum_i \delta(k - k_i), \tag{6}$$

where $N = \dim(\mathcal{H})$.

This distribution will serve as the primary object in the subsequent construction.

2.4 Interpretation

The transition from ρ to K, and from K to $p(k)$, represents a shift from a state-based description to a distributional description of informational content.

In this framework:

- the density operator ρ encodes the full state of the system,
- the modular generator K provides a logarithmic representation of this state,
- the distribution $p(k)$ captures the global structure of modular energies.

This shift allows one to describe physical properties in terms of statistical features of the spectrum, rather than in terms of individual components.

2.5 Scope and Status

At this stage, no physical interpretation beyond the informational level is imposed.

The construction defines a mapping

$$\rho \longrightarrow K \longrightarrow p(k), \tag{7}$$

which will serve as the starting point for the emergence of higher-level structures in the following sections.

All subsequent results will be derived from properties of $p(k)$ and its evolution under modular dynamics.

3 Spectral Information as Primary Structure

3.1 Spectral Distribution

Given the modular generator

$$K = -\log \rho, \tag{8}$$

with eigenvalues $\{k_i\}$, we associate to the state a spectral distribution

$$p(k) = \frac{1}{N} \sum_i \delta(k - k_i), \tag{9}$$

where $N = \dim(\mathcal{H})$.

This distribution encodes the global structure of modular energies associated with the state.

3.2 Statistical Descriptors

The distribution $p(k)$ can be characterized by standard statistical quantities:

$$\mu_K = \int k\, p(k)\, dk = \frac{1}{N} \sum_i k_i, \tag{10}$$

$$\sigma_K^2 = \int (k - \mu_K)^2 p(k)\, dk = \frac{1}{N} \sum_i (k_i - \mu_K)^2. \tag{11}$$

In addition, quantile-based descriptors capture the structure of the distribution beyond its central region. For a given $\alpha \in (0, 1)$, we define the quantile q_α by

$$\int_{-\infty}^{q_\alpha} p(k)\, dk = \alpha. \tag{12}$$

In particular, the width

$$\Delta_K = q_{90} - q_{10} \tag{13}$$

provides a robust measure of spectral spread.

3.3 Distributional Representation of Information

The distribution $p(k)$ provides a representation of the informational content of the state at a global level.

Rather than focusing on individual eigenvalues, the structure of $p(k)$ captures:

- the overall scale of modular energies through μ_K,
- fluctuations through σ_K^2,
- the influence of rare configurations through tail behavior.

This perspective allows one to describe physical properties in terms of statistical features of the spectrum.

3.4 Equivalence Class of States

Different density operators may share the same spectral distribution $p(k)$.

Such states form an equivalence class with respect to distributional structure:

$$\rho_1 \sim \rho_2 \quad \text{if} \quad p_1(k) = p_2(k). \tag{14}$$

Within this class, quantities that depend only on $p(k)$ are invariant.

This observation motivates the interpretation of $p(k)$ as a primary descriptor of the state at the level of spectral information.

3.5 Robustness Under Perturbations

Small perturbations of the state $\rho \to \rho + \delta\rho$ typically lead to small deformations of the distribution $p(k)$.

Statistical descriptors such as μ_K, σ_K^2, and Δ_K are therefore stable under such perturbations.

This robustness makes $p(k)$ a suitable object for describing emergent structures.

3.6 Scope and Status

At this stage, the construction remains purely informational.

No geometric, dynamical, or physical interpretation is imposed. The distribution $p(k)$ is introduced as a structural object derived from the state.

In the following sections, we will show how progressively richer structures can emerge from properties of $p(k)$ and its evolution.

4 Emergence of Structure (Spectral Modes)

4.1 From Distribution to Structure

The spectral distribution $p(k)$ introduced in the previous section encodes the global informational content of the state.

We now investigate how internal structure can emerge from this distribution.

Rather than treating $p(k)$ as a homogeneous object, we analyze its internal organization.

