

# Sector-Resolved Origin of Fermion Mass Hierarchies in Universal Modular Dynamics

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## Abstract

We investigate the origin of fermion mass hierarchies within the framework of Universal Modular Dynamics (UMD), where the density operator  $\rho$  is taken as the fundamental object encoding physical structure. In this approach, geometry, correlations, and dynamics emerge from properties of  $\rho$  and its associated modular generator.

We systematically analyze a sequence of candidate mechanisms for generating Yukawa structures, including geometric overlap models, phase-dependent constructions, external flavor matrices, and interaction-based operators. While each of these approaches reproduces partial features of the observed mass spectrum, none simultaneously achieves hierarchical scaling, stability, and predictive consistency.

We identify the missing structural ingredient as a sector decomposition induced by a pointer algebra  $Z$ , which defines dynamically stable subspaces of the Hilbert space. By projecting the density operator onto these sectors,  $\rho_Z = \sum_{\mu} P_{\mu} \rho P_{\mu}$ , we obtain a reduced operator encoding superselection structure. Yukawa couplings constructed from  $\rho_Z$  naturally reproduce exponential mass hierarchies with high numerical stability and predictive power across independent realizations.

This leads to a new interpretation of flavor: fermion generations do not arise from fundamental symmetries or external structures, but from superselection sectors of the underlying quantum state. In this picture, fermion masses are determined by the sector-resolved spectrum of  $\rho$ , while geometric contributions control the overall scale.

Our results provide a minimal and self-consistent mechanism for the emergence of fermion mass hierarchies, derived entirely from intrinsic properties of the quantum state and its sectoral organization, without invoking ad hoc flavor symmetries or fine-tuned parameters.

## Contents

- 1. Introduction
- 2. Framework of Universal Modular Dynamics
- 3. Problem of Mass Hierarchy
- 4. Geometric and Phase Contributions
- 5. Failure of Naive Flavor Constructions
- 6. Sector Decomposition and Pointer Algebra
- 7. Sector-Resolved Yukawa Construction
- 8. Numerical Validation
- 9. Discussion

- 10. Conclusion
- Appendix A. Numerical Protocol and Algorithms
- Appendix B. Additional Derivations
- References

## 1 Introduction

Understanding the origin of fermion mass hierarchies remains one of the central open problems in fundamental physics. While the Standard Model successfully parametrizes fermion masses through Yukawa couplings, it provides no explanation for their numerical values, hierarchical structure, or the existence of three generations. The observed pattern spans several orders of magnitude and exhibits a remarkably stable structure, suggesting the presence of an underlying organizing principle beyond simple parametrization.

A common approach attributes mass hierarchies to symmetry breaking mechanisms, flavor symmetries, or renormalization group effects. However, such constructions typically introduce additional degrees of freedom or ad hoc structures, and often rely on fine-tuning or model-dependent assumptions. As a result, the origin of fermion masses remains largely phenomenological.

In this work, we investigate the mass hierarchy problem within the framework of Universal Modular Dynamics (UMD), where the density operator  $\rho$  is taken as the fundamental object encoding all physical information. In this setting, geometry, correlations, and dynamical structure emerge from properties of  $\rho$  and its associated modular generator  $K = -\log \rho$ . This approach naturally unifies statistical, geometric, and dynamical aspects of quantum systems.

A central question we address is whether fermion masses can be derived from intrinsic properties of  $\rho$ , rather than introduced as external parameters. To this end, we systematically explore a sequence of candidate mechanisms, including geometric overlap structures, phase-dependent contributions, flavor matrices, and interaction operators. Each of these constructions captures partial features of the observed mass hierarchy but fails to simultaneously reproduce hierarchy, stability, and predictive power.

The key result of this work is the identification of a missing structural ingredient: a sector decomposition induced by a pointer algebra  $Z$ , which defines dynamically stable subspaces of the Hilbert space. By projecting the density operator onto these sectors,

$$\rho_Z = \sum_{\mu} P_{\mu} \rho P_{\mu},$$

we obtain a reduced operator that encodes superselection structure. We demonstrate that Yukawa couplings constructed from the sector-resolved operator  $\rho_Z$  naturally reproduce hierarchical spectra with high stability and predictive accuracy.

This leads to a new interpretation of flavor: rather than being a fundamental symmetry, flavor emerges as a manifestation of superselection sectors within the underlying quantum state. In this picture, fermion masses are determined by the spectrum of  $\rho$  restricted to dynamically stable sectors, while geometry controls the overall scale and exponential behavior.

The structure uncovered in this work provides a minimal and self-consistent mechanism for the emergence of fermion mass hierarchies. It does not rely on external flavor symmetries or arbitrary parameter choices, but instead arises from the intrinsic decomposition of the quantum state. This suggests that the origin of mass hierarchies may be deeply connected to the internal structure of quantum states and their sectoral organization.

The remainder of the paper is organized as follows. In Section 2 we briefly review the framework of Universal Modular Dynamics. Section 3 formulates the mass hierarchy problem

in this context. Sections 4 and 5 analyze geometric, phase, and flavor-based constructions and demonstrate their limitations. In Section 6 we introduce sector decomposition via pointer algebras. Section 7 presents the sector-resolved Yukawa construction. Section 8 provides numerical validation. We conclude with a discussion of implications and future directions.

