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Towards the analytical approximation of the hydrothermal vents / black smokers in the field conditions using "TAFELN HÖHERER FUNKTIONEN" database

Letter to the Editor

Alexander Bolkhovitinow*

Abstract

This report can be interpreted as the "bonus example" for our paper "*Towards the analytical approximation of weathering forms based on fitting of the geomorphological structures by the "Tafeln Höherer Funktionen" profile database*"; any hydrothermal vents / black smokers and white smokers are described in this report in the framework of our previous publication.

Keywords:

morphology, approximation, special functions, Euler–Riemann zeta function, Jacoby function, Mathieu function, Weierstrass function, Hankel function, hydrothermal vents, black smokers.

1. Introduction

It the work "*Towards the analytical approximation of weathering forms based on fitting of the geomorphological structures by the "Tafeln Höherer Funktionen" profile database*" our group report, that the "weathering activity and morphogenesis of the Earth surfaces can be qualitatively correlated and spatially colocalized (Govindarajan, Murthy, 1969; Modenesi, 1983; Pavich, 1985; Tokuyama, 1986; Le Pera, Sorriso-Valvo, 2000; Rochette Cordeiro, 2014). The form of the surface peculiarities can be analyzed and approximated using multifactor analysis, including the influence of some geophysical and geochemical factors such as temperature and mineralization of the geographical environment and also surface reactions on the solid state rock interfaces and within the pores (Velbel, 1990)" (Bolkhovitinov et al., 2018). But it is obvious, that such simulations were not realized for hydrothermal vents, despite the fact, that they are very interesting

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morphological / geohydrodynamical and geochemical (chemical weathering and biogeochemical processes) objects (Arp, Childress, 1981; Beaulieu et al., 2013; Chevaldonné et al., 1991; Cowen et al., 1990; Foustoukos, Seyfried, 2004; Jamtveit et al. 2004; John et al., 2008; Johnson et al., 1986; Lilley et al. 2003; Little et al., 1987; Little, Vrijenhoek, 2003; McCollom, 2000; Planke et al., 2005; Tivey, 1995, 2007; Tunnicliffe, Fowler, 1996).

2. Materials and methods

Methodological bases of this study include special function tables (by Jahnke E., Emde F. and Lösch F.), also known as a "Tafeln Höherer Funktionen" in original language editions (Jahnke, Emde und Lösch, 1960 [etc.]), initially provided by "B.G. Teubner Verlagsgesellsch" AFT, Leipzig.

The study is based on the comparative analysis between the approximations (by some special functions from this book) and different images of the hydrothermal vents, provided by the artificial intelligence or machine learning-assisted Web searching using "black smokers"-like keywords. Some notations of such illustrations are introduced into the article body for clarification of the theoretical principles proposed and visual recognizing of similar objects in different figures. Some types of hydrothermal fields / hydrothermal vents were analyzed in 3D directly (see fig. 1).