4.2 Clustering of the Spectrum

A generic distribution $p(k)$ may exhibit non-uniform features such as peaks, plateaus, or separated regions of support.

We define a spectral cluster as a subset of eigenvalues $\{k_i\}$ forming a locally dense region in the spectrum:

$$\mathcal{C}_\alpha = \{k_i \mid k_i \in \text{region of enhanced density}\}. \tag{15}$$

Such clusters can be identified through standard density-based criteria or through stability under perturbations.

4.3 Definition of Spectral Modes

We define a spectral mode as a cluster of modular energies that remains stable under small deformations of the state.

More precisely, a subset $\mathcal{C}_\alpha$ defines a mode if:

- it persists under perturbations $\rho \to \rho + \delta\rho$,
- its statistical descriptors (mean, variance) vary smoothly,
- it is separated from other clusters by regions of lower spectral density.

Each spectral mode can be characterized by its internal distribution

$$p_\alpha(k) = \frac{1}{|\mathcal{C}_\alpha|} \sum_{k_i \in \mathcal{C}_\alpha} \delta(k - k_i). \tag{16}$$

4.4 Structural Interpretation

Spectral modes provide a natural notion of internal structure.

Instead of assigning structure externally, we interpret each mode as a coherent sector of the state.

In this sense:

$$\text{structure} \equiv \text{stable spectral mode.} \tag{17}$$

This identification is purely internal and does not rely on spatial or dynamical assumptions.

4.5 Stability Under Evolution

Under modular evolution $\rho(\lambda)$, the distribution $p(k;\lambda)$ changes.

A spectral mode $\mathcal{C}_\alpha$ is said to be dynamically stable if its identity persists over a range of λ:

$$\mathcal{C}_\alpha(\lambda_1) \approx \mathcal{C}_\alpha(\lambda_2). \tag{18}$$

Such stability indicates that the corresponding structure is robust under the flow.

4.6 Interaction Between Modes

Different spectral modes may interact through the evolution of the distribution.

This interaction is reflected in:

- merging or splitting of clusters,
- transfer of spectral weight between regions,
- changes in relative prominence of modes.

These processes provide a mechanism for structural reorganization.

4.7 Minimal Structural Hierarchy

The existence of multiple spectral modes induces a hierarchical organization of the state.

At the lowest level, individual eigenvalues define microscopic structure.

At an intermediate level, spectral modes define mesoscopic structure.

At the global level, the full distribution $p(k)$ defines macroscopic structure.

4.8 Scope and Status

At this stage, structure is defined purely in spectral terms.

No reference to spatial organization, locality, or physical interactions has been introduced.

The construction shows that structured organization can emerge directly from the distribution $p(k)$, without additional assumptions.

In the following section, we will show how locality emerges from relations between such spectral structures.

5 Emergence of Locality

5.1 From Structure to Partition

The spectral modes introduced in the previous section define internal structure within the state.

We now consider how such structure can give rise to a notion of locality.

To this end, we introduce a partition P of the system into subsystems:

$$P = \{X_1, X_2, \dots, X_m\}. \tag{19}$$

Each subset $X \in P$ corresponds to a subsystem with reduced state

$$\rho_X = \mathrm{Tr}_{\bar{X}}(\rho). \tag{20}$$

5.2 Distinguishability-Based Criterion

To quantify the quality of a partition, we define a functional based on relative entropy:

$$T_P(\rho) = D\left(\rho \,\middle\|\, \bigotimes_{X \in P} \rho_X\right). \tag{21}$$

This quantity measures the deviation of the state from a product structure with respect to the partition P.

A small value of $T_P(\rho)$ indicates that correlations between subsystems are weak.

5.3 Optimal Partition

We define the optimal partition P^* as the one minimizing the distinguishability functional, with a penalty term controlling the complexity:

$$P^* = \arg\min_P \Big(T_P(\rho) + \eta\,|P|\Big), \tag{22}$$

where $|P|$ is the number of subsystems and $\eta > 0$ is a fixed parameter.

The penalty term prevents trivial over-fragmentation.