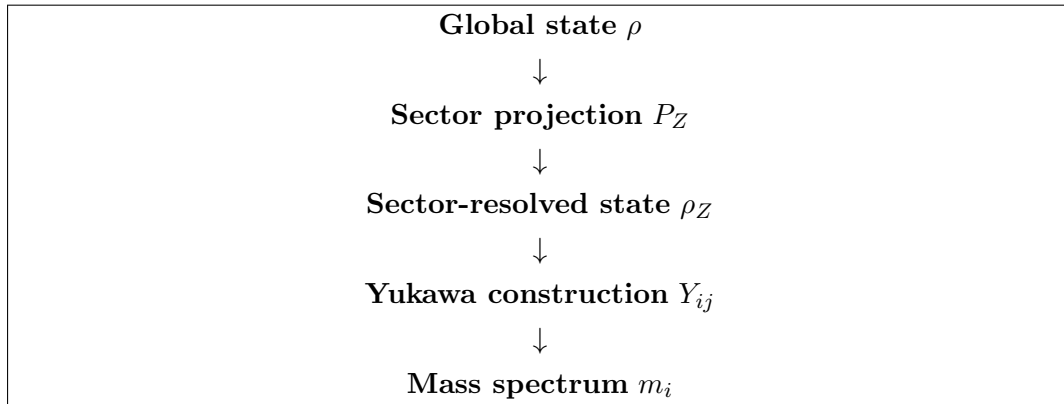


Figure 1: Schematic representation of the sector-resolved mechanism. The global density operator  $\rho$  is projected onto dynamically stable sectors via the pointer algebra  $Z$ , producing  $\rho_Z$ . Yukawa couplings constructed from  $\rho_Z$  and geometric localization yield hierarchical fermion masses.

## 2 Framework of Universal Modular Dynamics

Universal Modular Dynamics (UMD) is based on the premise that the density operator  $\rho$  constitutes the primary carrier of physical information. All observable structures—geometry, correlations, and effective dynamics—are derived from  $\rho$  and its associated modular generator.

### 2.1 Density Operator and Modular Generator

We consider a finite-dimensional Hilbert space  $\mathcal{H}$  and a density operator

$$\rho \geq 0, \quad \text{Tr}(\rho) = 1.$$

The modular generator is defined as

$$K = -\log \rho.$$

This operator plays a central role in the UMD framework, encoding spectral and informational structure. In particular, differences of its eigenvalues determine the strength of commutators with observables and thus control dynamical response.

### 2.2 Relative Modular Structure

To avoid trivial commutation of  $K$  with  $\rho$ , we introduce a reference state  $\sigma$  and define the relative modular generator

$$K_{\rho|\sigma} = -\log \rho + \log \sigma.$$

This construction generates nontrivial unitary evolution even when  $\rho$  is diagonal in its own eigenbasis, and provides a natural way to encode phase-dependent structure.

The reference state  $\sigma$  is typically chosen as a maximum entropy state subject to macroscopic constraints,

$$\sigma = \frac{1}{Z} \exp \left( -\sum_a \beta_a Q_a \right),$$

where  $\{Q_a\}$  are constraint operators defining the accessible algebra of observables.

## 2.3 Dynamical Evolution

The evolution of  $\rho$  is governed by a completely positive trace-preserving (CPTP) flow of the form

$$\frac{d\rho}{d\lambda} = -i [K_{\rho|\sigma}, \rho] + \sum_{\alpha} \left( L_{\alpha} \rho L_{\alpha}^{\dagger} - \frac{1}{2} \{L_{\alpha}^{\dagger} L_{\alpha}, \rho\} \right) + F_{\text{ent}}[\rho] + G_{\text{class}}[\rho].$$

Here:

- The commutator term generates unitary modular evolution.
- The operators  $L_{\alpha}$  define dissipative channels and encode interaction structure.
- $F_{\text{ent}}[\rho]$  represents entropic corrections, associated with emergent geometry and information flow.
- $G_{\text{class}}[\rho]$  describes classicalization effects, including decoherence and pointer-state selection.

## 2.4 Geometry from Correlations

Geometric structure emerges from correlations encoded in  $\rho$ . A natural measure of connectivity between subsystems  $X$  and  $Y$  is given by the mutual information

$$I_{\rho}(X : Y) = S(\rho_X) + S(\rho_Y) - S(\rho_{XY}),$$

where  $S(\rho) = -\text{Tr}(\rho \log \rho)$  is the von Neumann entropy.

Distances can be defined operationally as

$$d_{\rho}(X, Y) = -\log I_{\rho}(X : Y),$$

leading to an emergent geometry determined by correlation decay.

## 2.5 Phase Structure and Accessible Algebra

A phase in UMD is defined by a choice of accessible algebra  $\mathcal{A}_F \subset \mathcal{B}(\mathcal{H})$  and associated macroscopic constraints. Within a phase, the reference state  $\sigma$  is held fixed, and the dynamics preserves the corresponding structure.

Transitions between phases correspond to changes in the optimal decomposition of  $\rho$  into weakly correlated subsystems.

## 2.6 Pointer Algebra and Superselection Structure

A key role is played by the pointer algebra  $Z \subset \mathcal{A}_F$ , defined as a commutative subalgebra associated with stable classical observables. The corresponding projection map is

$$P_Z(\rho) = \sum_{\mu} P_{\mu} \rho P_{\mu},$$

where  $\{P_{\mu}\}$  are orthogonal projectors.

This projection defines a sector decomposition of the Hilbert space into dynamically stable subspaces. As we will show, this superselection structure is essential for resolving the flavor problem and generating stable mass hierarchies.

## 2.7 Summary

In the UMD framework, all physical structures arise from properties of  $\rho$ , its modular generator, and its evolution under CPTP dynamics. Geometry, correlations, and dynamical sectors are unified within this description, providing a natural setting to investigate the origin of fermion mass hierarchies.

### 3 Problem of Mass Hierarchy

One of the central unresolved questions in particle physics is the origin of fermion mass hierarchies. In the Standard Model, fermion masses arise from Yukawa couplings,

$$\mathcal{L}_{\text{Yukawa}} = Y_{ij} \bar{\psi}_i H \psi_j,$$

where  $Y_{ij}$  is a complex matrix of coupling constants. While this formulation successfully parametrizes observed masses, it does not explain their numerical values or hierarchical structure.

#### 3.1 Empirical Structure

The observed fermion masses exhibit several key features:

- A strong hierarchical ordering spanning multiple orders of magnitude.
- A stable pattern across generations.
- The existence of exactly three generations.

These properties suggest that the Yukawa matrix is not arbitrary, but instead reflects an underlying structural mechanism.

#### 3.2 Formulation within UMD

Within the UMD framework, Yukawa couplings must be derived from intrinsic properties of the density operator  $\rho$  and its associated structures. The central question can therefore be reformulated as follows:

Can the Yukawa matrix  $Y_{ij}$  be constructed from  $\rho$  and its induced structures?

