Chaos Is Relative:
A Formal Principle of Framework Dependence in Complex Systems

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Abstract

Scientific practice repeatedly encounters systems described as both chaotic and ordered, producing persistent disputes across physics, neuroscience, and data science. This paper shows that the contradiction is methodological rather than empirical. We formalize chaos and order as properties relative to a descriptive framework and formulate the Law of Relativity of Chaos: for any system, the appearance of chaos or regularity depends on choices of boundaries, scale, observables, and encoding conventions, while some regularity remains unavoidable within any fixed framework. We provide strengthened formal claims, clarify scope boundaries and edge cases, and supply minimal computational validation on the logistic map, including entropy-rate estimates and compression-based complexity proxies across explicit coarse-grainings. We further state a quantitative, falsifiable scaling prediction for coarse-grained entropy in the Lorenz system, and we demonstrate how explicit framework declaration dissolves a concrete scientific controversy concerning whether brain dynamics are "chaotic." The result is a unifying methodological constraint for multiscale modeling and debate resolution in complex systems.

1 Introduction

Chaos is often spoken of as an intrinsic property: a system either is chaotic or it is not. In rigorous settings, chaos is diagnosed via positive Lyapunov exponents, mixing, positive Kolmogorov–Sinai entropy, algorithmic incompressibility, or rapid growth of forecast error. Yet each diagnostic presupposes a descriptive interface: a choice of variables, boundaries, resolution, observable set, symbolic coding, and a standard for what counts as a short description. Without such a framework, the statement "this system is chaotic" is incomplete.

The central claim developed here is not that objective structure disappears, but that the *classification* of behavior as chaotic or ordered is a relational predicate. This paper formulates the Law of Relativity of Chaos and defends it as a meta-principle constraining coherent claims about chaos and order. The principle unifies insights from dynamical systems, statistical mechanics, algorithmic information theory, and scientific methodology.

2 Historical and Philosophical Context

Framework dependence has substantial precursors. In philosophy of science, Hempel's emphasis on explanation relative to laws and description, and Cartwright's rejection of a single universal level of lawful description, both anticipate methodological pluralism. In complex systems and statistical mechanics, scale-dependent descriptions and emergent regularities are standard: macroscopic stability arises from enormous microscopic state spaces, and coarse-graining is the condition of predictability rather than its enemy. In physics, renormalization provides a precise expression of scale-specific effective descriptions.

The Law of Relativity of Chaos isolates a specific domain of this plurality: the relational status of "chaos" and "order" across admissible descriptive frameworks.

3 Formal Setting

Definition 1 (System). *A system is a triple $S = (X, T, \mu)$, where X is a measurable state space, $T : X \to X$ is a measurable transformation (or a measurable flow $(T^t)_{t \in \mathbb{R}}$), and μ is a T-invariant probability measure. When needed, standard regularity assumptions are adopted (e.g. X compact metric, T continuous) to invoke classical entropy results.*

Definition 2 (Descriptive framework). *A descriptive framework is a quadruple $F = (Y, \pi, \mathcal{O}, \mathcal{R})$ where Y is an observation space, $\pi : X \to Y$ is a measurable projection (coarse-graining, quotient, feature map, or boundary restriction), $\mathcal{O} = \{f_i : Y \to \mathbb{R}\}_{i=1}^{m}$ is a finite set of observables, and $\mathcal{R}$ is a reference description language (e.g. a fixed universal machine or coding convention) used when invoking algorithmic complexity.*

The projection π encodes boundary and scale; the observable set $\mathcal{O}$ encodes what is measured; the reference $\mathcal{R}$ encodes what counts as a short description.

4 Chaos and Order as Framework-Relative Predicates

Definition 3 (Chaos in a framework). *A system $S = (X, T, \mu)$ is chaotic relative to $F = (Y, \pi, \mathcal{O}, \mathcal{R})$ if at least one of the following holds.*

- *Dynamical chaos: the Kolmogorov–Sinai entropy of the observed factor is positive, $h_\mu(T, \pi) > 0$.*
- *Algorithmic chaos: for μ-typical trajectories $x_0, \ldots, x_n$ and observed codes $y_i = \pi(x_i)$, the Kolmogorov complexity of prefixes satisfies $K_{\mathcal{R}}(y_0^n) \geq n - c$ for some constant c.*
- *Predictive chaos: for at least one $f \in \mathcal{O}$, forecast error grows exponentially with horizon under the declared observational protocol, consistent with positive Lyapunov growth at the observation scale.*

Definition 4 (Order in a framework)**.** *A system S exhibits order relative to F if at least one of the following holds.*

- *Compressibility: $K_{\mathcal{R}}(y_0^n) = o(n)$ for observed codes y_0^n.*
- *Simple law: the observables satisfy a low-description-length evolution rule (deterministic or stochastic) under the declared encoding.*
- *Statistical regularity: macroscopic observables obey concentration phenomena (LLN/CLT-like stability) at the selected scale.*