5.4 Definition of Locality

We define locality as a phase property of the state:

> **A state is local if there exists a partition P^* such that $T_{P^*}(\rho)$ is sufficiently small and stable under perturbations.**

Stability here means that the optimal partition does not change under small deformations of the state.

5.5 Relation to Spectral Structure

The optimal partition is not imposed externally, but emerges from the internal organization of the state.

In particular, spectral modes identified in Section 3 tend to align with subsystems in the optimal partition.

Thus, locality arises as a relational property between spectral structures.

5.6 Locality as a Phase

Different states may exhibit qualitatively different partition structures.

We interpret these differences as distinct phases:

- *Local phase*: stable partition with weak inter-subsystem correlations,
- *Non-local phase*: no stable partition, or strong global correlations,
- *Critical regime*: competing partitions with comparable values of the functional.

5.7 Dynamical Stability

Under modular evolution $\rho(\lambda)$, the optimal partition may change.

A local phase is characterized by the persistence of P^* over a range of λ:

$$P^*(\lambda_1) \approx P^*(\lambda_2). \tag{23}$$

Transitions between phases correspond to qualitative changes in the partition structure.

5.8 Scope and Status

Locality is defined here without reference to spatial coordinates.

It emerges as a property of the correlation structure of the state.

In the next section, we will show how geometric notions can be constructed from such locality relations.

6 Emergent Geometry

6.1 From Locality to Geometry

The notion of locality introduced in the previous section provides a partition of the system into subsystems with weak mutual correlations.

We now show how geometric structure can emerge from these locality relations.

6.2 Correlation-Based Distance

Given two subsystems X and Y in the optimal partition P^*, we define a measure of correlation using mutual information:

$$I(X:Y) = S(\rho_X) + S(\rho_Y) - S(\rho_{XY}), \tag{24}$$

where $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$ is the von Neumann entropy.

We define an effective distance between subsystems by

$$d(X,Y) = \frac{1}{\big(I(X:Y) + \varepsilon\big)^{\nu}}, \tag{25}$$

where $\varepsilon > 0$ is a regularization parameter and $\nu > 0$ controls the scaling.

6.3 Metric Properties

The function $d(X,Y)$ satisfies the following properties:

- $d(X,Y) \geq 0$,
- $d(X,Y) = d(Y,X)$,
- $d(X,Y) \to \infty$ as $I(X:Y) \to 0$.

While the triangle inequality is not guaranteed in general, the function defines a meaningful notion of relational distance.

6.4 Graph Representation

The set of subsystems $X \in P^*$, together with the distances $d(X,Y)$, defines a weighted graph:

$$G = (V, E), \tag{26}$$

where vertices correspond to subsystems and edges are weighted by $d(X,Y)$.

This graph encodes the relational structure of the system.

6.5 Emergence of Geometric Structure

Under suitable conditions, such as smooth variation of correlations and approximate locality, the graph G can be embedded into a continuous space.

In this case, the distance function $d(X, Y)$ induces an effective metric on this space.

Thus, geometry emerges as an effective description of the correlation structure.

6.6 Geometric Phases

Different states may give rise to different graph structures and, consequently, to different effective geometries.

We interpret these as distinct geometric phases.

Transitions between such phases correspond to qualitative changes in the correlation structure.

6.7 Stability and Coarse-Graining

Geometric structure is meaningful only if it is stable under coarse-graining and perturbations.

This requires that the distance function $d(X, Y)$ varies smoothly under changes of the state.

Such stability ensures that the emergent geometry is not an artifact of microscopic details.

6.8 Scope and Status

At this stage, geometry is defined as an emergent, relational construct.

It is derived entirely from the correlation structure of the state, without reference to any background spacetime.

In the next section, we will introduce dynamics in this framework and show how temporal ordering arises from modular evolution.

7 Emergent Dynamics (Ordering)

7.1 Absence of Fundamental Time

Up to this point, no notion of time has been introduced. The construction has been entirely based on the informational structure of the state and its spectral properties.

We now address the emergence of dynamics within this framework.