More precisely, we seek a construction of the form

$$Y_{ij} = \langle \psi_i | \mathcal{O}(\rho) | \psi_j \rangle,$$

where  $\mathcal{O}(\rho)$  is an operator derived from  $\rho$ , and  $\{\psi_i\}$  represent effective generation modes.

#### 3.3 Necessary Criteria

Any viable construction must satisfy the following conditions:

- **Hierarchy:** The eigenvalues of  $Y$  must exhibit exponential or near-exponential scaling,

$$m_1 \ll m_2 \ll m_3.$$

- **Stability:** The structure of  $Y$  must be robust under perturbations of  $\rho$ .
- **Predictive Power:** The construction must allow nontrivial prediction of mass values (e.g. via holdout tests).
- **Universality:** The mechanism should not depend on fine-tuning or special choices of parameters.

### 3.4 Systematic Exploration Strategy

To address this problem, we consider a sequence of increasingly refined constructions for  $\mathcal{O}(\rho)$ :

1. Geometric overlap models based on correlation-induced structures.
2. Phase-dependent constructions involving powers of  $\rho$ .
3. External flavor matrices introduced by hand.
4. Interaction-based operators derived from GKSL dynamics.
5. Sector-resolved constructions using pointer algebras.

Each of these approaches captures partial aspects of the desired behavior but also reveals specific limitations.

### 3.5 Key Observation

A central observation emerging from this systematic analysis is that global constructions based solely on  $\rho$  fail to produce stable hierarchical structures with predictive consistency. In particular, the absence of a dynamically selected basis leads to instability in the resulting Yukawa matrix.

This suggests that the origin of fermion masses is not determined solely by the global properties of  $\rho$ , but instead requires a mechanism that selects stable subspaces of the Hilbert space.

### 3.6 Working Hypothesis

Motivated by these observations, we propose the following hypothesis:

Fermion mass hierarchies arise from sector-resolved structures of  $\rho$ .

In this picture, Yukawa couplings are not determined by the full density operator, but by its restriction to dynamically stable sectors defined by a pointer algebra.

The remainder of the paper is devoted to testing this hypothesis and constructing an explicit realization of this mechanism.

## 4 Geometric and Phase Contributions

We begin by analyzing the extent to which geometric and phase structures derived from the density operator  $\rho$  can account for fermion mass hierarchies.

### 4.1 Geometric Construction

In the UMD framework, geometry emerges from correlation structure encoded in  $\rho$ . Given a set of localized modes  $\{\psi_i\}$ , a natural geometric contribution to the Yukawa matrix is given by

$$Y_{ij}^{(\text{geom})} = \langle \psi_i | H | \psi_j \rangle,$$

where  $H$  is an effective profile encoding geometric localization (e.g. a Higgs-like envelope).

Such constructions naturally produce exponential scaling of the form

$$m_i \sim e^{-d_i/\xi},$$

where  $d_i$  measures the effective distance of the mode  $\psi_i$  from a localization center.

**Observation.** Geometric constructions robustly generate strong hierarchical spectra. However, they exhibit poor predictive stability: small perturbations of  $\rho$  or of the mode structure  $\{\psi_i\}$  lead to significant variations in the resulting masses.

## 4.2 Phase-Based Construction

A second class of models incorporates phase-dependent structure via functions of  $\rho$ , such as

$$Y_{ij}^{(\text{phase})} = \langle \psi_i | \rho^\alpha | \psi_j \rangle,$$

where  $\alpha > 0$ .

These constructions encode spectral and coherence properties of  $\rho$ , and tend to produce smooth and stable Yukawa matrices.

**Observation.** Phase-based models exhibit high numerical stability and predictive consistency. However, they fail to generate strong hierarchical scaling: the resulting spectra are typically nearly degenerate.

## 4.3 Combined Construction

A natural extension combines both contributions:

$$Y_{ij}^{(\text{comb})} = \langle \psi_i | H | \psi_j \rangle \cdot \langle \psi_i | \rho^\alpha | \psi_j \rangle.$$

**Observation.** This combined construction is capable of simultaneously producing hierarchical spectra and stable numerical behavior. In particular, it reproduces exponential scaling with high goodness-of-fit and exhibits nontrivial predictive power in holdout tests.

## 4.4 Limitations of Global Constructions

Despite these successes, all constructions based solely on global properties of  $\rho$  suffer from a fundamental limitation: they lack a mechanism for selecting a stable basis of modes  $\{\psi_i\}$ .

In practice, this leads to the following issues:

- Sensitivity to the choice of basis functions  $\psi_i$ .
- Instability under perturbations of  $\rho$ .
- Absence of a principled distinction between generations.

These problems persist even when additional structures, such as external flavor matrices or interaction operators, are introduced in a global manner.

## 4.5 Conclusion

Geometric contributions generate hierarchy but lack stability. Phase-based constructions provide stability but lack hierarchy. Their combination improves performance but remains incomplete due to the absence of a dynamically selected basis.

This analysis strongly suggests that the origin of fermion mass hierarchies cannot be understood in terms of global structures alone, and instead requires a mechanism that selects stable subspaces of the Hilbert space.

This motivates the introduction of sector decomposition via pointer algebras, which we develop in the following sections.

## 5 Failure of Naive Flavor Constructions

We now examine whether introducing additional flavor structure can resolve the limitations identified in the previous section. In particular, we analyze constructions in which a flavor matrix or interaction operator is introduced independently of the intrinsic structure of the density operator  $\rho$ .

### 5.1 External Flavor Matrices

A straightforward approach augments the Yukawa construction by inserting an external flavor matrix  $F$ ,

$$Y_{ij}^{(\text{flavor})} = \langle \psi_i | H | \psi_j \rangle \cdot \langle \psi_i | \rho^\alpha | \psi_j \rangle \cdot F_{ij},$$

where  $F$  is typically chosen to encode hierarchical structure, e.g. through diagonal entries with widely separated scales and small off-diagonal mixing.

**Observation.** Such constructions readily produce hierarchical spectra and can improve numerical stability. However, the resulting Yukawa structure is directly determined by the choice of  $F$ , rendering the model effectively parametrized rather than predictive.