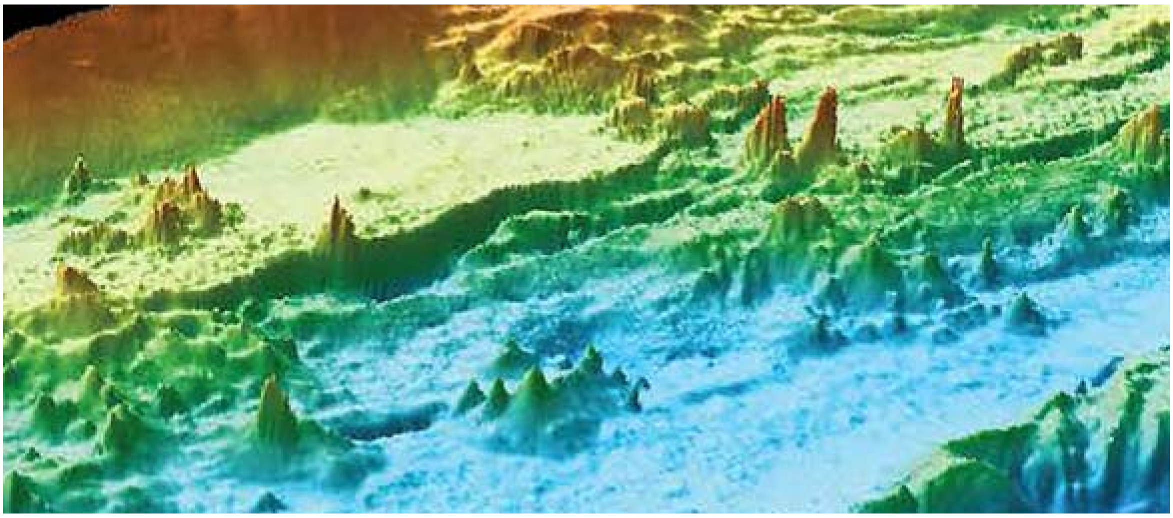

Fig. 1: Volume representations of hydrothermal vent / fields in pseudocolor mode.

3. Results

The results of the comparative morphological studies are presented in Table 1.

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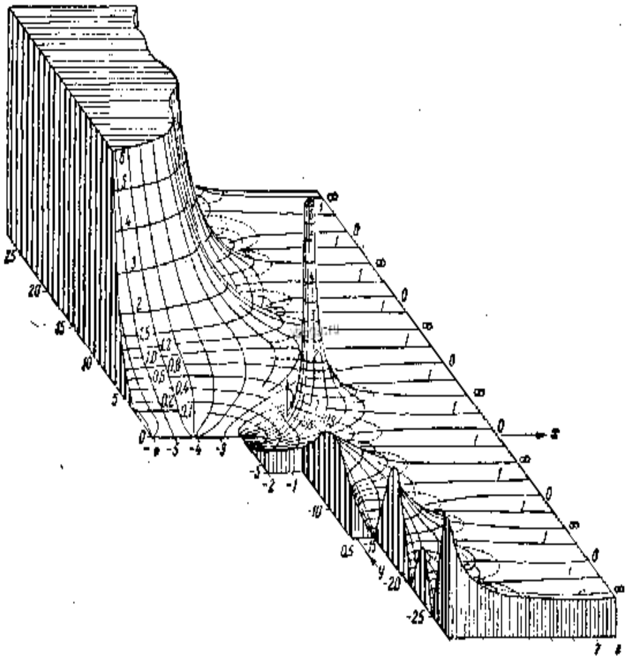
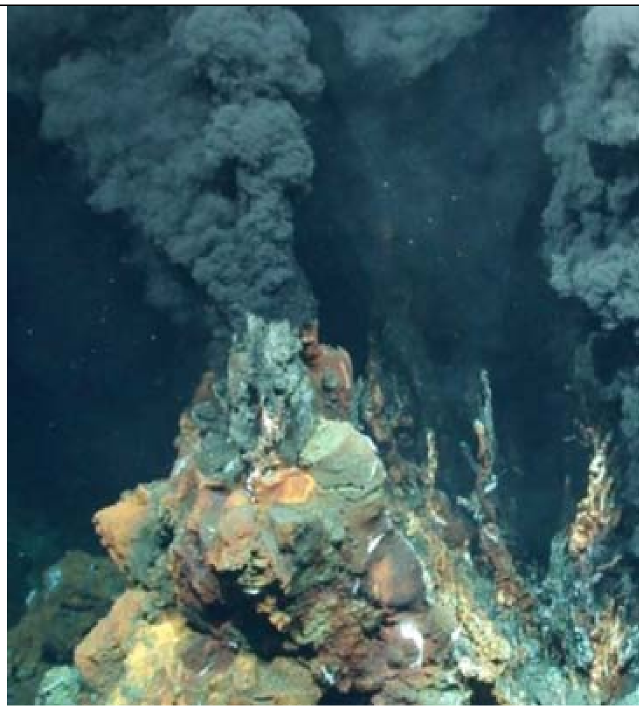
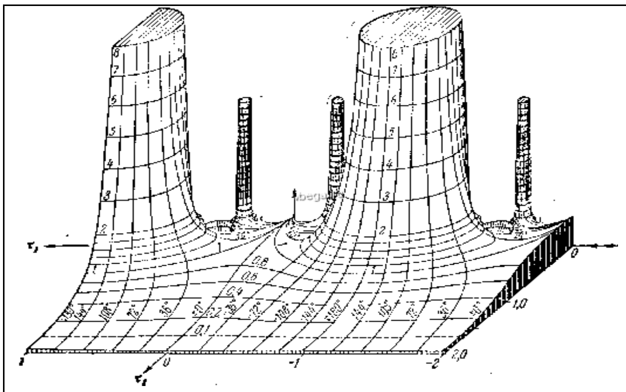
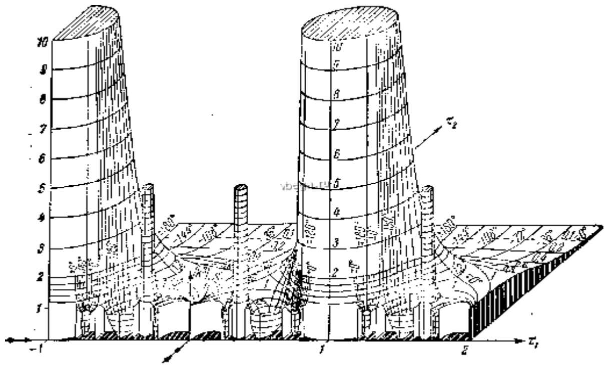
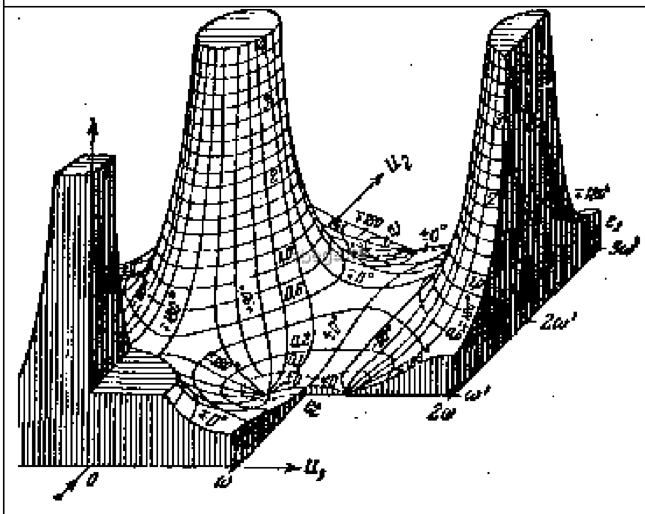
<tr><th>Special functions</th><th>Geostructures</th></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">Riemann zeta function or Euler–Riemann zeta function ζ(s) is a function of a complex variable s that analytically continues the sum of the Dirichlet series:
$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$
when the real part of s is not greater than 1. More general representations of ζ(s) for all s are given below.
The values of the Riemann zeta function at even positive integers were computed by Euler. The first of them, ζ(2), provides a solution to the Basel problem. The values at negative integer points, also found by Euler, are rational numbers and play an important role in the theory of modular forms. Many generalizations of the Riemann zeta function (Dirichlet series, Dirichlet L-functions) are known.</td></tr>
</table>