7.2 Modular Evolution

Given a density operator ρ, the modular generator

$$K = -\log \rho \tag{27}$$

provides a natural candidate for defining an intrinsic flow.

However, since $[K, \rho] = 0$, this generator alone does not produce non-trivial evolution.

To obtain a non-degenerate dynamics, we introduce a reference state σ and define the relative modular generator

$$K_{\rho|\sigma} = -\log \rho + \log \sigma. \tag{28}$$

We then define the evolution equation

$$\frac{d\rho}{d\lambda} = -i[K_{\rho|\sigma}, \rho] + \mathcal{D}[\rho], \tag{29}$$

where $\mathcal{D}$ is a completely positive trace-preserving (CPTP) dissipative term.

7.3 Ordering Parameter

The parameter λ is not interpreted as physical time.

Instead, it provides an ordering of states along a trajectory in the space of density operators:

$$\rho \to \rho(\lambda). \tag{30}$$

Thus, λ defines a direction of evolution without presupposing a temporal background.

7.4 Monotonic Quantities

The evolution is constrained by the monotonicity of informational quantities.

In particular, for a fixed reference state σ, the relative entropy satisfies

$$\frac{d}{d\lambda} D(\rho(\lambda)\|\sigma) \leq 0, \tag{31}$$

under CPTP dynamics.

This provides a notion of irreversibility and establishes a preferred direction of evolution.

7.5 Emergence of Temporal Structure

The existence of a monotonic functional allows one to define an effective notion of time.

We interpret the parameter λ as an ordering variable that induces a temporal structure through the sequence of states.

In this sense:

$$\text{time} \;\equiv\; \text{ordered evolution of states under modular flow.} \tag{32}$$

7.6 Relation to Spectral Evolution

Under the evolution $\rho(\lambda)$, the modular generator becomes

$$K(\lambda) = -\log \rho(\lambda), \tag{33}$$

and the spectral distribution evolves:

$$p(k;\lambda). \tag{34}$$

Changes in $p(k;\lambda)$ correspond to dynamical processes.

Thus, dynamics can be described in terms of spectral redistribution.

7.7 Stability and Fixed Points

A state ρ^* is a fixed point of the evolution if

$$\frac{d\rho}{d\lambda} = 0. \tag{35}$$

Such states correspond to stationary spectral distributions and define equilibrium configurations.

7.8 Scope and Status

Dynamics is introduced here without reference to external time or Hamiltonian evolution.

It arises as an intrinsic property of the state space, defined by modular and dissipative structures.

In the next section, we will show how renormalization emerges as a spectral property of this dynamics.

8 Renormalization as Spectral Law

8.1 Renormalization in the UMD Framework

Within the modular formulation of dynamics introduced in the previous section, the evolution of the state $\rho(\lambda)$ induces a corresponding evolution of the modular generator

$$K(\lambda) = -\log \rho(\lambda), \tag{36}$$

and its spectral distribution

$$p(k; \lambda). \tag{37}$$

Renormalization is interpreted as a transformation of this spectral distribution along the flow parameter λ.

8.2 Spectral Redistribution

The evolution of $p(k; \lambda)$ typically involves redistribution of spectral weight, leading to changes in its statistical structure.

In particular, the variance

$$\mathrm{Var}(K(\lambda)) = \int (k - \mu_K)^2 p(k; \lambda)\, dk \tag{38}$$

provides a measure of the effective spectral width.

Changes in this quantity reflect large-scale transformations in the structure of the state.

8.3 Critical Scale

We define the critical scale λ^* as the point at which the spectral redistribution undergoes a qualitative change.

Operationally, this can be identified through the behavior of the variance:

$$\frac{d}{d\lambda} \mathrm{Var}(K(\lambda)) \Big|_{\lambda^*} = 0. \tag{39}$$

This condition marks a transition in the evolution of the spectral structure.

8.4 Spectral Law of Renormalization

The analysis presented in the companion work establishes that the critical scale is determined by the spectral distribution of the modular generator.

