

Spectral and Computational Structure of Probability in Universal Modular Dynamics

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Abstract

We develop a spectral and computational framework for probability and dynamics within the theory of Universal Modular Dynamics (UMD), where physical structure is encoded in a transition operator rather than in a predefined spacetime background.

Starting from a discrete set of states with transition probabilities, we introduce an information-theoretic distance based on the logarithm of transition weights and show that this structure induces an emergent geometry. In the continuum limit, this construction yields a Riemannian metric defined directly from the local behavior of the transition operator.

We establish a distinction between structural probability and dynamical probability, and introduce the concept of dynamic drift, which quantifies their deviation. A rigorous bound is derived showing that the magnitude of this drift is controlled by the spectral gap of the transition matrix. This leads to a natural renormalization group (RG) interpretation, where the subleading eigenvalue governs flow toward criticality.

We further demonstrate that UMD exhibits a well-defined phase structure characterized by spectral properties, including critical regimes with universal scaling behavior. Finite-size scaling and universality classes are identified, and the role of information retention is clarified.

Extending the geometric construction, we define curvature directly from non-factorization of transitions and show that an effective gravitational dynamics emerges. A variational principle is formulated, leading to field equations in tensor form, where curvature is determined by information density and its gradients.

We prove that the framework is generally covariant and establish a rigorous continuum limit connecting discrete transition dynamics to smooth manifolds. Consistency tests confirm the recovery of flat space, weak-field behavior, diffusion dynamics, and critical scaling.

These results suggest that geometry, dynamics, and gravitational behavior can be understood as emergent phenomena arising from the spectral structure of probabilistic transitions, providing a unified perspective on probability, information, and physical law.

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1 Introduction

The problem of deriving geometry, dynamics, and physical law from more primitive principles remains one of the central challenges in theoretical physics. In conventional approaches, space-time geometry is assumed a priori, and physical processes are described as fields evolving on a fixed or dynamical geometric background. However, this framework leaves open the question of whether geometry itself can emerge from a deeper, non-geometric structure.

In this work, we develop a formulation of Universal Modular Dynamics (UMD) in which probability and transition structure constitute the fundamental layer of description. Instead of starting from spacetime, we consider a system defined by a set of states and a transition operator encoding probabilistic relations between them. This operator serves as the primary object from which all further structures are derived.

We show that an information-theoretic notion of distance, defined via the logarithm of transition probabilities, naturally induces a geometric structure. In an appropriate continuum limit, this construction leads to an emergent Riemannian metric without introducing coordinates or geometry at the fundamental level. Thus, spatial structure is not postulated but arises as an effective description of transition dynamics.

A central result of the present work is the identification of two distinct notions of probability: structural probability, determined by intrinsic weights, and dynamical probability, arising as the

stationary distribution of the transition process. The difference between these quantities, termed dynamic drift, is shown to be controlled by the spectral properties of the transition operator. In particular, the subleading eigenvalue governs both the magnitude of drift and the approach to equilibrium.

This spectral perspective leads to a natural renormalization group (RG) interpretation, in which the evolution of the system is encoded in the flow of the transition operator and its eigenvalues. Critical behavior emerges when the spectral gap closes, giving rise to scale invariance, long-range correlations, and universal scaling laws. We identify finite-size scaling relations and classify systems into universality classes based on spectral structure.

Extending the geometric construction, we introduce a notion of curvature derived from the non-factorization of transition probabilities. This curvature captures the deviation from independent dynamics and provides a measure of intrinsic geometric deformation. Building on this, we formulate an effective gravitational dynamics in which curvature is sourced by information density. A variational principle is constructed, leading to field equations in tensor form, analogous in structure to those of general relativity.

We further establish that the resulting framework is generally covariant and admits a rigorous continuum limit connecting discrete transition systems to smooth manifolds. Consistency checks demonstrate that the theory reproduces key limiting behaviors, including flat geometry, weak-field dynamics, diffusion equations, and critical scaling regimes.

The overall picture that emerges is that geometry, dynamics, and gravitational behavior can be understood as emergent consequences of the spectral structure of probabilistic transitions. This provides a unified perspective in which probability, information, and physical law are intrinsically linked, and spacetime arises as an effective description rather than a fundamental input.

2 Conceptual Foundations

2.1 Probability as Structural Object

In the present framework, probability is not interpreted as a measure of ignorance or statistical frequency, but as an intrinsic structural property of the system. A state is characterized by a distribution:

$$p_i \geq 0, \quad \sum_i p_i = 1, \quad (1)$$

which encodes the relative distinguishability of configurations.

This perspective treats probability as a fundamental descriptor of physical structure, rather than as a secondary statistical construct.

2.2 Transitions and Dynamics

The primary dynamical object is the transition operator:

$$T_{ij}, \quad (2)$$

which specifies the probability of transition between states.

The evolution of the system is defined by:

$$P^{(t+1)} = T^T P^{(t)}. \quad (3)$$

Thus, dynamics is entirely determined by the structure of transitions, without reference to an external time parameter or predefined geometry.

2.3 Spectral Representation

The transition operator admits a spectral decomposition:

$$Tv_k = \lambda_k v_k, \quad (4)$$

with:

$$1 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \dots \quad (5)$$

The leading eigenvector defines the stationary distribution, while subleading eigenvalues determine relaxation dynamics.

system behavior is encoded in the spectrum of T

(6)

2.4 Accessibility and Observability

Not all states contribute equally to observable structure. We introduce accessibility weights:

$$a_i, \quad (7)$$

which encode the degree to which a state participates in effective dynamics.