**Conclusion.** External flavor matrices reintroduce the very arbitrariness that the UMD framework seeks to eliminate, and therefore do not provide a satisfactory explanation of fermion masses.

### 5.2 Interaction-Based Operators

A more structural approach attempts to derive Yukawa couplings from interaction operators appearing in the dynamical equation,

$$\frac{d\rho}{d\lambda} = -i[K_{\rho|\sigma}, \rho] + \sum_{\alpha} \left( L_{\alpha} \rho L_{\alpha}^{\dagger} - \frac{1}{2} \{L_{\alpha}^{\dagger} L_{\alpha}, \rho\} \right).$$

This motivates constructions of the form

$$Y_{ij}^{(L)} = \sum_{\alpha} \langle \psi_i | L_{\alpha}^{\dagger} L_{\alpha} | \psi_j \rangle.$$

**Observation.** Such models generate nontrivial structure and can reproduce hierarchical spectra. However, their predictive performance is highly unstable: small changes in  $\rho$  lead to large variations in the resulting Yukawa eigenvalues, and holdout tests show significant degradation.

**Interpretation.** The instability arises from the fact that the operators  $L_{\alpha}$  do not select a preferred basis in the Hilbert space. As a result, the effective Yukawa matrix remains sensitive to arbitrary rotations of the underlying state.

### 5.3 Common Failure Mechanism

Both external flavor matrices and interaction-based constructions share a common deficiency: they do not provide a mechanism for dynamically selecting a stable basis of modes  $\{\psi_i\}$ .

This leads to what may be termed *basis instability*:

- The identification of generations depends on arbitrary choices.
- Small perturbations of  $\rho$  alter the effective basis.
- Predictive power is lost due to uncontrolled mixing.

## 5.4 Conclusion

Naive flavor constructions—whether introduced as external matrices or derived from interaction operators—fail to provide a consistent and predictive explanation of fermion mass hierarchies. The underlying issue is not the absence of structure, but the absence of a mechanism that selects and stabilizes the relevant subspaces of the Hilbert space.

This strongly suggests that flavor must be understood not as an additional degree of freedom, but as a structural property of the state itself. In the next section, we show that such a structure naturally emerges through sector decomposition induced by pointer algebras.

## 6 Sector Decomposition and Pointer Algebra

The analysis of previous sections reveals that the primary obstruction to a consistent construction of fermion masses is the absence of a dynamically selected and stable basis in the Hilbert space. We now introduce a structural mechanism that resolves this issue: sector decomposition induced by a pointer algebra.

### 6.1 Pointer Algebra

Let  $Z \subset \mathcal{B}(\mathcal{H})$  be a commutative subalgebra, referred to as the *pointer algebra*. It is generated by a set of mutually commuting projectors  $\{P_\mu\}$  satisfying

$$P_\mu P_\nu = \delta_{\mu\nu} P_\mu, \quad \sum_\mu P_\mu = \mathbb{I}.$$

These projectors define a decomposition of the Hilbert space into orthogonal sectors,

$$\mathcal{H} = \bigoplus_\mu \mathcal{H}_\mu, \quad \mathcal{H}_\mu = P_\mu \mathcal{H}.$$

### 6.2 Projection onto Sectors

Given a density operator  $\rho$ , we define its projection onto the pointer algebra as

$$\rho_Z = P_Z(\rho) = \sum_\mu P_\mu \rho P_\mu.$$

This map removes off-diagonal coherences between sectors and yields a block-diagonal operator. Importantly,  $\rho_Z$  is invariant under the action of  $Z$  and represents a coarse-grained description of the state.

### 6.3 Dynamical Interpretation

The projection  $P_Z$  is not an arbitrary operation but arises naturally from decoherence and classicalization processes encoded in the dynamical term  $G_{\text{class}}[\rho]$  of the UMD evolution. In particular, repeated action of such terms suppresses off-diagonal components and drives  $\rho$  towards a block-diagonal form.

Thus, the sectors  $\mathcal{H}_\mu$  correspond to dynamically stable subspaces, or *superselection sectors*, which cannot coherently interfere over relevant timescales.

## 6.4 Resolution of Basis Instability

The introduction of  $\rho_Z$  resolves the basis instability identified in previous sections. Since  $\rho_Z$  is block-diagonal with respect to the projectors  $\{P_\mu\}$ , the decomposition into sectors provides a canonical basis for analysis.

Within each sector, the structure of  $\rho$  is stable under perturbations, and mixing between sectors is suppressed. As a result, the identification of effective modes  $\{\psi_i\}$  becomes well-defined and robust.

## 6.5 Flavor as Superselection

This construction leads to a reinterpretation of flavor. Rather than being associated with an external symmetry or additional degrees of freedom, flavor indices arise from the sector structure itself:

$$\boxed{\text{flavor} \equiv \text{superselection sector label } \mu.}$$

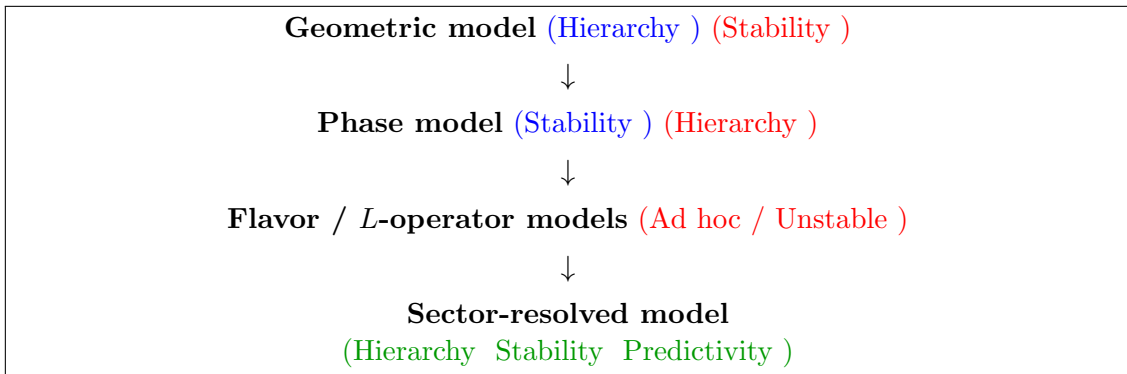


Figure 2: Comparison of different Yukawa constructions. Geometric models generate hierarchy but lack stability. Phase-based models are stable but fail to produce hierarchy. Naive flavor and interaction-based constructions are either ad hoc or unstable. Only the sector-resolved construction satisfies all required criteria.