In particular, to leading order, one finds the relation

$$\lambda^*(\rho) \sim \frac{1}{\sqrt{\mathrm{Var}(-\log \rho)}}. \tag{40}$$

This expression defines a spectral law of renormalization, relating the emergence of a characteristic scale to the statistical properties of the modular spectrum.

8.5 Interpretation

The above relation implies that renormalization is not governed by external scale transformations, but by intrinsic properties of the state.

In particular:

- a broader spectral distribution leads to a smaller critical scale,
- a narrower distribution delays the onset of structural change,
- critical behavior is determined by collective spectral features.

8.6 Relation to Emergent Structure

The critical scale λ^* marks the point at which previously identified structures undergo reorganization.

This affects:

- spectral modes (Section 3),
- locality structure (Section 4),
- geometric relations (Section 5).

Thus, renormalization acts as a unifying mechanism governing transitions across multiple levels of emergent structure.

8.7 Scope and Status

The spectral law of renormalization is used here as an established result.

Its detailed derivation, including operator formulations and entropy-based interpretations, is presented in a separate work.

In the present context, it serves as a central link connecting spectral information to large-scale structural behavior.

In the following sections, we explore how this mechanism extends to higher levels of organization.

9 Emergence of Matter

9.1 Motivation

The previous sections have established a sequence of emergent structures derived from the spectral properties of the modular generator $K = -\log \rho$.

We now address whether higher-level physical structures, commonly associated with matter, can be related to this framework.

9.2 Spectral Stability as a Criterion

We propose that physically relevant structures correspond to stable configurations in the modular spectrum.

A spectral configuration is said to be stable if it satisfies:

- persistence under modular evolution $\rho(\lambda)$,
- robustness under perturbations $\rho \to \rho + \delta\rho$,
- structural coherence across multiple scales of λ.

Such configurations are naturally associated with the spectral modes introduced in Section 3.

9.3 Definition of Matter-Like Structures

We define a matter-like structure as a spectral mode or a collection of modes that remains stable under both dynamical evolution and structural reorganization.

Formally, a subset $\mathcal{C} \subset \mathrm{Spec}(K)$ defines a matter-like structure if

$$\mathcal{C}(\lambda_1) \approx \mathcal{C}(\lambda_2) \tag{41}$$

for a range of λ, and its statistical properties remain bounded and continuous.

9.4 Interpretation

In this framework, matter is not introduced as a primitive concept.

Instead, it emerges as a manifestation of stable spectral organization.

In particular:

- spectral modes correspond to coherent structures,
- stability under flow provides persistence,
- robustness under perturbations ensures physical relevance.

This interpretation aligns matter with the existence of dynamically stable configurations in the space of states.

9.5 Stability and Lifetime of Matter-Like Structures

The stability criterion introduced in Section 3 allows one to refine the interpretation of matter-like structures.

Given a spectral mode $\mathcal{C}$, we define its effective leakage rate as

$$\Gamma \sim \sum_{i\in\mathcal{C},\, j\notin\mathcal{C}} |K_{ij}|^2, \tag{42}$$

where K is the modular generator in its eigenbasis.

This quantity characterizes the strength of coupling between the mode and the rest of the spectrum.

A small value of Γ indicates that transitions out of the mode are strongly suppressed.

This naturally defines an effective lifetime:

$$\tau \sim \frac{1}{\Gamma}. \tag{43}$$

9.6 Interpretation of Lifetime

Within this framework, long-lived structures correspond to spectrally isolated and dynamically stable configurations.

Such configurations remain localized in the space of modular energies and are weakly coupled to other spectral regions.

In particular:

- strong spectral separation leads to small Γ,
- weak coupling suppresses transitions,
- suppressed transitions imply long persistence.

This provides a structural mechanism for the emergence of long-lived physical entities.

9.7 Scope of the Result

The present formulation does not aim to reproduce specific lifetimes of known particles.

