

# On the Absence of Automatic Linear Lorentz-Violating Dispersion from discreteness Alone

*A note on emergent-metric discrete spacetime models*

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## Abstract

Discrete models of spacetime are often assumed to generically predict a first-order, energy-dependent (linear-in-E) Lorentz-violating dispersion of high-energy photons, of the kind tightly constrained by gamma-ray burst (GRB) timing observations. We show that this implication does not follow from discreteness alone: it requires an additional, independent structural assumption - namely, that local phase contributions accumulated along propagation are coherently aligned with a fixed, globally preferred frame. In any discrete substrate where the physical metric is emergent and reconstructed (rather than given by a fixed embedding), and where local phase contributions are not long-range correlated, the accumulated phase grows only as the square root of the number of traversed elements, suppressing any linear dispersion signal. This result is structural and substrate-independent: it applies to any discrete model satisfying the stated decorrelation property, and is intended as a general consistency tool for the broader class of emergent-geometry approaches to quantum gravity, not as evidence for any specific model.

The mechanism discussed here corresponds to the nonsystematic Lorentz-violation regime, where microscopic fluctuations average statistically rather than accumulate coherently.

Previous analyses of nonsystematic Lorentz violation have shown that fluctuating microscopic structures need not generate a coherent propagation effect. The present theorem formalizes this observation by identifying coherent accumulation as a necessary condition for a linear-in-energy dispersion term to arise from discrete structure.

## 1. Motivation

A standard objection to discrete spacetime models is that discreteness should generically introduce a preferred reference frame, leading to an energy-dependent speed of light and hence a measurable dispersion in the arrival times of photons of different energies from distant sources. Observations of gamma-ray bursts, in particular GRB 090510, 221009A, etc. have placed stringent bounds on any such linear-in-energy dispersion, pushing the relevant quantum-gravity energy scale at or above the Planck energy. This is frequently treated as a near-generic constraint on “discrete spacetime” as a category.

This inference is too fast. It conflates two logically distinct properties of a discrete model: discreteness of the underlying substrate, and the existence of a fixed, rigid embedding of that substrate into a metric space with a privileged orientation. The latter, not the former, is what produces a generic linear dispersion signal. Causal set theory has long illustrated this distinction at the level of the substrate itself - a Lorentz-invariant discretization (a Poisson sprinkling) avoids a preferred frame by construction. The present note generalizes the underlying mechanism to any model in which the metric itself, not merely the embedding, is emergent and dynamically reconstructed, and isolates the precise structural condition under which linear dispersion is or is not avoided.

## 2. Setup

Let  $G$  be a discrete substrate (a graph, causal set, or similar combinatorial structure) without a primitive metric.

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Physical distance  $D$  and duration are assumed to be reconstructed quantities, obtained from  $G$  via some statistical or dynamical procedure  $R$  (e.g. diffusion processes, correlation decay, or information-theoretic measures on  $G$ ), rather than read off a fixed embedding chosen in advance. Propagation of a massless excitation between two reconstructed points is modeled as traversal of  $N$  local configurations of  $G$ , each contributing a phase  $\delta\varphi_i$  that depends on the local orientation of that configuration relative to some reference direction.

A clarification is in order before stating the theorem, since the decorrelation condition used below is easy to misread as an appeal to chaotic or random underlying dynamics. It is not. Decorrelation of phase contributions,  $\langle \delta\varphi_i \delta\varphi_j \rangle \approx 0$ , is a statement about equidistribution - the statistical spread of orientations sampled along a path - not about sensitivity to initial conditions. A fully deterministic, non-chaotic, unitary dynamics can still produce a sequence of orientations that is equidistributed and effectively uncorrelated in the relevant statistical sense; the digits of an irrational rotation on a torus are a standard example of a deterministic, non-chaotic sequence with exactly this property. The decorrelation assumed here is therefore compatible with - and, in a fully unitary microscopic theory, should be understood as arising from an epistemic coarse-graining performed by any observer or propagating excitation that cannot resolve the full microscopic configuration of  $G$ , rather than from any genuine, ontic chaos in the underlying dynamics. This distinction matters: it keeps the present argument consistent with microscopic theories that are deterministic and non-chaotic at the ontic level, while still producing apparent randomness at the coarse-grained, epistemic level relevant to propagation.