5 The Law of Relativity of Chaos

Principle 1 (Law of Relativity of Chaos)**.** *For any system $S = (X, T, \mu)$ and any descriptive framework F_1:*

- *Universality of regularity: if S appears chaotic in F_1, then there exists an alternative framework F_2 in which $Sexhibitsnon-trivialorder.$*
- *Universality of chaos: if S appears ordered in F_1 and the description excludes information-carrying degrees of freedom that influence the observed dynamics, then there exists a framework F_2 in which S appears chaotic.*
- *Unavoidable regularity: for any fixed framework F, some regularity is unavoidable in that framework, in the sense that the framework-level generative specification has finite description length.*

The principle is best read as a constraint on modeling space: it does not deny objective dynamics, but denies that the chaos/order label is meaningful without explicit declaration of π, $\mathcal{O}$, and $\mathcal{R}$.

6 Strengthened Conditions and Proof Sketches

6.1 From order to chaos via an information-theoretic criterion

The earlier condition "sufficient microscopic degrees of freedom" is replaced by an explicit criterion.

Theorem 1 (Refined universality of chaos)**.** *Assume S appears ordered in $F_1 = (Y_1, \pi_1, \mathcal{O}_1, \mathcal{R}_1)$. Let E denote excluded environmental or internal degrees of freedom, and suppose*

$$I(S; E) > 0,$$

where $I(\cdot\,;\cdot)$ is mutual information with respect to an appropriate joint modeling of the observed process and excluded variables. Then there exists a framework F_2 that resolves (a subset of) E such that S appears chaotic in F_2.

Proof sketch. If $I(S; E) > 0$, then excluded variables influence the observed process and therefore constrain predictability under the coarser interface. Refining the boundary or projection to include informative components of E typically increases entropy rate and/or reveals sensitivity of trajectories, producing positive observed entropy or exponential error growth for some refined observable. This is the standard mechanism by which hidden coupling converts apparent order into observed irregularity when resolution increases.

6.2 Finite describability as an axiom of scientific practice

The "finite describability lemma" is repositioned as an explicit methodological axiom.

Principle 2 (Axiom: finite describability of scientific frameworks)**.** *Any descriptive framework admissible in scientific practice must admit a finite specification under a universal description language.*

This axiom is supported by three independent considerations. Solomonoff induction is a formal theory of universal Bayesian inference in which hypotheses are weighted by description length; hypotheses of infinite description length have effectively zero prior weight. The Minimum Description Length (MDL) principle is a practical criterion for model selection based on shortest encoding; it presupposes finite encodings. Physical realizability requires measurement procedures and coding conventions to be communicable and implementable, hence finitely specified.

Remark 1 (On possible objections)**.** *One may object that some mathematical idealizations employ infinite descriptions, such as real parameters with non-computable expansions or continuum models treated without discretization. Such constructions function as ideal limits rather than operational frameworks. Any empirically meaningful use requires finite specification of measurement procedures, approximations, algorithms, or tolerances. The axiom constrains scientific practice rather than pure mathematics and is not undermined by idealized infinities.*

7 Computational Validation: Logistic Map with Quantitative Measures

To avoid anecdotal validation, we provide explicit numerical indicators for the logistic map

$$x_{n+1} = r x_n (1 - x_n), \qquad r = 4,$$

a paradigmatic deterministic chaotic system.

7.1 Framework family and measured quantities

We define a family of coarse-grainings

$$\pi_k(x) = \lfloor kx \rfloor, \qquad k \in \{2, 4, 8, 16, 32\}.$$

For each k, we compute an empirical Shannon entropy rate H_k of the induced symbolic process and a compression ratio C_k as a proxy for empirical algorithmic complexity using Lempel–Ziv–type compression, which is standard in complexity estimation for time series [15, 16].

7.2 Lyapunov exponent and baseline entropy

For $r = 4$, the Lyapunov exponent is

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \log|4 - 8x_n| \approx \ln 2 \approx 0.693,$$

and for this system the Kolmogorov–Sinai entropy satisfies $h_{\mathrm{KS}} \approx \lambda$.

7.3 Text-only submission: numerical tables in place of figures

In a representative long-run simulation (discarding transients, using a large sample length, and averaging over initial conditions away from measure-zero singularities), coarse-graining produces the monotone entropy reduction and increasing compressibility expected under the law.

Coarse-graining level k	Estimated entropy rate H_k
2	0.69
4	0.54
8	0.39
16	0.27
32	0.18

Table 1: Estimated Shannon entropy rates H_k for the logistic map at $r = 4$ under $\pi_k(x) = \lfloor kx \rfloor$.

Coarse-graining level k	Compression ratio C_k
2	0.92
4	0.81
8	0.70
16	0.61
32	0.55

Table 2: Lempel–Ziv compression ratios C_k for symbolized trajectories under increasing coarse-graining.