Rather, it establishes a general principle:

> *Long-lived structures correspond to spectrally isolated configurations with suppressed transition amplitudes.*

This principle is consistent with the broader framework and suggests a pathway for connecting spectral structure to physical stability.

9.8 Renormalization and Stability

The stability of matter-like structures is closely related to the renormalization behavior introduced in Section 7.

As the state evolves under the modular flow $\rho(\lambda)$, the spectral distribution $p(k;\lambda)$ undergoes redistribution. This process affects both the internal structure of spectral modes and their coupling to the rest of the spectrum.

In particular, the effective leakage rate

$$\Gamma(\lambda) \sim \sum_{i\in\mathcal{C},\, j\notin\mathcal{C}} |K_{ij}(\lambda)|^2 \tag{44}$$

becomes a function of the flow parameter.

Near the critical scale λ^*, defined by

$$\frac{d}{d\lambda}\mathrm{Var}(K(\lambda))\Big|_{\lambda^*} = 0, \tag{45}$$

the spectral structure undergoes qualitative reorganization.

9.9 Critical Behavior of Stability

This reorganization has direct implications for stability:

- spectral modes may merge or split,
- couplings between modes can increase or decrease,
- previously isolated configurations may become unstable.

As a result, the leakage rate $\Gamma(\lambda)$ can change significantly in the vicinity of λ^*.

9.10 Interpretation

The critical scale λ^* therefore marks a transition not only in structural organization, but also in stability properties.

In particular:

- for $\lambda < \lambda^*$, stable structures may persist with small Γ,
- near λ^*, transitions between modes become enhanced,
- for $\lambda > \lambda^*$, new stable configurations may emerge.

This establishes a direct link between renormalization and the lifetime of matter-like structures.

9.11 Scope

The above relation provides a structural mechanism by which stability is controlled by spectral evolution.

While no quantitative lifetime predictions are made, the framework suggests that critical phenomena in the spectral distribution govern both the formation and the decay of stable configurations.

9.12 Relation to Dynamics and Geometry

Matter-like structures interact with previously defined levels of organization:

- through dynamics, as they persist or transform under modular evolution,
- through geometry, as they are embedded in the correlation-based structure defined in Section 5,
- through renormalization, as they undergo reorganization near critical scales.

Thus, matter appears as a higher-level manifestation of the same underlying spectral structure.

9.13 Limitations

The present formulation does not identify specific particle species or field content.

It provides a structural definition of matter based on stability, rather than a detailed phenomenological model.

Further work is required to establish explicit connections to known physical systems.

9.14 Scope and Status

This section introduces a controlled interpretation extending the previous formal results.

The definition of matter-like structures is consistent with the framework but does not constitute a complete physical theory of matter.

It should be understood as a structural step in the chain of emergence.

10 Large-Scale Structures

10.1 From Local Structure to Global Organization

The previous sections have established a sequence of emergent structures, from spectral distributions to matter-like configurations.

We now consider how such structures may extend to large-scale organization.

At this level, we do not introduce new principles. Instead, we analyze how previously defined mechanisms behave when applied to extended systems.

10.2 Scaling and Hierarchical Organization

The existence of stable spectral modes suggests a natural hierarchy of structures.

Under modular evolution and renormalization, these structures may undergo coarse-graining:

$$\mathcal{C}_\alpha \longrightarrow \tilde{\mathcal{C}}_\beta, \tag{46}$$

where $\tilde{\mathcal{C}}_\beta$ represents an effective structure at a larger scale.

This process generates a hierarchy in which:

- microscopic spectral modes define local structures,
- intermediate modes define mesoscopic organization,
- large-scale modes correspond to global structure.

10.3 Emergent Macroscopic Geometry

As discussed in Section 5, geometric structure arises from correlations between subsystems.

At large scales, this structure is determined by the collective behavior of many interacting spectral modes.

The resulting geometry reflects the coarse-grained correlation structure, rather than microscopic details.

10.4 Collective Effects

Large-scale organization is characterized by collective phenomena, including:

- long-range correlations,
- alignment of spectral modes,
- emergence of dominant structures.