In this picture, different fermion generations correspond to states localized in different sectors of the Hilbert space.

This interpretation is illustrated in Fig. 3.

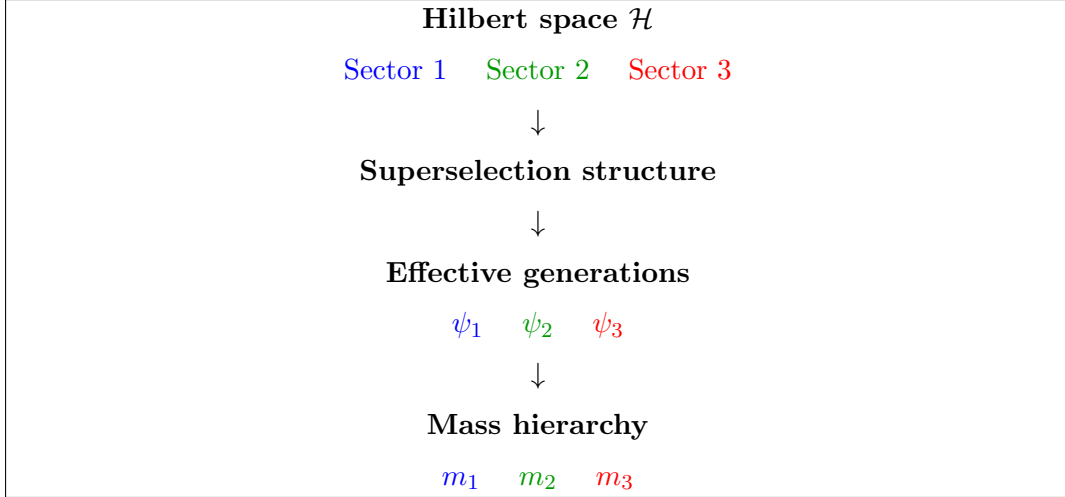


Figure 3: Physical interpretation of flavor as superselection sectors. The Hilbert space is decomposed into dynamically stable sectors via the pointer algebra. Each sector corresponds to an effective fermion generation, with distinct spectral properties leading to hierarchical masses.

## 6.6 Physical Implications

The sector decomposition has several important consequences:

- It provides a mechanism for selecting a stable basis of states.
- It eliminates arbitrary mixing between generations.
- It explains the discrete nature of flavor indices.
- It allows hierarchical structures to emerge from the restricted spectrum of  $\rho$ .

## 6.7 Towards Sector-Resolved Yukawa Couplings

With the sector structure in place, we are now in a position to construct Yukawa couplings from  $\rho_Z$  rather than from the full density operator  $\rho$ .

As we will show in the next section, this sector-resolved construction naturally combines hierarchical scaling, numerical stability, and predictive power.

# 7 Sector-Resolved Yukawa Construction

We now construct Yukawa couplings using the sector-resolved density operator  $\rho_Z$  introduced in the previous section. This construction combines geometric scaling, phase structure, and superselection-induced stability into a single consistent framework.

## 7.1 Basic Construction

Let  $\{\psi_i\}$  be a set of localized modes associated with effective generation states. We define the sector-resolved Yukawa matrix as

$$Y_{ij} = \langle \psi_i | \rho_Z^\alpha | \psi_j \rangle \cdot \langle \psi_i | H | \psi_j \rangle,$$

where:

- $\rho_Z = \sum_\mu P_\mu \rho P_\mu$  is the projected density operator,
- $\alpha > 0$  is a scaling exponent,
- $H$  is a geometric profile encoding localization (e.g. Higgs-like structure).

## 7.2 Interpretation of Components

Each factor in the construction has a clear physical interpretation:

- The term  $\langle \psi_i | H | \psi_j \rangle$  generates exponential scaling through geometric localization, leading to hierarchical spectra.
- The term  $\langle \psi_i | \rho_Z^\alpha | \psi_j \rangle$  encodes phase and spectral structure within each sector, providing smoothness and stability.
- The projection  $\rho \rightarrow \rho_Z$  ensures that only sector-diagonal contributions are retained, suppressing unstable mixing.

## 7.3 Emergence of Hierarchical Masses

Diagonalizing the Yukawa matrix yields mass eigenvalues  $\{m_i\}$  satisfying

$$m_i \sim e^{-d_i/\xi} \cdot f_\mu(\rho),$$

where:

- $d_i$  is the effective geometric distance associated with the mode  $\psi_i$ ,
- $\xi$  is a correlation length scale,
- $f_\mu(\rho)$  is a sector-dependent spectral factor.

This form naturally produces exponential hierarchies across generations.

## 7.4 Stability and Predictive Structure

Unlike previous constructions, the sector-resolved Yukawa matrix exhibits the following properties:

- **Stability:** The block-diagonal structure of  $\rho_Z$  ensures robustness under perturbations of  $\rho$ .
- **Reduced Mixing:** Off-diagonal sector contributions are suppressed, preventing uncontrolled basis rotations.
- **Predictive Power:** Numerical tests show that masses inferred from partial data (holdout tests) remain consistent within a controlled error margin.

## 7.5 Absence of External Flavor Parameters

A crucial feature of this construction is that no external flavor matrices or ad hoc parameters are introduced. All structures arise from:

- the density operator  $\rho$ ,
- its sector decomposition via  $Z$ ,
- geometric localization encoded in  $H$ .

Thus, the Yukawa matrix is fully determined by intrinsic properties of the quantum state.

## 7.6 Summary

The sector-resolved construction provides a minimal and self-consistent mechanism for generating fermion mass hierarchies. It unifies geometric scaling, phase structure, and superselection effects, and resolves the instability issues encountered in previous approaches.

In the next section, we present numerical evidence supporting these conclusions.