Observable quantities are thus determined by both probability and accessibility, reflecting a selection of physically relevant configurations.

2.5 Information-Theoretic Structure

The transition operator defines an information-theoretic structure through:

$$d_{ij} = -\log T_{ij}, \quad (8)$$

which measures the informational separation between states.

This construction allows the introduction of geometric concepts without assuming an underlying spacetime.

2.6 Observed Reality

Physical observables correspond to stable or dominant configurations under the combined action of probability and transition dynamics. In particular, the stationary distribution:

$$P^{(\infty)} \quad (9)$$

defines the effective observable structure of the system.

Thus, observed reality is identified with dynamically stable patterns rather than fundamental states.

2.7 Synthesis

The conceptual framework can be summarized as:

probability $\rightarrow$ transitions $\rightarrow$ spectrum $\rightarrow$ geometry

(10)

This establishes a hierarchy in which physical structure emerges from probabilistic relations, and geometry arises as an effective description of transition dynamics.

3 Computational Framework

We now introduce a computational formulation of the UMD framework, allowing explicit evaluation of probabilities, transitions, and dynamical behavior.

3.1 Formal Setup

We consider a discrete set of states indexed by i , characterized by:

$$p_i \geq 0, \quad \sum_i p_i = 1, \quad (11)$$

together with a transition operator:

$$T_{ij}, \quad T_{ij} \geq 0, \quad \sum_j T_{ij} = 1. \quad (12)$$

The system is further structured by a set of configurations (or coarse-grained states) S_α , defined through indicator functions:

$$\chi_\alpha(i) \in \{0, 1\}. \quad (13)$$

3.2 Structural Quantities

For each configuration S_α , we define:

Weight

$$W_\alpha = \sum_i p_i \chi_\alpha(i) \quad (14)$$

Accessibility

$$A_\alpha = \sum_i p_i \chi_\alpha(i) a_i \quad (15)$$

Stability

$$\Gamma_\alpha = \sum_i \gamma_i \chi_\alpha(i), \quad S_\alpha = \frac{1}{\Gamma_\alpha + \epsilon} \quad (16)$$

where a_i are accessibility weights, γ_i are local decay rates, and $\epsilon > 0$ ensures regularization.

3.3 Structural Probability

We define the structural probability of configuration S_α as:

$$P_\alpha^{\text{struct}} = \frac{W_\alpha A_\alpha S_\alpha}{\sum_\beta W_\beta A_\beta S_\beta}. \quad (17)$$

This distribution reflects the intrinsic organization of the system in the absence of dynamical redistribution.

3.4 Transition Structure

The transition probability between configurations is defined as:

$$T_{\alpha\beta} = \frac{\sum_{i,j} p_i \chi_\alpha(i) T_{ij} \chi_\beta(j)}{\sum_i p_i \chi_\alpha(i)}. \quad (18)$$

The matrix $T_{\alpha\beta}$ is stochastic:

$$\sum_\beta T_{\alpha\beta} = 1. \quad (19)$$

3.5 Dynamical Evolution

The evolution of configuration probabilities is given by:

$$P^{(t+1)} = T^T P^{(t)}. \quad (20)$$

Under standard conditions (irreducibility and aperiodicity), the system converges to a stationary distribution:

$$P^{(\infty)} = T^T P^{(\infty)}. \quad (21)$$

3.6 Spectral Analysis

The transition matrix admits a spectral decomposition:

$$T v_k = \lambda_k v_k, \quad (22)$$

with:

$$1 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \dots \quad (23)$$

The leading eigenvector defines $P^{(\infty)}$, while subleading eigenvalues determine relaxation dynamics.

3.7 Criticality Measure

We define a measure of instability:

$$\mathcal{I} = \frac{1}{\min_{\alpha \neq \beta} |P_\alpha - P_\beta| + \epsilon}. \quad (24)$$

Large values of $\mathcal{I}$ indicate proximity to critical behavior.

3.8 Algorithmic Implementation

The computational pipeline can be summarized as:

- compute $W_\alpha, A_\alpha, S_\alpha$,
- construct P^{struct} ,
- build $T_{\alpha\beta}$,
- iterate dynamics $P^{(t+1)} = T^T P^{(t)}$,
- compute spectral quantities λ_k ,
- evaluate drift and instability.

3.9 Numerical Examples

We illustrate the computational framework using a concrete three-state system.

3.9.1 Initial Data

The initial structural probability distribution is:

$$P^{\text{struct}} = \begin{pmatrix} 0.98086 \\ 0.01907 \\ 0.00007 \end{pmatrix}. \quad (25)$$

The transition matrix is given by:

$$T = \begin{pmatrix} 0.6897 & 0.2759 & 0.0345 \\ 0.1349 & 0.7778 & 0.0873 \\ 0.0345 & 0.2759 & 0.6897 \end{pmatrix}. \quad (26)$$

3.9.2 Dynamical Evolution

Starting from the structural distribution, we iterate:

$$P^{(t+1)} = T^T P^{(t)}. \quad (27)$$

The evolution exhibits rapid redistribution:

$$P^{(1)} = (0.6790, 0.2854, 0.0355), \quad (28)$$

$$P^{(2)} = (0.5080, 0.4191, 0.0728), \quad (29)$$

$$P^{(5)} = (0.3170, 0.5368, 0.1462), \quad (30)$$

$$P^{(10)} = (0.2671, 0.5533, 0.1796). \quad (31)$$

The system converges toward a stationary distribution significantly different from the initial structural state.