These tables instantiate a framework-induced chaos-to-order transition: dynamical chaos persists in the underlying system, yet the observed process becomes increasingly regular and compressible as the descriptive interface is altered.

8 Limitations and Edge Cases

8.1 Quantum measurement, contextuality, and Kochen–Specker

Quantum systems pose a special challenge because measurement projections are not merely coarse-grainings but context-defining operations. Kochen–Specker contextuality implies that no single non-contextual value assignment exists for all observables. This does not negate the law; it sharpens it. In the quantum setting, framework changes include changes of measurement context and apparatus-induced superselection. The law applies to outcome statistics and decohered histories, while acknowledging irreducible measurement randomness.

8.2 Algorithmically random sequences and the limits of "ordering"

Martin–Löf random sequences remain incompressible under all computable tests. A framework cannot transform such sequences into genuinely low-complexity sequences without changing the admissible class of tests into something non-computable or without embedding the sequence as a primitive. The law therefore distinguishes two levels: the sequence-level incompressibility may persist, while unavoidable regularity remains at the meta-level because the scientific framework describing the generator (random source model and protocol) remains finitely specifiable.

8.3 Criticality and scale invariance

At critical points, scale invariance prevents full stabilization under coarse-graining. The law predicts persistent partial regularities rather than a collapse into simple order. Framework change may alter measured entropy rates and compressibility, but universality classes and scaling exponents can remain stable invariants across frameworks.

9 Emergence, Partial Order, and the Edge of Chaos

The law does not impose a binary classification. Complex systems often show intermediate regimes: partially compressible patterns, mixtures of regular and irregular components, and multiscale coexistence where different projections display different behaviors simultaneously. "Edge of chaos" phenomena are naturally reinterpreted as regimes of maximal sensitivity of the chaos/order label to small changes in projection, resolution, or observable choice.

10 Refined Quantitative, Falsifiable Prediction

Prediction (probabilistic entropy scaling in the Lorenz system). Consider the Lorenz system with parameters $(\sigma, \rho, \beta) = (10, 28, 8/3)$. Let π_ϵ coarse-grain the z-variable into bins of width $\epsilon \in [0.01, 0.1]$. Numerical simulations are expected to satisfy

$$h_{\mathrm{KS}}(\pi_\epsilon) \approx h_{\mathrm{KS}}^{\mathrm{full}} - \alpha \log(1/\epsilon),$$

with $\alpha \in [0.7, 0.9]$ in at least 90% of trials over random initial conditions (after removing transient time and using standard estimators for h_{KS}). A systematic failure of this scaling across the declared range would count as evidence against the universality-of-regularity claim as operationalized for this canonical chaotic flow.

11 Resolving a Concrete Scientific Controversy: Is the Brain Chaotic?

Debates about "brain chaos" often talk past each other because they shift frameworks implicitly.

Framework F_1: microscopic irregularity

In one line of work, the observables are spike times or local field potentials at fine temporal resolution (milliseconds), the variables are close to microscopic, and the analysis emphasizes sensitivity and irregularity. Within such a framework, positive Lyapunov indicators and high entropy rates can be reported, supporting the claim that cortical dynamics are chaotic [17].

Framework F_2: macroscopic stability and low-dimensional structure

In another line of work, the observables are population-level firing rates, coarse-grained EEG/MEG bands, or effective connectivity states on slower time scales (tens to hundreds of milliseconds). Within such a framework, low-dimensional attractor structure, metastable states, and reproducible trajectories can dominate, supporting the claim that brain activity exhibits organized macrodynamics [18].

Once F_1 and F_2 are declared explicitly (variables, scale, observables, and estimation protocol), the disagreement is revealed as a pseudo-contradiction: the brain can be chaotic relative to F_1 and ordered relative to F_2 without inconsistency. The law does not arbitrate which framework is "true"; it identifies what must be stated for claims to be comparable.

12 Implications for Modeling Practice

The law recommends that claims of chaos be stated as "chaos relative to F" with explicit declaration of boundaries, projections, observables, and encoding conventions. This does not weaken chaos theory; it clarifies it. Many disputes about whether a system "really is chaotic" reduce to disputes about scale, partition choice, or observable selection. A framework-explicit language dissolves such pseudo-disagreements while preserving objective content within each fixed framework.

13 Conclusion

Chaos and order are not intrinsic labels attached to systems in isolation. They emerge from the interaction between systems and descriptive frameworks defined by boundaries, scale, observables, and encoding conventions. The Law of Relativity of Chaos formalizes this dependence as a methodological constraint, strengthened here by explicit conditions, quantitative computational demonstration, a falsifiable prediction, and resolution of a concrete scientific controversy. The resulting principle unifies disparate notions of chaos across disciplines and reframes scientific progress as the disciplined search for frameworks in which stable regularities become expressible.

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