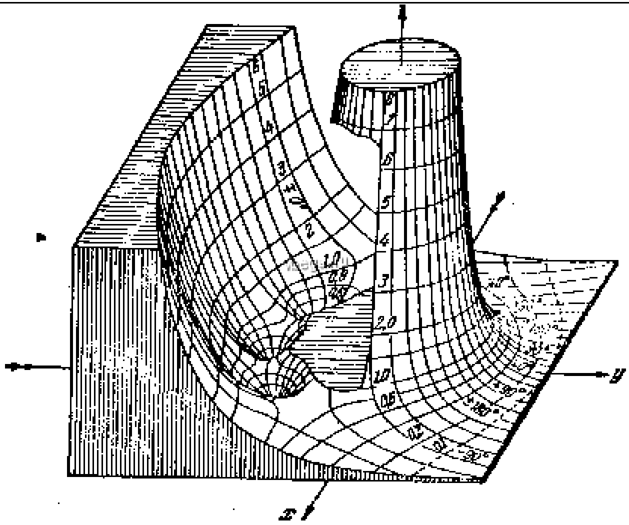
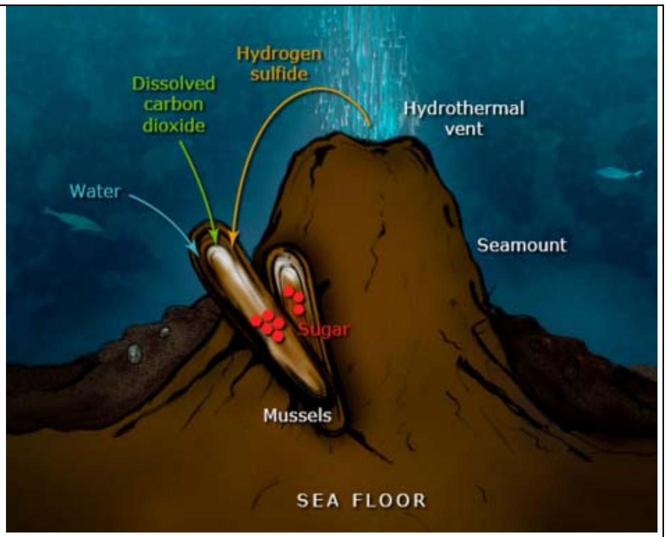
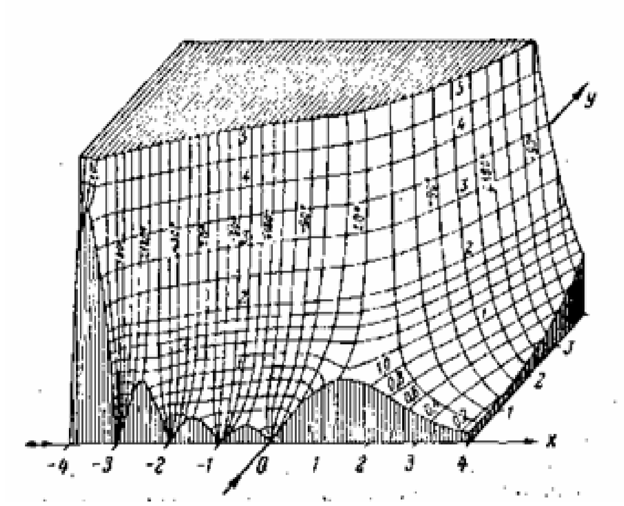
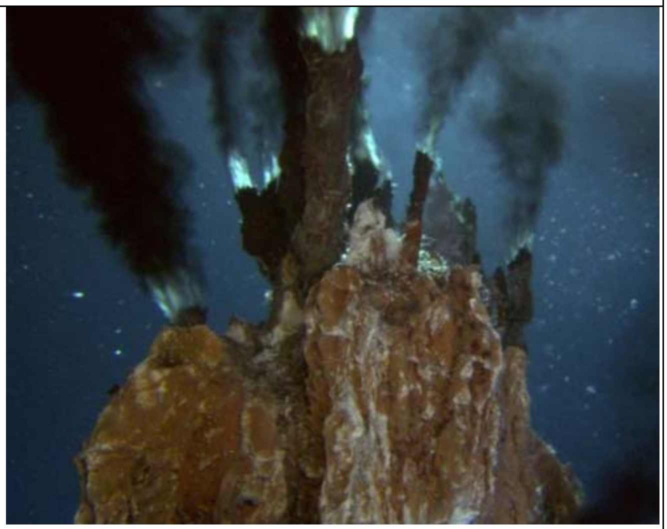
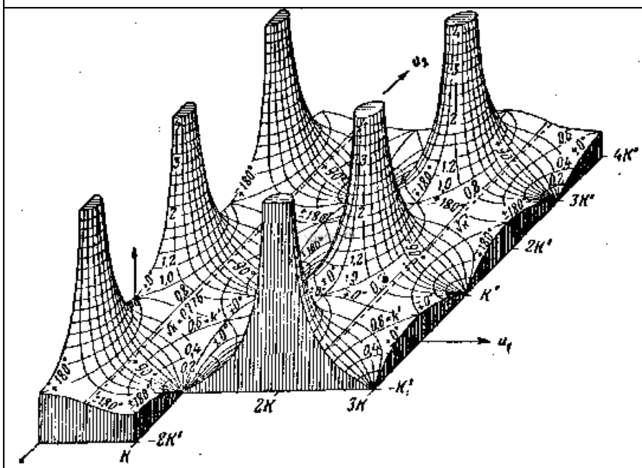
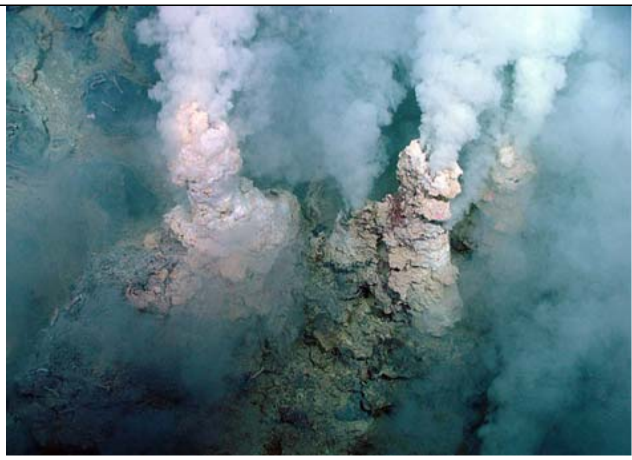