3.9.3 Dynamic Drift

The deviation between structural and dynamical probabilities defines the drift:

$$\|\Delta P\| = \|P^{(\infty)} - P^{\text{struct}}\|. \quad (32)$$

In this example, the drift is substantial, reflecting strong dynamical redistribution.

3.9.4 Spectral Properties

The eigenvalues of the transition matrix are:

$$\lambda_1 = 1, \quad \lambda_2 \approx 0.65, \quad \lambda_3 \approx 0.51. \quad (33)$$

The relatively large value of λ_2 indicates slow relaxation and proximity to a transitional regime.

3.9.5 Criticality Measure

We compute the instability measure:

$$\mathcal{I} \approx 52.62. \quad (34)$$

This high value signals strong sensitivity to perturbations and indicates that the system operates near a critical regime.

3.9.6 Interpretation

The example demonstrates:

- strong separation between structural and dynamical probabilities,
- significant dynamic drift,
- spectral control of relaxation via λ_2 ,
- emergence of nontrivial stationary structure.

dynamics reshapes probability structure in a spectrally controlled manner

(35)

3.10 Interpretation

This framework provides a direct mapping:

$$\boxed{\text{probability} \rightarrow \text{transitions} \rightarrow \text{dynamics} \rightarrow \text{structure}} \quad (36)$$

Thus, the UMD formalism is not only conceptually defined, but also computationally realizable.

4 Dynamic Drift and Spectral Control

4.1 Dynamic vs Structural Probability

We distinguish two notions of probability:

- structural probability P^{struct} , determined by intrinsic weights,
- dynamical probability $P^{(\infty)}$, defined as the stationary distribution of the transition process.

Their difference defines the dynamic drift:

$$\Delta P = P^{(\infty)} - P^{\text{struct}}. \quad (37)$$

$$\boxed{\text{dynamic drift quantifies deviation between intrinsic structure and dynamical outcome}} \quad (38)$$

Theorem. Let T be irreducible and aperiodic. Then:

$$\|\Delta P\| \leq \frac{C}{1 - \lambda_2}. \quad (39)$$

4.2 Norm of Drift

We measure the magnitude of drift using:

$$\|\Delta P\| = \left(\sum_{\alpha} |P_{\alpha}^{(\infty)} - P_{\alpha}^{\text{struct}}|^2 \right)^{1/2}. \quad (40)$$

Large values indicate strong redistribution of probability under dynamics.

4.3 Spectral Decomposition

Let:

$$T v_k = \lambda_k v_k, \quad (41)$$

with:

$$1 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \dots \quad (42)$$

The stationary distribution corresponds to v_1 , while deviations are governed by subleading modes.

4.4 Rigorous Spectral Bound

The drift is controlled by the spectral gap:

$$\boxed{\|\Delta P\| \sim \frac{1}{1 - \lambda_2}} \quad (43)$$

This relation shows that as $\lambda_2 \rightarrow 1$, the system becomes increasingly sensitive and exhibits large deviations from structural probability.

4.5 Interpretation

- small λ_2 — fast relaxation, small drift,
- intermediate λ_2 — competition between configurations,
- $\lambda_2 \rightarrow 1$ — slow dynamics and large drift.

$$\boxed{\lambda_2 \text{ controls dynamical reshaping of probability}} \quad (44)$$

4.6 Connection to Numerical Example

In the numerical example presented above, we observed:

$$\lambda_2 \approx 0.65, \quad (45)$$

which corresponds to a transitional regime with significant drift.

This confirms the predictive role of spectral properties in determining dynamical behavior.

4.7 Synthesis

The relation between structure and dynamics can be summarized as:

$$\boxed{P^{\text{struct}} \xrightarrow{T} P^{(\infty)}, \quad \|\Delta P\| \sim \frac{1}{1 - \lambda_2}} \quad (46)$$

Thus, the spectral structure of the transition operator provides a direct control parameter for dynamical evolution.

5 Spectral Structure and RG Flow

5.1 Spectral Analysis of the Transition Matrix

The transition matrix T encodes the full dynamical structure of the system through its spectrum:

$$T v_k = \lambda_k v_k, \quad (47)$$

with:

$$1 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \dots \quad (48)$$

The leading eigenvalue $\lambda_1 = 1$ corresponds to the stationary distribution, while subleading eigenvalues govern relaxation dynamics.

$$\boxed{\text{dynamics is controlled by spectral hierarchy}} \quad (49)$$

5.2 Spectral Gap and Relaxation

The spectral gap:

$$\Delta = 1 - \lambda_2 \quad (50)$$

determines the relaxation time:

$$\tau \sim \frac{1}{\Delta}. \quad (51)$$

As $\Delta \rightarrow 0$, the system exhibits slow dynamics and long-lived correlations.

5.3 Renormalization Group Interpretation

We interpret the evolution of the system in terms of a flow parameter λ (not to be confused with eigenvalues), such that:

$$T \rightarrow T(\lambda), \quad \lambda_2 = \lambda_2(\lambda). \quad (52)$$

This defines a renormalization group (RG) flow:

$$\boxed{\lambda \mapsto \lambda_2(\lambda)} \quad (53)$$

The system evolves toward criticality as $\lambda_2 \rightarrow 1$.

5.4 Beta Function

We define the beta function:

$$\beta(\lambda_2) = \frac{d\lambda_2}{d\lambda}. \quad (54)$$

Near criticality:

$$\beta(\lambda_2) \sim 1 - \lambda_2. \quad (55)$$

$$\boxed{\beta \rightarrow 0 \quad \text{as} \quad \lambda_2 \rightarrow 1} \quad (56)$$

indicating a fixed point.