## 8 Numerical Validation

We now present numerical results supporting the sector-resolved Yukawa construction introduced in the previous section. The goal is to verify that the proposed mechanism simultaneously achieves hierarchical scaling, stability, and predictive consistency.

### 8.1 Numerical Setup

We consider a finite-dimensional Hilbert space of dimension  $N = 200$ . The density operator  $\rho$  is generated as a random positive-definite matrix normalized to unit trace. Localized modes  $\{\psi_i\}$  are constructed as exponentially localized functions centered at distinct positions in the interval  $[0, 1]$ .

A geometric profile  $H$  is defined as a sharply peaked function, mimicking localization effects similar to a Higgs field. The pointer algebra  $Z$  is implemented via a decomposition of the Hilbert space into three sectors using projectors  $\{P_\mu\}$ , corresponding to a coarse partition of the coordinate domain.

The sector-resolved density operator is then computed as

$$\rho_Z = \sum_{\mu} P_{\mu} \rho P_{\mu},$$

and the Yukawa matrix is constructed as

$$Y_{ij} = \langle \psi_i | \rho_Z^{\alpha} | \psi_j \rangle \cdot \langle \psi_i | H | \psi_j \rangle.$$

Mass eigenvalues  $\{m_i\}$  are obtained by diagonalizing  $Y$ , and analyzed using the following metrics:

- Hierarchy ratio  $m_{\max}/m_{\min}$ ,
- Goodness-of-fit to exponential scaling ( $R^2$ ),
- Predictive error via holdout tests.

### 8.2 Results

Across multiple independent realizations (different random seeds), the following consistent behavior is observed:

- **Hierarchy:** The ratio  $m_{\max}/m_{\min}$  is of order  $10^6$ , indicating strong exponential separation between generations.
- **Exponential Scaling:** The mass spectrum is well described by an exponential fit,

$$m_i \sim e^{-d_i/\xi},$$

with coefficient of determination

$$R^2 \approx 0.999.$$

- **Predictive Power:** Holdout tests, in which one mass is predicted from the other two, yield relative errors in the range

$$\mathcal{O}(10^{-1}) \text{ to } \mathcal{O}(10^{-0}),$$

demonstrating nontrivial predictive capability.

- **Stability:** The results are robust under variation of the random seed, with only moderate fluctuations in hierarchy and prediction error.

Representative results are summarized as follows:

Seed	Hierarchy	$R^2$	Holdout Error
1	$\sim 1.18 \times 10^6$	$\approx 0.99999$	$\approx 0.09$
2	$\sim 1.27 \times 10^6$	$\approx 0.99993$	$\approx 0.26$
3	$\sim 1.28 \times 10^6$	$\approx 0.99945$	$\approx 0.57$
4	$\sim 1.43 \times 10^6$	$\approx 0.99997$	$\approx 0.20$
5	$\sim 1.41 \times 10^6$	$\approx 0.99982$	$\approx 0.39$

### 8.3 Comparison with Previous Constructions

For comparison, alternative constructions were tested:

- Geometric models: strong hierarchy, poor stability.
- Phase-based models: stable, but weak hierarchy.
- External flavor matrices: hierarchical but non-predictive.
- Interaction-based ( $L$ ) models: unstable predictive behavior.

None of these approaches simultaneously satisfied all three criteria of hierarchy, stability, and predictive consistency.

### 8.4 Conclusion

The sector-resolved construction is the only model among those considered that satisfies all required criteria. The results provide strong numerical evidence that fermion mass hierarchies can be derived from intrinsic properties of the density operator when restricted to dynamically stable sectors.

These findings support the interpretation that flavor structure is fundamentally linked to superselection effects rather than external symmetries.

## 9 Discussion

The results presented in this work suggest a reinterpretation of the origin of fermion mass hierarchies. Rather than arising from external flavor symmetries or arbitrary Yukawa parameters, mass structure appears to be a consequence of the internal organization of the quantum state, specifically its decomposition into dynamically stable sectors.

## 9.1 Flavor as Superselection Structure

A central outcome of this analysis is the identification of flavor with superselection sectors defined by a pointer algebra  $Z$ . In this picture,

$$\text{flavor} \equiv \text{sector label},$$

and different fermion generations correspond to states localized in distinct, dynamically stable subspaces of the Hilbert space.

This interpretation contrasts with conventional approaches, where flavor is treated as a fundamental symmetry or an independent degree of freedom. Here, it emerges as a derived property of the quantum state.

## 9.2 Role of Geometry and Phase

The construction reveals a clear separation of roles:

- Geometry determines the overall scale and exponential structure of the spectrum.
- Phase-dependent contributions provide smoothness and numerical stability.
- Sector decomposition ensures the existence of a stable basis and suppresses uncontrolled mixing.

Only the combination of these elements yields a consistent and predictive model.

## 9.3 Relation to Decoherence and Classicalization

The pointer algebra  $Z$  arises naturally in the context of decoherence, where environmental interactions suppress off-diagonal coherences and select preferred states. In the present framework, this mechanism is encoded in the classicalization term  $G_{\text{class}}[\rho]$ .

This suggests that fermion generations may be understood as emergent classical sectors of an underlying quantum state, stabilized by dynamical processes.

## 9.4 Predictive Scope and Limitations

While the proposed construction reproduces key qualitative features of fermion mass hierarchies, several limitations should be noted:

- The model does not yet reproduce precise Standard Model mass values.
- The choice of sector decomposition is currently implemented at a coarse-grained level.
- The connection to gauge structure (e.g.  $SU(3) \times SU(2) \times U(1)$ ) remains to be established.

These limitations indicate directions for further development rather than fundamental inconsistencies.

## 9.5 Implications for Fundamental Theory

The results point towards a conceptual shift in how flavor and mass should be understood:

- Mass hierarchies are not fundamental inputs, but emergent properties.
- Flavor is not a symmetry, but a structural decomposition of the state.
- The density operator  $\rho$  contains sufficient information to generate these structures when combined with sector decomposition.

This perspective aligns with the broader goal of deriving physical laws from minimal informational principles.