### **Theorem on the Absence of Automatic Linear Dispersion from Discreteness**

A first-order Lorentz-violating dispersion relation - i.e., an accumulated phase  $\Phi_N = \sum_{n=1}^N \delta\varphi_i$  growing linearly,  $\Phi_N \propto N$ , equivalently a dispersive term linear in  $\frac{E}{E_{Pl}}$  - is **not** a generic consequence of the discreteness of  $G$  alone. It requires an additional, independent structural assumption: that the local phase contributions  $\delta\varphi_i$  are coherently aligned along the propagation direction, i.e.,  $\langle \delta\varphi_i \rangle \neq 0$  with a consistent sign set by a globally preferred frame.

The theorem does not exclude Lorentz-violating dispersion in all discrete spacetime models; it proves that discreteness alone is insufficient to derive a universal linear dispersion law.

**Proof sketch.** If  $R$  does not single out a preferred frame - i.e., there is no canonical, fixed embedding of  $G$  into a metric space, consistent with the metric being emergent - then the orientation of local configurations sampled along a reconstructed path is not fixed relative to the propagation direction. Suppose, in addition, the local phase contributions are uncorrelated at long range:

$$\langle \delta\varphi_i \delta\varphi_j \rangle \approx 0, \quad i \neq j.$$

This decorrelation condition is itself a separate, checkable property of the reconstruction procedure  $R$  - specifically, an equidistribution property of orientations under  $R$ , recovered at the epistemic, coarse-grained level rather than presupposed as ontic randomness or chaos (see the remark above) and not an assumption about discreteness per se. Under this condition, the variance of the accumulated phase is additive,

$$Var(\Phi_N) = N \cdot Var(\delta\varphi), \quad \text{so } \sigma_\Phi \sim \sqrt{N},$$

rather than  $\sigma_\Phi \sim N$ . A linear-in- $N$  (coherent) signal therefore requires either (a) a fixed background embedding, which breaks the emergent-metric assumption, or (b) explicit long-range phase correlation  $\langle \delta\varphi_i \delta\varphi_j \rangle \neq 0$  surviving reconstruction. Neither follows from discreteness of  $G$  by itself. ■

### 3. Corollary and Scope

**Corollary.** Observed linear Lorentz-violating dispersion (or its absence) is informative about discrete models *only* to the extent that those models additionally assume a fixed, rigid embedding with a privileged orientation. Models with emergent, reconstructed, and dynamically reconfiguring metrics are not automatically constrained by GRB linear-dispersion bounds in the way a static crystalline lattice would be; the bound constrains the coherence assumption, not discreteness as such.

**Scope.** The argument is purely structural: it uses no feature specific to any particular discrete-gravity proposal beyond the absence of a fixed embedding and the decorrelation property  $\langle \delta\varphi_i \delta\varphi_j \rangle \approx 0$ . It therefore applies to any discrete substrate with an emergent, reconstructed metric satisfying this property, including (but not limited to) causal set theory and related emergent-geometry approaches to quantum gravity. The theorem is accordingly a *consistency result* protecting an entire class of theories from a specific objection - it does not, by itself, provide evidence for any individual member of that class, and does not by itself constitute a falsifiable prediction. Discriminating between specific models in this class requires additional, model-specific structure beyond what is assumed here.

**Conditional result.** The theorem establishes that linear dispersion is not *forced* by discreteness; it does not establish that it is absent in any specific model. For a given discrete substrate and reconstruction procedure  $R$ , verifying the decorrelation property  $\langle \delta\varphi_i \delta\varphi_j \rangle \approx 0$  explicitly, and ruling out any residual preferred frame surviving reconstruction, remains a separate, model-specific task.

### 4. Relation to Observational Bounds

Current GRB-based bounds constrain the quantum-gravity energy scale associated with linear-in-energy photon dispersion within phenomenological models that assume coherent phase accumulation. The theorem above clarifies the logical status of these bounds with respect to discrete spacetime models in general: they constrain models that assume coherent phase accumulation relative to a fixed frame, and are silent - to leading structural order on models in which the metric is emergent and the decorrelation property holds. This does not weaken the observational bounds; it sharpens what class of theoretical models they actually constrain.

### 5. A Second, Distinct Prediction: Stochastic Spreading vs. Systematic Dispersion

It is important not to conflate two logically distinct observables. Systematic dispersion is a shift in the mean arrival-time difference between photons of different energies, growing as  $\langle \Delta t \rangle \propto E$  and tested directly by the GRB timing bounds discussed above. Stochastic wavefront spreading (sometimes called spacetime “fuzziness”) is a different quantity: the random spread of arrival times around the mean,  $\sigma(\Delta t)$ , which can grow with distance even when the mean shift is exactly zero. This has been tested separately, for instance by modeling photon speeds as normally distributed around  $c$  with an energy-dependent standard deviation.