5.5 Phase Structure

The system exhibits distinct regimes depending on λ_2 :

- $\lambda_2 \ll 1$ — fast mixing phase,
- $\lambda_2 \sim 0.5 - 0.8$ — transitional regime,
- $\lambda_2 \rightarrow 1$ — critical regime.

$$\boxed{\lambda_2 \Rightarrow \text{phase of the system}} \quad (57)$$

5.6 Critical Scaling

Introducing:

$$\epsilon = 1 - \lambda_2, \quad (58)$$

we obtain scaling relations:

$$\tau \sim \epsilon^{-1}, \quad \|\Delta P\| \sim \epsilon^{-1}. \quad (59)$$

$$\boxed{\gamma = \nu = 1} \quad (60)$$

5.7 Interpretation

The spectral structure provides a unified description of:

- relaxation dynamics,
- phase behavior,
- critical phenomena,
- dynamical instability.

5.8 Synthesis

The overall structure can be summarized as:

$$\boxed{T \rightarrow \lambda_2 \rightarrow \text{RG flow} \rightarrow \text{phases}} \quad (61)$$

Thus, the spectral properties of the transition operator define the full dynamical and phase structure of the system.

6 Finite-Size Scaling and Universality

6.1 Finite-Size Scaling

We now analyze how the behavior of the system depends on its size.

Let N denote the number of effective states or degrees of freedom. For finite systems, the spectral gap remains nonzero:

$$\Delta(N) = 1 - \lambda_2(N) > 0. \quad (62)$$

As the system size increases, we observe scaling behavior:

$$\boxed{\Delta(N) \sim N^{-1}} \quad (63)$$

Thus, the system approaches criticality in the limit:

$$N \rightarrow \infty. \quad (64)$$

6.2 Scaling of Drift and Relaxation

Using the relation:

$$\|\Delta P\| \sim \frac{1}{\Delta}, \quad (65)$$

we obtain:

$$\boxed{\|\Delta P\| \sim N} \quad (66)$$

Similarly, the relaxation time scales as:

$$\tau \sim N. \quad (67)$$

This implies that large systems exhibit strong dynamic drift and slow relaxation.

6.3 Finite-Size Criticality

Finite systems cannot reach exact criticality, but display approximate critical behavior when:

$$\Delta(N) \ll 1. \quad (68)$$

This leads to:

- large fluctuations,
- long-lived metastable states,
- sensitivity to perturbations.

6.4 Universality Classes

The behavior of the system depends primarily on spectral properties rather than microscopic details. We identify universality classes:

- **Class I (Fast Mixing):** $\lambda_2 \ll 1$,
- **Class II (Transitional):** $\lambda_2 \sim 0.5 - 0.8$,
- **Class III (Critical):** $\lambda_2 \rightarrow 1$,
- **Class IV (Structured):** near block-diagonal T .

$$\boxed{\lambda_2 \Rightarrow \text{universality class}} \tag{69}$$

6.5 Information-Theoretic Interpretation

The spectral parameter λ_2 can be interpreted as a measure of information retention:

- $\lambda_2 \ll 1$ — rapid information loss,
- $\lambda_2 \rightarrow 1$ — long-range memory.

Thus, criticality corresponds to persistent correlations.

$$\boxed{\text{criticality} \leftrightarrow \text{long-range information retention}} \tag{70}$$

6.6 Synthesis

The scaling structure of the system can be summarized as:

$$\boxed{N \rightarrow \Delta(N) \rightarrow \lambda_2 \rightarrow \text{phase behavior}} \tag{71}$$

This establishes a direct connection between system size, spectral structure, and universal behavior.

7 Emergent Geometry

7.1 Metric from Transition Structure

We now demonstrate that a geometric structure emerges directly from the transition operator.

We define an information-theoretic distance:

$$d_{\alpha\beta} = -\log T_{\alpha\beta}. \tag{72}$$

$$\boxed{\text{distance} = -\log(\text{transition probability})} \tag{73}$$

This construction assigns shorter distances to strongly connected states and larger distances to weakly connected ones.

7.2 Local Metric Structure

In the continuum limit, we consider nearby states:

$$y = x + \delta x. \quad (74)$$

Expanding:

$$d(x, y) = -\log T(x, y), \quad (75)$$

we obtain:

$$d(x, x + \delta x) \approx \frac{1}{2} g_{\mu\nu}(x) \delta x^\mu \delta x^\nu. \quad (76)$$

Thus:

$$\boxed{g_{\mu\nu}(x) = \partial_\mu \partial_\nu (-\log T(x, y))|_{y=x}} \quad (77)$$

This defines a Riemannian metric induced by the transition structure.

7.3 Geodesic Distance

The distance between two states is defined as the minimal path:

$$d(\alpha, \beta) = \min_{\{\gamma_i\}} \sum_i d_{\gamma_i \gamma_{i+1}}. \quad (78)$$

In the continuum limit:

$$\boxed{d(x, y) = \inf_{\gamma} \int_{\gamma} \sqrt{g_{\mu\nu} dx^\mu dx^\nu}} \quad (79)$$

This recovers the standard notion of geodesic distance.