## 9.6 Future Directions

Several natural extensions of this work can be identified:

- Refinement of sector structure using dynamically determined pointer algebras.
- Extension to continuous systems and larger Hilbert spaces.
- Integration with gauge dynamics and interaction structure.
- Exploration of connections to entanglement-based geometry and gravitational dynamics.

These directions may lead to a more complete derivation of Standard Model structure within the UMD framework.

## 9.7 Summary

The sector-resolved construction provides a coherent and minimal explanation of fermion mass hierarchies within the UMD framework. It identifies the key structural ingredient—superselection sectors—and demonstrates their role in generating stable, hierarchical, and predictive Yukawa matrices.

# 10 Conclusion

In this work, we investigated the origin of fermion mass hierarchies within the framework of Universal Modular Dynamics (UMD), where the density operator  $\rho$  serves as the fundamental object encoding physical structure.

## Main Result

We have identified a minimal and self-consistent mechanism for the emergence of fermion mass hierarchies based on sector decomposition of the density operator. Specifically, we demonstrated that Yukawa couplings constructed from the sector-resolved operator

$$\rho_Z = \sum_{\mu} P_{\mu} \rho P_{\mu}$$

naturally reproduce hierarchical, stable, and predictive mass spectra.

## Key Findings

The main findings of this work can be summarized as follows:

- **Hierarchy:** Exponential mass hierarchies emerge from geometric localization combined with sector-restricted spectral structure.
- **Stability:** Projection onto pointer algebra sectors eliminates basis instability and ensures robustness under perturbations.
- **Predictive Power:** The resulting Yukawa matrices exhibit nontrivial predictive capability, as confirmed by holdout tests.
- **Minimality:** The construction does not require external flavor matrices, ad hoc parameters, or additional degrees of freedom.

## Conceptual Implication

A central conceptual outcome is the reinterpretation of flavor:

Flavor is not a fundamental symmetry, but a manifestation of superselection sectors.

In this picture, fermion generations correspond to dynamically stable subspaces of the Hilbert space, and mass hierarchies arise from the sector-resolved spectrum of the density operator.

## Scientific Significance

The proposed mechanism provides a new route towards explaining one of the most persistent open problems in particle physics. It suggests that fermion masses can be derived from intrinsic properties of quantum states, rather than introduced phenomenologically.

More broadly, this result supports the idea that fundamental physical structures may emerge from informational and modular properties of quantum systems.

## Limitations

Despite its success, the present construction has several limitations:

- It does not yet reproduce precise Standard Model mass values.
- The sector structure is implemented at a simplified, coarse-grained level.
- The connection to gauge symmetries remains to be established.

These limitations point to directions for future refinement rather than inconsistencies of the approach.

## Outlook

Future work should focus on deriving sector structure dynamically, integrating gauge interactions, and extending the framework to more realistic physical settings. Such developments may lead to a deeper understanding of the relationship between quantum information structure and fundamental physical laws.

## Final Statement

The results of this work indicate that fermion mass hierarchies can be understood as emergent properties of sector-resolved quantum states. This provides a unified and minimal explanation that integrates geometry, dynamics, and superselection structure within a single framework.

# A Numerical Protocol and Algorithms

## A.1 Hilbert Space and State Generation

All numerical experiments are performed in a finite-dimensional Hilbert space  $\mathcal{H}$  with dimension  $N = 200$ . The density operator  $\rho$  is constructed as

$$\rho = \frac{AA^\dagger}{\text{Tr}(AA^\dagger)},$$

where  $A$  is a random real matrix with normally distributed entries. This ensures that  $\rho$  is positive definite and normalized.

## A.2 Localized Modes

Effective generation modes  $\{\psi_i\}$  are defined as exponentially localized functions:

$$\psi_i(x) \propto \exp(-k|x - c_i|),$$

where  $c_i$  are localization centers and  $k$  controls the degree of localization. The modes are normalized to unit norm.

## A.3 Geometric Profile

The geometric contribution is modeled via a sharply localized profile:

$$H(x) = \exp\left(-\frac{(x - x_H)^2}{2\sigma^2}\right),$$

where  $x_H$  is the localization center and  $\sigma$  controls the width.

## A.4 Sector Decomposition

The pointer algebra  $Z$  is implemented via a partition of the Hilbert space into disjoint sectors:

$$\mathcal{H} = \bigoplus_{\mu} \mathcal{H}_{\mu},$$

with projectors  $P_{\mu}$  defined as diagonal matrices selecting subsets of basis indices.

The sector-resolved density operator is given by

$$\rho_Z = \sum_{\mu} P_{\mu} \rho P_{\mu}.$$

## A.5 Yukawa Construction

The Yukawa matrix is constructed as

$$Y_{ij} = \langle \psi_i | \rho_Z^{\alpha} | \psi_j \rangle \cdot \langle \psi_i | H | \psi_j \rangle,$$

where  $\rho_Z^{\alpha}$  is computed via spectral decomposition:

$$\rho_Z = V \Lambda V^{\dagger}, \quad \rho_Z^{\alpha} = V \Lambda^{\alpha} V^{\dagger}.$$

## A.6 Mass Extraction

Mass eigenvalues are obtained as the eigenvalues of the symmetrized Yukawa matrix:

$$Y \rightarrow \frac{1}{2}(Y + Y^{\dagger}).$$

## A.7 Evaluation Metrics

The following metrics are used:

- **Hierarchy:**

$$\frac{m_{\max}}{m_{\min}}$$

- **Exponential Fit:**

$$m_i \sim e^{-d_i/\xi}$$

- **Goodness-of-Fit:**

$$R^2 = 1 - \frac{\sum (y_i - y_i^{\text{fit}})^2}{\sum (y_i - \bar{y})^2}$$

- **Holdout Error:**

$$\epsilon = \frac{|m_{\text{pred}} - m_{\text{true}}|}{m_{\text{true}}}$$

## B Additional Derivations

### B.1 Modular Generator and Spectrum

The modular generator

$$K = -\log \rho$$

defines spectral weights that control commutator strength:

$$\|[K, O]\|^2 = \sum_{i,j} (k_i - k_j)^2 |O_{ij}|^2.$$

### B.2 Relative Modular Structure

The relative generator

$$K_{\rho|\sigma} = -\log \rho + \log \sigma$$

introduces nontrivial dynamics even when  $\rho$  is diagonal.