The theorem above bears directly on this second quantity, and the connection is worth making explicit rather than left implicit. The decorrelation condition  $\langle \delta\varphi_i \delta\varphi_j \rangle \approx 0$  does not merely suppress the *mean* accumulated phase to zero; it leaves a nonzero *variance*,  $\sigma_\phi \sim \sqrt{N}$ , growing with the number of traversed configurations. This residual variance is precisely the stochastic counterpart of wavefront spreading: the theorem predicts not an exactly null result for  $\sigma(\Delta t)$ , but a specific, suppressed scaling for it, distinct from - and weaker than the naive coherent estimate that would follow from a fixed embedding.

Two consequences follow. First, the suppression of systematic dispersion established here does not, by itself, establish suppression of stochastic spreading to the same order; the two must be bounded separately, by translating  $\sigma_\phi$  into a time-domain quantity  $\sigma(\Delta t)$  with correct dimensional factors and comparing it against the empirical fuzziness bound. Second, because  $\sigma_\phi$  is manifestly nonzero, the

theorem does not predict an exact absence of spreading - only a strongly suppressed one, consistent with current non-detection but leaving, in principle, a residual, statistically recoverable signal at higher sensitivity. This sharpens the falsifiability of the framework: a future detection of stochastic spreading at a level inconsistent with the  $\sqrt{N}$  suppression derived here, but inconsistent also with a null result, would constrain the decorrelation assumption itself.

## 6. Boost Invariance of the Decorrelation Condition: A Meta-Theorem

The theorem established above suppresses linear Lorentz-violating dispersion in any given inertial frame. A natural and deeper question is whether the decorrelation condition itself is the structural assumption on which the theorem rests - preserved under Lorentz boosts. If it is not, then a boosted observer could in principle see coherent phase accumulation even when a stationary observer does not, restoring a frame-dependent dispersion signal. We show here that this does not occur, provided the metric is emergent and the observer's measuring apparatus is composed of the same dynamical fields as the substrate itself.

**Lemma (Boost Invariance of Decorrelation).** Let each observer define the measure  $\mu$  on the space of local plaquette orientations relative to their own reconstructed coordinates and measuring apparatus. Then the decorrelation condition

$$\langle \delta\varphi_i \delta\varphi_j \rangle_{\mu_{obs}} = 0, i \neq j,$$

holds in the reference frame of every observer, independently of their velocity. Consequently, the  $\sqrt{N}$  suppression of dispersion established in the main theorem applies in all inertial frames. ■

**Proof sketch.** Let  $S$  be a stationary observer and  $S'$  a boosted observer with boost parameter  $\gamma$ . The local phase deviation at plaquette  $i$  transforms as  $\delta\varphi'_i = g(o_i, \Lambda)$ , where  $o_i$  is the local orientation of plaquette  $i$  and  $\Lambda$  is the boost (a deterministic, global parameter, not a random variable). Since  $\delta\varphi'_i$  depends only on local data at  $i$  and the global parameter  $\Lambda$ , and since  $\Lambda$  is the same for all  $i$ , the independence of  $o_i$  and  $o_j$  for  $i \neq j$  (decorrelation in  $S$ ) implies:

$$\langle g(o_i, \Lambda) \cdot g(o_j, \Lambda) \rangle = \langle g(o_i, \Lambda) \rangle \cdot \langle g(o_j, \Lambda) \rangle.$$

For  $\langle \delta\varphi'_i \rangle = 0$  to hold in  $S'$ , the measure  $\mu' = (\Lambda)_+\mu$  (the push-forward of  $\mu$  under  $\Lambda$ , defined as  $(\Lambda)_+\mu(A) = \mu(\Lambda^{-1}(A))$  for measurable  $A$  - standard measure theory, unrelated to the non-standard ‘\*’ notation of hyperreal analysis) must be zero-mean for  $g$ . Here the emergent-metric condition is essential: because the observer's measuring apparatus consists of the same dynamical fields as the substrate, the boost transforms both the substrate *and* the apparatus coherently. The observer  $S'$  defines orientation relative to their own reconstructed frame, so their effective measure is  $\mu' = (\Lambda)_+\mu$  expressed in their own coordinates which, relative to those coordinates; is again equidistributed by the same argument as in  $S$ . No observer-independent preferred orientation survives, because there is no observer-independent background against which to define one.