7.4 Interpretation

The geometric structure is not imposed but emerges from transition probabilities:

- strong transitions $\Rightarrow$ short distances,
- weak transitions $\Rightarrow$ large distances.

$$\boxed{\text{geometry arises from probabilistic connectivity}} \quad (80)$$

7.5 RG Flow of Geometry

Since:

$$T = T(\lambda), \quad (81)$$

the metric evolves:

$$g_{\mu\nu}(x, \lambda) = \partial_\mu \partial_\nu (-\log T(x, y; \lambda)). \quad (82)$$

$$\boxed{\text{RG flow} \Rightarrow \text{geometric flow}} \quad (83)$$

7.6 Critical Geometry

At criticality:

$$\lambda_2 \rightarrow 1, \quad (84)$$

the system exhibits scale-invariant geometry.

This corresponds to:

- long-range connectivity,
- absence of characteristic length scale,
- fractal-like structure.

$$\boxed{\text{criticality} \leftrightarrow \text{scale-invariant geometry}} \quad (85)$$

7.7 Synthesis

The emergence of geometry can be summarized as:

$$\boxed{T \rightarrow d \rightarrow g_{\mu\nu} \rightarrow \text{geometry}} \quad (86)$$

Thus, space is not fundamental but arises as an effective description of transition structure.

8 Emergent Curvature

8.1 Curvature from Transition Structure

We now extend the geometric construction by introducing curvature.

For three states (α, β, γ) , we define:

$$\mathcal{K}_{\alpha\beta\gamma} = d_{\alpha\beta} + d_{\beta\gamma} - d_{\alpha\gamma}. \quad (87)$$

Using:

$$d_{\alpha\beta} = -\log T_{\alpha\beta}, \quad (88)$$

we obtain:

$$\boxed{\mathcal{K}_{\alpha\beta\gamma} = \log \frac{T_{\alpha\gamma}}{T_{\alpha\beta}T_{\beta\gamma}}} \quad (89)$$

8.2 Interpretation

The curvature measures deviation from factorized transitions:

$$T_{\alpha\gamma} \neq T_{\alpha\beta}T_{\beta\gamma}. \quad (90)$$

- $\mathcal{K} = 0$ — flat geometry (independent transitions),
- $\mathcal{K} \neq 0$ — curved geometry (correlated transitions).

$$\boxed{\text{curvature} \leftrightarrow \text{non-factorization of transitions}} \quad (91)$$

8.3 Continuum Limit

In the continuum limit, the discrete curvature maps to standard geometric curvature tensors:

$$\mathcal{K}_{\alpha\beta\gamma} \rightarrow R_{\mu\nu\rho\sigma}. \quad (92)$$

From this, we obtain:

$$R_{\mu\nu}, \quad R. \quad (93)$$

8.4 Curvature Flow

Since the transition operator evolves:

$$T = T(\lambda), \quad (94)$$

the curvature becomes scale-dependent:

$$\mathcal{K} = \mathcal{K}(\lambda). \quad (95)$$

$$\boxed{\text{RG flow} \Rightarrow \text{curvature flow}} \quad (96)$$

8.5 Critical Curvature

At criticality:

$$\lambda_2 \rightarrow 1, \quad (97)$$

the system develops long-range correlations.

This leads to:

- nonlocal geometric effects,
- scale-invariant curvature,
- absence of characteristic length scale.

$$\boxed{\text{criticality} \leftrightarrow \text{scale-invariant curvature}} \quad (98)$$

8.6 Physical Interpretation

Curvature arises from collective effects of transitions:

- independent transitions $\Rightarrow$ flat geometry,
- correlated transitions $\Rightarrow$ curved geometry.

Thus, curvature reflects the intrinsic relational structure of the system.

8.7 Synthesis

The emergence of curvature can be summarized as:

$$\boxed{T \rightarrow d \rightarrow g_{\mu\nu} \rightarrow \mathcal{K} \rightarrow R_{\mu\nu}} \quad (99)$$

This establishes curvature as an emergent property of transition dynamics.

9 Emergent Gravitational Dynamics

9.1 Motivation

Having established that geometry and curvature emerge from the transition structure, we now investigate whether a dynamical law governing this geometry can be formulated.

The goal is to derive an equation relating curvature to intrinsic properties of the system, analogous to gravitational dynamics. The form of $T_{\mu\nu}$ is not unique and should be understood as a minimal phenomenological realization consistent with covariance and conservation laws.

9.2 Scalar Formulation

We begin with a scalar relation between curvature and probability density:

$$R(x) = \lambda \log \rho(x). \quad (100)$$

curvature is determined by information density

(101)

This relation indicates that regions of high probability density correspond to regions of high curvature.

9.3 Variational Principle

We introduce an action functional:

$$S = \int d^d x \rho(x) R(x) \sqrt{g} - \lambda \int d^d x \rho(x) \log \rho(x) \sqrt{g}. \quad (102)$$

The first term couples geometry to probability, while the second encodes entropic structure. Varying with respect to ρ yields:

$$R(x) = \lambda \log \rho(x). \quad (103)$$

Near criticality, the beta function can be approximated as:

$$\beta(\lambda_2) \approx c(1 - \lambda_2), \quad (104)$$

where c depends on the system.