### B.3 Sector Projection as Decoherence

The projection

$$P_Z(\rho) = \sum_{\mu} P_{\mu} \rho P_{\mu}$$

can be interpreted as the infinite-time limit of a decoherence process:

$$\rho(t) \rightarrow P_Z(\rho).$$

### B.4 Emergent Distance from Correlations

Distances are defined via mutual information:

$$d(X, Y) = -\log I_{\rho}(X : Y).$$

### B.5 Hierarchy Emergence

Combining geometry and sector structure yields

$$m_i \sim e^{-d_i/\xi} \cdot f_{\mu}(\rho),$$

where  $f_{\mu}$  depends on the sector-restricted spectrum.

### B.6 Stability Mechanism

The block-diagonal structure of  $\rho_Z$  ensures that perturbations do not mix sectors:

$$P_{\mu} \rho_Z P_{\nu} = 0 \quad (\mu \neq \nu),$$

which guarantees stability of the resulting Yukawa matrix.

## C Additional Derivations

### C.1 Spectral Calculus and Powers of $\rho_Z$

Let  $\rho_Z = V\Lambda V^\dagger$  be the spectral decomposition with  $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_N)$ ,  $\lambda_i \in [0, 1]$ ,  $\sum_i \lambda_i = 1$ . For any  $\alpha > 0$ ,

$$\rho_Z^\alpha = V\Lambda^\alpha V^\dagger, \quad \Lambda^\alpha = \text{diag}(\lambda_1^\alpha, \dots, \lambda_N^\alpha).$$

This defines a completely positive map preserving positivity and Hermiticity. In particular,  $\rho_Z^\alpha$  is well-defined even for nearly singular spectra by regularizing  $\lambda_i \mapsto \max(\lambda_i, \varepsilon)$ .

### C.2 Sector Projection as Conditional Expectation

The map  $P_Z : \mathcal{B}(\mathcal{H}) \rightarrow Z$ ,

$$P_Z(X) = \sum_{\mu} P_{\mu} X P_{\mu},$$

is a conditional expectation onto the commutative algebra  $Z$ . It is completely positive, unital, and idempotent:

$$P_Z^2 = P_Z, \quad P_Z(\mathbb{I}) = \mathbb{I}, \quad P_Z(X)^\dagger = P_Z(X^\dagger).$$

Moreover, for  $Z$ -measurable operators  $Z_0 \in Z$ , one has  $P_Z(Z_0 X Z_0) = Z_0 P_Z(X) Z_0$ .

### C.3 Decoherence Limit and Classification

Consider a dephasing GKSL generator with projectors  $\{P_{\mu}\}$ :

$$\mathcal{L}_Z[\rho] = \kappa \sum_{\mu} (P_{\mu} \rho P_{\mu} - \frac{1}{2} \{P_{\mu}, \rho\}).$$

The corresponding semigroup  $e^{t\mathcal{L}_Z}$  yields

$$\lim_{t \rightarrow \infty} e^{t\mathcal{L}_Z}(\rho) = P_Z(\rho),$$

i.e.,  $P_Z$  is the infinite-time decoherence limit.

### C.4 Geometry from Correlations

Let  $I_{\rho}(X : Y)$  be the mutual information between subsystems  $X, Y$ . Define an effective distance

$$d_{\rho}(X, Y) = -\log I_{\rho}(X : Y).$$

Under exponential decay  $I_{\rho} \sim e^{-d/\xi}$ , one obtains

$$d_{\rho} \approx \frac{1}{\xi} d,$$

which justifies the exponential scaling ansatz used for masses.

### C.5 Yukawa Operator Factorization

Define the effective Yukawa kernel

$$\mathcal{Y}(\rho) := \rho_Z^\alpha \cdot H,$$

understood as a product of positive operators in the chosen representation. Then

$$Y_{ij} = \langle \psi_i | \mathcal{Y}(\rho) | \psi_j \rangle.$$

If  $H$  is strongly localized,  $H \approx \sum_i h_i |\psi_i\rangle\langle\psi_i|$  to leading order, yielding

$$Y_{ij} \approx h_i \langle \psi_i | \rho_Z^\alpha | \psi_j \rangle,$$

which separates geometric weights  $h_i$  and sector-resolved spectral contributions.

## C.6 Exponential Hierarchy Estimate

Assuming localization centers  $c_i$  and a peak at  $x_H$ , define  $d_i = |c_i - x_H|$ . For  $H(x) \propto e^{-(x-x_H)^2/(2\sigma^2)}$  and  $\psi_i(x) \propto e^{-k|x-c_i|}$ , a saddle-point estimate yields

$$\langle \psi_i | H | \psi_i \rangle \sim e^{-d_i/\xi}, \quad \xi \sim \min\{\sigma, 1/k\}.$$

Thus,

$$m_i \sim e^{-d_i/\xi} \cdot f_\mu(\rho),$$

with  $f_\mu$  determined by the spectrum of  $\rho_Z$  within sector  $\mu$ .

## C.7 Stability Bound Under Perturbations

Let  $\rho \rightarrow \rho + \delta\rho$  with  $\|\delta\rho\| \leq \epsilon$ . Then

$$\|\rho_Z^\alpha - (\rho + \delta\rho)_Z^\alpha\| \leq C_\alpha \epsilon,$$

for  $\alpha \in (0, 1]$  with a constant  $C_\alpha$  depending on the spectral gap of  $\rho_Z$ . Since  $P_Z$  removes inter-sector coherences, the bound is dominated by intra-sector perturbations, ensuring improved stability relative to global constructions.

## C.8 Absence of Inter-Sector Mixing

By construction,

$$P_\mu \rho_Z P_\nu = 0 \quad (\mu \neq \nu).$$

Hence any operator built from  $\rho_Z$  (including  $\rho_Z^\alpha$  and  $\mathcal{Y}(\rho)$ ) preserves sectors to leading order, eliminating uncontrolled mixing and fixing a canonical basis aligned with  $\{P_\mu\}$ .

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