These effects are not imposed, but arise from the accumulation of local interactions and spectral redistribution.

10.5 Relation to Observed Structures

The framework suggests that large-scale physical structures can be interpreted as manifestations of stable and interacting spectral configurations.

This includes:

- extended spatial organization,
- clustering of matter-like structures,
- large-scale coherence patterns.

However, no direct identification with specific astrophysical or cosmological objects is made at this stage.

10.6 Interpretational Remarks

It is natural to consider whether phenomena such as dark sectors or cosmological structures may be related to hidden or weakly coupled spectral components.

Within the present framework, such possibilities correspond to:

- spectral sectors that do not significantly contribute to observable correlations,
- weakly interacting modes that influence global structure indirectly.

These considerations remain speculative and are not required for the internal consistency of the construction.

10.7 Limitations

The present analysis does not provide a quantitative model of cosmology.

It establishes a structural pathway by which large-scale organization may arise from spectral properties, but does not derive specific predictions.

10.8 Scope and Status

This section extends the chain of emergence to large-scale organization in a controlled manner.

The results should be understood as qualitative consequences of the framework, consistent with previous sections but not constituting a complete theory of large-scale physics.

11 Synthesis: Chain of Emergence

11.1 Summary of the Construction

The previous sections have established a sequence of structures derived from the spectral properties of the modular generator.

Each level of organization has been introduced as a consequence of the preceding one, without the need for additional external assumptions.

11.2 Chain of Emergence

The full construction can be summarized as the following chain:

$$\rho \longrightarrow K = -\log \rho \longrightarrow p(k) \longrightarrow \text{spectral modes} \longrightarrow \text{locality} \longrightarrow \text{geometry} \longrightarrow \text{dynamics} \longrightarrow \lambda^* \longrightarrow \tag{47}$$

Each transition in this chain corresponds to a well-defined operation or property:

- spectral transformation: $\rho \to K \to p(k)$,
- structural emergence: clustering of $p(k)$,
- relational organization: optimization of partitions,
- geometric construction: correlation-based distance,
- dynamical ordering: modular evolution,
- scale formation: spectral law of renormalization,
- stabilization: persistence of spectral configurations,
- hierarchical extension: coarse-grained organization.

11.3 Unifying Principle

The construction suggests a unifying principle:

> **Physical structure emerges from the spectral organization of informational states.**

In this view, all levels of physical description arise from properties of the distribution $p(k)$ and its evolution.

11.4 Internal Consistency

An important feature of the construction is its internal closure.

Each level is defined in terms of previously introduced quantities, and no additional primitives are required.

In particular:

- geometry is derived from correlations,

- dynamics is defined through modular flow,
- scale is determined by spectral properties,
- structure is identified through stability.

11.5 Interpretation

The resulting framework does not posit physical structures as fundamental.

Instead, it provides a mechanism by which such structures can arise from informational content.

The role of the density operator ρ is therefore dual:

- it encodes the state of the system,
- it serves as the origin of emergent physical organization.

11.6 Scope and Status

The chain of emergence presented here should be understood as a structural synthesis of the framework.

Each step is supported by definitions and constructions introduced in the previous sections.

At the same time, the full physical interpretation of the resulting structures remains an open direction for further investigation.

12 Discussion

12.1 Summary of Results

In this work, we have constructed a sequential framework in which physical structures emerge from the spectral properties of quantum states.

Starting from the density operator ρ, we introduced the modular generator $K = -\log \rho$ and its spectral distribution $p(k)$ as the primary informational object. We then demonstrated how increasingly complex structures arise from this distribution.

In particular, we showed that:

- spectral modes define internal structure,
- optimal partitions give rise to locality,
- correlation patterns induce geometric relations,
- modular evolution introduces an intrinsic notion of dynamics,
- spectral statistics determine characteristic scales through renormalization,
- stable configurations correspond to matter-like structures,
- hierarchical organization extends these structures to larger scales.

These elements combine into a coherent chain of emergence.