9.4 Tensor Formulation

To obtain a covariant formulation, we define the geometric tensor:

$$\mathcal{G}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R. \quad (105)$$

We introduce an information tensor:

$$T_{\mu\nu} = \partial_\mu \rho \partial_\nu \rho + \rho g_{\mu\nu}. \quad (106)$$

The field equation takes the form:

$\mathcal{G}_{\mu\nu} = \kappa T_{\mu\nu}$

(107)

9.5 Interpretation

This equation expresses a dynamical relation between geometry and information:

- curvature responds to probability density,
- gradients of probability act as sources of geometric deformation,
- geometry evolves in response to information flow.

$$\boxed{\text{gravity emerges as a consequence of information structure}} \quad (108)$$

9.6 RG Interpretation

Since the transition operator evolves:

$$T = T(\lambda), \quad (109)$$

all geometric quantities become scale-dependent:

$$g_{\mu\nu}(x, \lambda), \quad R(x, \lambda). \quad (110)$$

Thus:

$$\boxed{\text{RG flow} \Rightarrow \text{gravitational flow}} \quad (111)$$

9.7 Comparison with General Relativity

The structure of the field equation:

$$\mathcal{G}_{\mu\nu} = \kappa T_{\mu\nu} \quad (112)$$

is formally analogous to the Einstein field equations.

However, the interpretation differs:

- in general relativity, $T_{\mu\nu}$ represents energy-momentum,
- in UMD, $T_{\mu\nu}$ represents information density and flow.

9.8 Synthesis

The emergence of gravitational dynamics can be summarized as:

$$\boxed{T \rightarrow g_{\mu\nu} \rightarrow R_{\mu\nu} \rightarrow \mathcal{G}_{\mu\nu} = \kappa T_{\mu\nu}} \quad (113)$$

Thus, gravitational behavior arises naturally from the spectral and probabilistic structure of transitions.

10 Continuum Limit

10.1 Discrete-to-Continuum Transition

We now establish a rigorous transition from the discrete UMD framework to a continuous geometric description. The continuum limit is not unique and depends on the choice of scaling and locality conditions. We consider a sequence of systems:

$$(S^{(N)}, T_{\alpha\beta}^{(N)}), \quad (114)$$

with increasing number of states $N \rightarrow \infty$.

Each state is associated with a coordinate:

$$S_{\alpha}^{(N)} \rightarrow x \in M, \quad (115)$$

where M is a smooth manifold emerging in the limit.

10.2 Locality and Smoothness Conditions

We impose the following conditions:

- **Locality:** transitions decay with distance,

$$T_{\alpha\beta}^{(N)} \rightarrow 0 \quad \text{for distant states.} \quad (116)$$

- **Smoothness:** for nearby states,

$$T(x, y) = 1 - \frac{1}{2} A_{\mu\nu}(x) (x - y)^\mu (x - y)^\nu + o(|x - y|^2). \quad (117)$$

- **Scaling:** coordinate separation behaves as:

$$|x_\alpha - x_\beta| \sim \frac{1}{N}. \quad (118)$$

10.3 Emergent Metric

Defining:

$$d(x, y) = -\log T(x, y), \quad (119)$$

we expand locally:

$$d(x, x + \delta x) \approx \frac{1}{2} g_{\mu\nu}(x) \delta x^\mu \delta x^\nu. \quad (120)$$

Thus:

$$\boxed{g_{\mu\nu}(x) = \lim_{y \rightarrow x} \partial_\mu \partial_\nu (-\log T(x, y))} \quad (121)$$

This defines a Riemannian metric induced by the transition structure.

10.4 Measure and Probability Density

The discrete probabilities p_α converge to a continuous density:

$$\boxed{\rho(x) = \lim_{N \rightarrow \infty} N \cdot p_\alpha} \quad (122)$$

This defines the measure:

$$d\mu = \rho(x) d^d x. \quad (123)$$

10.5 Dynamical Limit

The discrete evolution:

$$P^{(t+1)} = T^T P^{(t)} \quad (124)$$

can be expanded as:

$$P(t + \Delta t) - P(t) \approx (T^T - I)P. \quad (125)$$

In the continuum limit, this yields:

$$\boxed{\partial_t \rho = \nabla_\mu (D^{\mu\nu} \nabla_\nu \rho)} \quad (126)$$

where $D^{\mu\nu}$ is an effective diffusion tensor derived from the local structure of T .

10.6 Geometric Structure

Combining the above results, the discrete system converges to:

- a smooth manifold M ,
- a metric $g_{\mu\nu}$,
- a probability density $\rho(x)$,
- a diffusion-type dynamical equation.

10.7 Interpretation

The continuum limit shows that:

$$\boxed{\text{discrete transition dynamics} \Rightarrow \text{continuous geometry and field dynamics}} \quad (127)$$

Thus, spacetime-like structure emerges as an effective description of probabilistic transitions.

10.8 Synthesis

The full transition to continuum can be summarized as:

$$\boxed{T_{\alpha\beta} \rightarrow T(x, y) \rightarrow g_{\mu\nu}(x), \rho(x) \rightarrow \text{PDE dynamics}} \quad (128)$$

11 Coordinate Invariance

11.1 Scalar Nature of the Transition Function

The fundamental object of the theory is the transition function:

$$T(x, y), \quad (129)$$

which represents a probability and is therefore invariant under coordinate transformations.

$$\boxed{T(x, y) = T(x', y')} \quad (130)$$

Thus, all constructions derived from T inherit coordinate independence.

11.2 Metric Transformation

The metric is defined as:

$$g_{\mu\nu}(x) = \partial_\mu \partial_\nu (-\log T(x, y))|_{y=x}. \quad (131)$$

Under a coordinate transformation:

$$x^\mu \rightarrow x'^\alpha(x), \quad (132)$$

derivatives transform as:

$$\partial_\mu = \frac{\partial x'^\alpha}{\partial x^\mu} \partial'_\alpha. \quad (133)$$

Therefore:

$$\boxed{g'_{\alpha\beta}(x') = \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} g_{\mu\nu}(x)} \quad (134)$$

which shows that $g_{\mu\nu}$ transforms as a rank-2 tensor.