This is the precise mathematical content of the physical picture: discrete elements do not sit inside space - they constitute it. When an observer performs a boost, their measuring instruments (wave packets of the same dynamical fields) undergo the same phase-gradient transformation as the substrate. The phase gradient steepens along the direction of motion; the observer sees Lorentz-contracted lengths precisely because more phase cycles fit between the endpoints of a boosted ruler, not because any fundamental ‘cell’ has been compressed. The decorrelation condition  $\langle \delta\varphi_i \delta\varphi_j \rangle = 0$  is a statement about local phase noise relative to the mean gradient, and since both the noise and the gradient transform together under the boost, their ratio and hence the decorrelation - is preserved.

**Remark on the measure.** For rotations the above argument is automatic:  $SO(3)$  is compact and its Haar measure is rotation-invariant, so  $(\Lambda)_+\mu = \mu$  exactly. For boosts the argument is different in character: we do not claim  $(\Lambda)_+\mu = \mu$  (this would require a finite invariant measure on the non-compact boost group, which does not exist). Instead, we claim that  $(\Lambda)_+\mu$ , when re-expressed in the coordinates of  $S'$ , is zero-mean for the same structural reason that  $\mu$  is zero-mean in  $S$ : the absence of a background preferred frame. This is a weaker but sufficient condition, and it is precisely what the emergent-metric assumption provides.

**Corollary.** Under the emergent-metric assumption, the  $\sqrt{N}$  suppression of Lorentz-violating photon dispersion is frame-independent: it holds for all inertial observers, provided that the decorrelation condition is preserved under reconstruction of the observer's frame. No preferred frame is introduced or implied. The reconstructed propagation speed remains invariant for all inertial observers, and Lorentz contraction of macroscopic lengths is fully consistent with the microscopic phase-averaging mechanism, because the measuring apparatus and the underlying degrees of freedom belong to the same dynamical structure.

## 7. Observational Signature: What Astronomers Can Look For.

The theorem establishes that no coherent, linear-in- $E$  dispersion signal is expected from the class of emergent-metric discrete theories satisfying the decorrelation condition. However, the residual statistical phase variance  $\sigma_\phi \sim \sqrt{N}$  is not zero: it represents a genuine, in-principle detectable signature. This section specifies what form that signature takes, distinguishing what can be said model-independently from what requires a specific theory.

### 7.1 The Model-Independent Formula Within the Emergent-Metric Class

For any theory in the emergent-metric class satisfying the conditions of the theorem, the residual stochastic spread in photon arrival times scales as:

$$\sigma(\Delta t) \sim \frac{E}{E_{Pl}} \frac{\sqrt{\ell_{eff} D}}{c},$$

where  $D$  is the source distance,  $\frac{E}{E_{Pl}} \sim 10^{-17}$  for high-energy GRB photons, and  $\ell_{eff}$  is the effective step scale of the discrete substrate - a free parameter at this level of generality, to be fixed by the specific theory. This signal grows as  $\sqrt{D}$  rather than as  $D$  (the coherent case) or as a constant (no effect). This  $\sqrt{D}$  scaling is the universal, model-independent prediction of the emergent-metric class, and it is what distinguishes this class from both coherent Lorentz-violating models and from trivially null predictions.

### 7.2 What Requires a Specific Theory

Three inputs are needed to convert  $\sigma(\Delta t)$  into a definite number:

**(i) The value of  $\ell_{eff}$ .** In any specific model this is fixed by the theory's own calibration procedure - for instance, by requiring the spectral dimension  $d_s \rightarrow 4$  at large scales, or by matching to the observed cosmological constant. Without this,  $\ell_{eff}$  is a free parameter ranging from the Planck length upward. ( $d_s \rightarrow 4$  a general property of the class, whereas the specific form of  $d_s(L)$  distinguishes between theories within the class).

**(ii) The correlation length  $\xi$  of residual phase correlations.** The decorrelation condition  $\langle \delta\varphi_i \delta\varphi_j \rangle \approx 0$  is an idealization. Realistic theories will have a finite correlation length  $\xi$  such that  $\langle \delta\varphi_i \delta\varphi_j \rangle \sim \exp\left(-\frac{|i-j|}{\xi}\right)$ . When  $\xi > 1$  (in units of  $\ell_{eff}$ ), the effective step size in the  $\sqrt{N}$  formula

becomes  $\ell_{eff} \cdot \xi$  rather than  $\ell_{eff}$  alone. The value of  $\xi$  is a dynamical property of the specific reconstruction procedure R.

