

Ternary Probability as a Natural Generalization of 3-Space

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Articles written by E. T. Whittaker in 1903 and 1904 suggest the natural generalization of 3-space as ternary probability. This is proven using basic mathematical statements and their associated transition as main eigenvalues from the product of the symmetric group S_2 and the Klein four-group $\mathbb{Z}_2^2$ to the dihedral of order 12 D_6 and related properties. A generalization of physical theory is suggested.

Ternary Probability

The work of E. T. Whittaker suggests that there are methods for computing wavefunctions by tetration, hyperoperations, interpolation and ternary probability.

Consider the statement:

$$\begin{aligned}x + y &= \sqrt{7} \\x - y &= \sqrt{3} \\xy &= 1\end{aligned}\tag{1}$$

Additionally consider the statement:

$$\sqrt[3]{8} = 2\tag{2}$$

Due to asymmetry in Cartesian coordinates (Whittaker, 1903), it is feasible to consider (1) as analogous to vector calculus in 3-space if xy is considered real as $\sqrt{-1}$ is imaginary, $x+y$ or $x-y$ are considered vectors in a 3-parameter space, and (2) is real. This and the property of infinite tetration unique to $\sqrt{2}$ suggest the implementation of ternary probability.

This is further suggested by (1) being commensurate with the transition as main eigenvalues from the product of the symmetric group S_2 and the Klein four-group $\mathbb{Z}_2^2$ (nilpotent, abelian-by-cyclic) to the dihedral of order 12 D_6 (non-abelian, solvable) in Galois theory (Du et al., 2021). The former permutes coordinates independently as a form of orthogonal sphericity $xy \perp zr$ (Titleman, 2026) while the latter combines a primary rotation of order 6 with a single axis reflection of order 2 as a form of wave propagation (Whittaker, 1904).

3-space is thus only one paradigm for modeling trajectories. Mathematical assumptions of cartesian symmetry and kinematics require matching physical assumptions, such as those made in the Clausius formulation for Root Mean Square (RMS) velocity.

References

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