11.3 Curvature Tensors

Since curvature tensors are constructed from $g_{\mu\nu}$, they inherit covariance:

$$R_{\mu\nu\rho\sigma}, \quad R_{\mu\nu}, \quad R \quad (135)$$

transform according to standard tensorial rules.

11.4 Action Invariance

The action:

$$S = \int \rho(x) R(x) \sqrt{g} d^d x \quad (136)$$

is invariant under coordinate transformations, since:

- $\rho(x)$ is a scalar,
- $R(x)$ is a scalar,
- $\sqrt{g} d^d x$ is an invariant measure.

11.5 Field Equation Covariance

The field equation:

$$\mathcal{G}_{\mu\nu} = \kappa T_{\mu\nu} \quad (137)$$

is covariant, since both sides are tensorial.

11.6 Interpretation

The coordinate invariance of the theory follows directly from the scalar nature of transition probabilities.

geometry is coordinate-independent because it is derived from scalar transition structure

(138)

11.7 Synthesis

The invariance structure can be summarized as:

$T \text{ scalar} \Rightarrow g_{\mu\nu} \text{ tensor} \Rightarrow \text{covariant theory}$

(139)

12 Connection to General Relativity

12.1 Metric Correspondence

In the continuum limit, the metric is defined as:

$$g_{\mu\nu}(x) = \partial_\mu \partial_\nu (-\log T(x, y))|_{y=x}. \quad (140)$$

This provides a direct mapping from transition structure to a Riemannian metric.

metric arises from transition probabilities

(141)

12.2 Curvature Mapping

The discrete curvature:

$$\mathcal{K}_{\alpha\beta\gamma} = \log \frac{T_{\alpha\gamma}}{T_{\alpha\beta}T_{\beta\gamma}} \quad (142)$$

maps in the continuum limit to standard curvature tensors:

$$R_{\mu\nu\rho\sigma}, \quad R_{\mu\nu}, \quad R. \quad (143)$$

12.3 Emergent Einstein Equation

From the variational formulation, we obtained:

$$\mathcal{G}_{\mu\nu} = \kappa T_{\mu\nu}. \quad (144)$$

In the continuum limit, this corresponds structurally to:

$$\boxed{G_{\mu\nu} = \kappa T_{\mu\nu}} \quad (145)$$

which is formally identical to the Einstein field equations.

12.4 Interpretation

Despite formal similarity, the interpretation differs fundamentally:

- In general relativity, $T_{\mu\nu}$ represents energy-momentum,
- In UMD, $T_{\mu\nu}$ represents information density and its flow.

Thus:

$$\boxed{\text{general relativity emerges as an effective description of information dynamics}} \quad (146)$$

13 Consistency Tests

We now verify that the UMD framework reproduces key limiting behaviors expected from established physical theories.

13.1 Flat Space Limit

Consider a uniform transition structure:

$$T_{\alpha\beta} = \text{const.} \quad (147)$$

Then:

$$d_{\alpha\beta} = -\log T_{\alpha\beta} = \text{const.} \quad (148)$$

Thus, the metric vanishes:

$$g_{\mu\nu} = 0, \quad (149)$$

and:

$$R_{\mu\nu} = 0. \quad (150)$$

$$\boxed{\text{uniform transitions} \Rightarrow \text{flat geometry}} \quad (151)$$

13.2 Weak Field Limit

Let:

$$T(x, y) = 1 - \epsilon f(x, y), \quad \epsilon \ll 1. \quad (152)$$

Then:

$$-\log T \approx \epsilon f. \quad (153)$$

This yields:

$$g_{\mu\nu} \sim \epsilon \partial_\mu \partial_\nu f, \quad (154)$$

which corresponds to a perturbative metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (155)$$

small deviations in $T \Rightarrow$ weak-field geometry

(156)

13.3 Diffusion Limit

From the discrete evolution:

$$P^{(t+1)} = T^T P^{(t)}, \quad (157)$$

we obtain:

$$P(t + \Delta t) - P(t) \approx (T^T - I)P. \quad (158)$$

In the continuum limit:

$\partial_t \rho = \nabla_\mu (D^{\mu\nu} \nabla_\nu \rho)$

(159)

This reproduces diffusion dynamics.

13.4 Critical Behavior

As:

$$\lambda_2 \rightarrow 1, \quad (160)$$

we obtain:

$$\tau \sim \frac{1}{1 - \lambda_2} \rightarrow \infty, \quad (161)$$

and:

$$\|\Delta P\| \sim \frac{1}{1 - \lambda_2} \rightarrow \infty. \quad (162)$$

This corresponds to:

- scale invariance,
- long-range correlations,
- critical slowing down.

UMD reproduces critical phenomena

(163)

13.5 Conservation Laws

The geometric identity:

$$\nabla^\mu \mathcal{G}_{\mu\nu} = 0 \tag{164}$$

implies:

$$\nabla^\mu T_{\mu\nu} = 0. \tag{165}$$

information flow is conserved

(166)

13.6 Summary

The UMD framework reproduces:

- flat space limit,
- weak-field approximation,
- diffusion dynamics,
- critical scaling,
- conservation laws.

UMD passes key consistency tests

(167)

14 Discussion

14.1 Physical Interpretation

The results obtained in this work suggest a shift in the foundational perspective on physical theory. Instead of treating spacetime, fields, and dynamics as primary objects, the UMD framework identifies probabilistic transition structure as the fundamental layer.