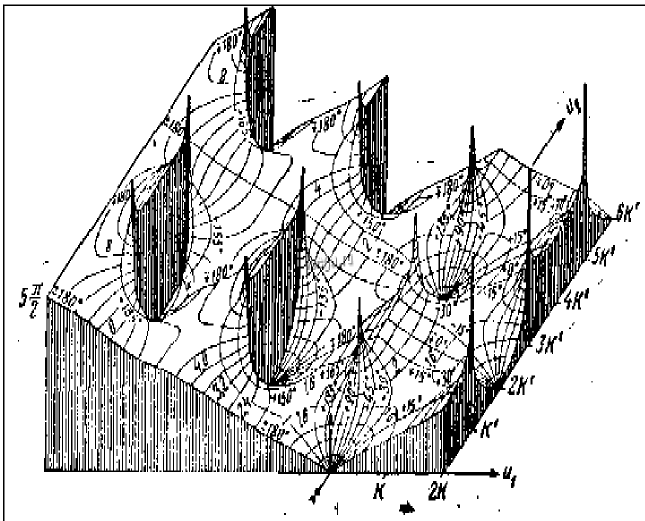
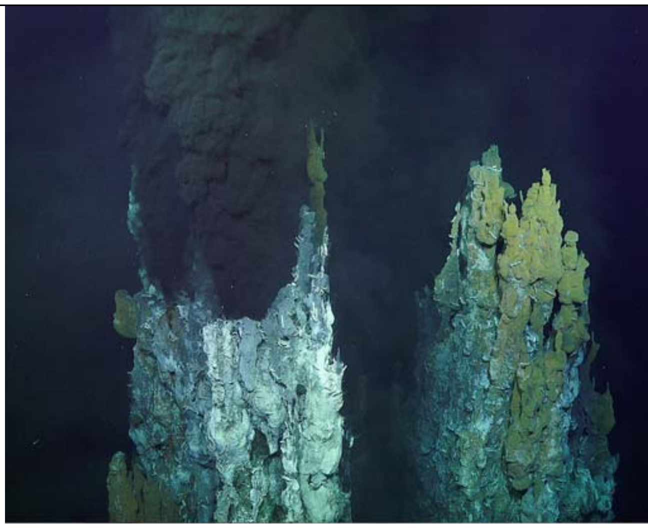