12.2 Conceptual Implications

The construction suggests a shift in perspective regarding the foundations of physical theory.

Rather than treating geometry, dynamics, or matter as fundamental, these structures appear as consequences of the informational content of the state.

In this framework, the spectral distribution $p(k)$ plays a central role, serving as the organizing principle underlying all higher-level structures.

12.3 Relation to Existing Approaches

The present framework is consistent with, but distinct from, existing approaches based on entanglement geometry and information-theoretic formulations of physics.

In particular, the emergence of geometry from correlations aligns with previous work, while the explicit role of the modular spectrum provides a new organizing principle.

12.4 Limitations

The construction is primarily structural and does not provide detailed phenomenological models.

In particular:

- no explicit identification of particle species is made,
- the geometric construction is effective rather than fundamental,
- large-scale structures are described qualitatively,
- quantitative comparison with experimental systems remains to be developed.

12.5 Outlook

The framework opens several directions for further investigation.

Future work may include:

- quantitative modeling of specific physical systems,
- extension to infinite-dimensional settings,
- refinement of the relation between spectral structure and geometry,
- exploration of observable consequences of the spectral law of renormalization.

Overall, the results suggest that spectral information provides a viable foundation for a unified description of physical structure.

A Appendix A: Spectral Representation

Let ρ be a density operator with spectral decomposition

$$\rho = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i|. \tag{48}$$

The modular generator is defined as

$$K = -\log\rho = \sum_i k_i |\psi_i\rangle\langle\psi_i|, \quad k_i = -\log\lambda_i. \tag{49}$$

For an observable O, the commutator takes the form

$$[K, O] = \sum_{i,j} (k_i - k_j) O_{ij} |\psi_i\rangle\langle\psi_j|. \tag{50}$$

The Frobenius norm is given by

$$\|[K, O]\|_F^2 = \sum_{i,j} (k_i - k_j)^2 |O_{ij}|^2. \tag{51}$$

This expression shows that dynamical response is governed by pairwise spectral differences. The variance of the modular spectrum is defined as

$$\mathrm{Var}(K) = \frac{1}{N} \sum_i (k_i - \mu_K)^2. \tag{52}$$

This quantity provides a measure of effective spectral width.

B Appendix B: Numerical Protocol

We summarize the numerical procedure used to analyze spectral evolution.

State Construction

A family of states $\rho(\lambda)$ is generated through modular evolution:

$$\frac{d\rho}{d\lambda} = -i[K_{\rho|\sigma}, \rho] + \mathcal{D}[\rho]. \tag{53}$$

Spectral Analysis

At each value of λ, the modular generator is computed:

$$K(\lambda) = -\log \rho(\lambda). \tag{54}$$

The spectral distribution $p(k; \lambda)$ is obtained from the eigenvalues of $K(\lambda)$.

Statistical Quantities

We compute:

- mean μ_K,
- variance $\mathrm{Var}(K)$,
- quantiles q_{10}, q_{90}.

Critical Scale

The critical scale λ^* is identified through the condition

$$\frac{d}{d\lambda} \mathrm{Var}(K(\lambda)) = 0. \tag{55}$$

Stability Checks

The results are verified under:

- perturbations of the initial state,
- variations of parameters,
- smoothing of spectral data.

C Conclusion

We have presented a framework in which physical structures emerge from the spectral properties of quantum states.

Starting from the density operator ρ, we introduced the modular generator $K = -\log \rho$ and its spectral distribution as the primary object of analysis. We demonstrated that a sequence of structures—ranging from internal organization to large-scale behavior—can be derived from this spectral information.

The resulting construction defines a coherent chain of emergence, in which each level arises from properties of the preceding one without introducing external assumptions.

A central outcome of this work is the identification of spectral organization as a unifying principle underlying physical structure. In this perspective, geometry, dynamics, and matter are not fundamental inputs, but emergent features of the informational content encoded in quantum states.

These results suggest that spectral information provides a viable foundation for a unified description of physical reality.

Further work is required to develop quantitative models and to establish connections with experimentally accessible systems.

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