**(iii) The angular dependence.** The residual signal is statistical and manifests as a correlation between arrival-time spreads of GRBs in nearby sky directions. The angular profile of this correlation depends on whether the theory has large-scale anisotropy in its ensemble structure. A fully isotropic emergent-metric theory predicts no preferred direction in the sky correlation; a theory with residual anisotropy predicts a direction-dependent enhancement. This is a model-specific, not a universal, prediction.

### 7.3 Observational Strategy

The following observational programme follows directly from the theorem, independent of any specific model:

**Do not look for a systematic delay in a single GRB.** The theorem rules out a coherent, linear-in-E delay at the level of current sensitivity. A detection in a single source would falsify the decorrelation condition, not confirm the emergent-metric framework.

**Look for  $\sqrt{D}$ -scaling of the arrival-time spread across a large ensemble of GRBs.** Collect  $\sigma(\Delta t)$  for many sources at different distances D and fit to  $\sigma(\Delta t) \propto \sqrt{D}$ . A detection of this scaling, distinct from both  $\sigma \propto D$  (coherent LIV) and  $\sigma = \text{const}$  (no effect), would be positive evidence for the emergent-metric class as a whole.

**Look for sky-direction correlations in the spread, not in the mean.** Sources in nearby sky directions should show correlated  $\sigma(\Delta t)$  values if the residual phase correlations have a spatial structure in the emergent geometry. Sources in orthogonal directions should be uncorrelated. This is the angular analogue of the  $\sqrt{N}$  formula.

The predicted signal is unlikely to be detectable in a single GRB event. Its possible observation requires statistical analysis over a sufficiently large population of sources, where future observatories such as CTA, AMEGO-X, and space-based GRB monitors may provide the necessary sensitivity. The key quantity is not the per-burst sensitivity but the ensemble statistical power across hundreds or thousands of GRBs at known distances a regime that becomes accessible as GRB catalogues grow.

A null result would therefore not rule out discreteness itself. Instead, it would show that any viable discrete spacetime model must generate Lorentz invariance through a mechanism stronger than simple statistical phase averaging during the emergence of geometry.

**Summary.** The theorem predicts a specific, falsifiable observational signature:  $\sqrt{D}$ -scaling of the stochastic arrival-time spread in GRB ensembles, with no systematic per-energy delay in individual sources. The amplitude of this signal requires a specific theory to fix  $\ell_{eff}$  and  $\xi$ ; the scaling law itself is model-independent and follows from the decorrelation condition alone. A detection of coherent linear delay would falsify the decorrelation condition. A detection of  $\sqrt{D}$ -scaling spread would support the emergent-metric class. An exact null result in both channels would constrain  $\ell_{eff} \cdot \xi$  to be exponentially small, placing a strong bound on any specific model in this class.

This theorem and lemma identifies the missing assumption required for discreteness-induced Lorentz-violating dispersion to become observationally relevant.

## 8. Relation to nonsystematic Lorentz violation in discrete quantum gravity approaches

Discrete approaches to quantum gravity have previously considered the possibility that microscopic fluctuations of spacetime may induce Lorentz-violating effects. A key distinction in this context is

between systematic and nonsystematic Lorentz violation. In a systematic scenario, microscopic corrections accumulate coherently along the propagation path, producing an observable first-order dispersion term proportional to  $\frac{E}{E_{Pl}}$ . In contrast, theories without a preferred microscopic frame may produce nonsystematic Lorentz violation, where local fluctuations do not share a common orientation and therefore do not accumulate linearly. Instead, the net effect behaves statistically and may average out over large distances, leaving only stochastic residual fluctuations. The theorem presented here formalizes this distinction. It shows that discreteness alone is insufficient to imply a coherent linear dispersion signal. Such a signal requires an additional assumption: correlated accumulation of microscopic phase corrections along the propagation path. Therefore, observational constraints from gamma-ray bursts do not exclude discrete spacetime models in general. They constrain models in which microscopic discreteness generates systematic, coherent propagation effects. The remaining observable signature in the nonsystematic regime is not a linear time delay but a statistical fluctuation, whose scaling depends on the correlation structure of the underlying discrete geometry.

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