A modular form is a complex analytic function on the upper half-plane satisfying a certain kind of functional equation with respect to the group action of the modular group, and also satisfying a growth condition. The theory of modular forms therefore belongs to complex analysis but the main importance of the theory has traditionally been in its connections with the number theory. A modular form can equivalently be defined as a function F from the set of lattices in C to the set of complex numbers which satisfies certain conditions: I) If we consider the lattice $\Lambda = Z\alpha + Zz$ generated by a constant α and a variable z, then $F(\Lambda)$ is an analytic function of z. II) If α is a non-zero complex number and $\alpha\Lambda$ is the lattice obtained by multiplying each element of Λ by α, then $F(\alpha\Lambda) = \alpha-kF(\Lambda)$ where k is a constant (typically a positive integer) called the weight of the form. III) The absolute value of $F(\Lambda)$ remains bounded above as long as the absolute value of the smallest non-zero element in Λ is bounded away from 0.

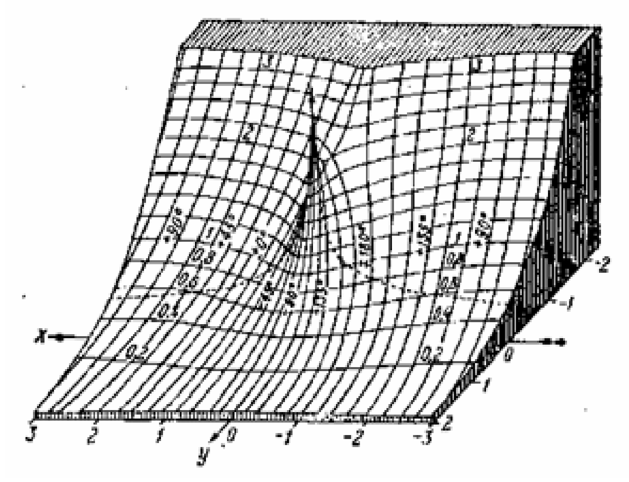
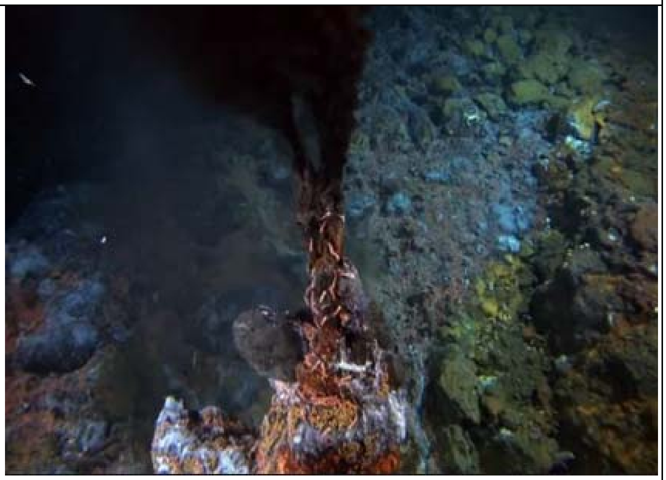


The example of the Jacoby function relief.

<table>
<tr><td></td><td>
</td></tr>
<tr><td colspan="2">The example of the Hankel function relief.</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">The map of the inversed values of the gamma function.</td></tr>
<tr><td></td><td></td></tr>
<tr><td>The relief of the Weierstrass function.</td><td></td></tr>
</table>

Another example of the Jacoby function relief with branching or bifurcation.

Example of the Hankel function relief.

4. Conclusion

Some results of this computations and comparative morphological studies very good confirm our hypothesis about approximability of different types of hydrothermal vents using different special functions, including Euler–Riemann zeta function, Jacoby function, Mathieu function, Weierstrass function, Hankel function etc.

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На пути к аналитической аппроксимации форм «черных курильщиков» на основе сопоставления с профилями специальных функций из справочников, каталогов, баз данных “Tafeln Höherer Funktionen”

Краткое сообщение

А.С. Болховитинов

Аннотация

В данном кратком сообщении автором предлагается новый подход к аналитической аппроксимации форм различных форм полей гетоермальных источников и сопутствующих им процессов морфогенеза. Новый метод, основанный на сопоставлении характерных типов профилей выветривания и специальных функций (в частности – таких, как дзета-функция Римана / функция Эйлера-Римана, функция Якоби, функция Матье, функция Вейерштрасса, функция Ганкеля, функция Ангера, эллиптические модулярные функции и т.д.) позволяет весьма эффективно моделировать и прогнозировать морфологию рельефа и ландшафтов на разных стадиях и при разных формах гидротермальной активности, подставляя корректные коэффициенты, при коих морфология «черных курильщиков» адекватно аппроксимируется визуализированными профилями соответствующих функций.

Keywords: морфология, черные курильщики, аппроксимация, специальные функции, дзета-функция Римана, функция Якоби, функция Матье, функция Вейерштрасса, функция Ганкеля.