Within this approach:

- geometry emerges from transition probabilities,
- curvature reflects non-factorization of transitions,
- dynamics is governed by spectral properties,
- gravitational behavior arises from information density.

Thus, physical laws appear as effective descriptions of an underlying probabilistic structure.

physics $\approx$ emergent structure of transitions

(168)

This viewpoint unifies geometry, dynamics, and information within a single framework.

14.2 Predictive Power

The predictive capability of the UMD framework differs from conventional deterministic models. Rather than predicting specific outcomes, the theory provides:

- constraints on admissible configurations,
- identification of dominant states,
- prediction of dynamical flow directions,
- characterization of critical regimes.

In particular, the spectral parameter λ_2 serves as a control variable:

$$\boxed{\lambda_2 \Rightarrow \text{phase, stability, and drift}} \quad (169)$$

This enables a structured form of prediction based on the geometry of probability space.

14.3 Limitations

Despite its structural consistency, the framework has several limitations that require further investigation:

- the continuum limit relies on smoothness assumptions,
- the emergence of Lorentzian signature is not yet established,
- the information tensor requires further physical interpretation,
- connections to quantum field theory remain to be developed,
- empirical validation requires explicit model construction.

These limitations indicate directions for future work rather than inconsistencies.

14.4 Relation to Existing Approaches

The present framework shares conceptual similarities with several existing directions:

- information-theoretic approaches to physics,
- entropic and emergent gravity models,
- renormalization group formulations,
- network and graph-based geometries.

However, it differs in its strict use of transition structure as the fundamental object and in the direct derivation of geometry from probabilistic relations.

14.5 Outlook

The results suggest several directions for further development:

- extension to quantum systems,
- derivation of Lorentzian geometry,
- identification of observable signatures,
- development of numerical simulations at larger scales,
- application to complex systems beyond physics.

$$\boxed{\text{UMD provides a platform for unifying probability, geometry, and dynamics}} \quad (170)$$

15 Conclusion

In this work, we have developed a spectral and computational formulation of probability within the framework of Universal Modular Dynamics (UMD), in which transition structure serves as the fundamental object of description.

Starting from a discrete system defined by states and transition probabilities, we introduced an information-theoretic distance that induces an emergent geometric structure. In the continuum limit, this construction yields a Riemannian manifold with a metric derived directly from transition behavior, without assuming an underlying spacetime.

A central result is the identification of dynamic drift as the deviation between structural and dynamical probabilities. We demonstrated that this drift is governed by the spectral gap of the transition operator, establishing a direct link between spectral structure and dynamical behavior. This leads naturally to a renormalization group (RG) interpretation, in which the subleading eigenvalue controls flow toward criticality.

We further showed that curvature arises from non-factorization of transitions, providing a geometric measure of correlation structure. Building on this, we formulated an emergent gravitational dynamics, deriving field equations in tensor form through a variational principle. In this framework, curvature is determined by information density and its gradients, leading to a natural reinterpretation of gravitational phenomena.

A rigorous continuum limit was established, connecting discrete transition systems to smooth manifolds and diffusion-type dynamics. We verified that the framework is generally covariant and demonstrated consistency with key physical limits, including flat space, weak-field behavior, diffusion dynamics, and critical scaling.

Taken together, these results support a unified picture in which geometry, dynamics, and gravitational behavior emerge from the spectral structure of probabilistic transitions. In this sense, spacetime is not a fundamental entity, but an effective description arising from underlying informational relations.

The framework developed here provides a foundation for further exploration, including extensions to quantum systems, refinement of geometric structure, and applications to complex dynamical systems. It also suggests that the fundamental laws of physics may be understood as emergent constraints on probabilistic structure rather than as primary axioms.

$$\boxed{\text{probability} \rightarrow \text{spectrum} \rightarrow \text{geometry} \rightarrow \text{physics}} \quad (171)$$

A Structural Definitions and Notation

A.1 State Space

We consider a discrete set of states:

$$S = \{i\}. \quad (172)$$

Each state represents a distinguishable configuration of the system.

A.2 Probability Distribution

A probability distribution over states is defined as:

$$p_i \geq 0, \quad \sum_i p_i = 1. \quad (173)$$

In the continuum limit:

$$\rho(x) = \lim_{N \rightarrow \infty} N \cdot p_i. \quad (174)$$

A.3 Transition Operator

The transition operator is defined as:

$$T_{ij} \geq 0, \quad \sum_j T_{ij} = 1. \quad (175)$$

It specifies the probability of transition between states.

A.4 Configuration Space

Configurations are defined by indicator functions:

$$\chi_\alpha(i) \in \{0, 1\}. \quad (176)$$

They define coarse-grained states:

$$S_\alpha = \{i \mid \chi_\alpha(i) = 1\}. \quad (177)$$

A.5 Structural Quantities

For each configuration S_α :

Weight

$$W_\alpha = \sum_i p_i \chi_\alpha(i). \quad (178)$$

Accessibility

$$A_\alpha = \sum_i p_i \chi_\alpha(i) a_i. \quad (179)$$

Stability

$$\Gamma_\alpha = \sum_i \gamma_i \chi_\alpha(i), \quad S_\alpha = \frac{1}{\Gamma_\alpha + \epsilon}. \quad (180)$$

A.6 Structural Probability

$$P_\alpha^{\text{struct}} = \frac{W_\alpha A_\alpha S_\alpha}{\sum_\beta W_\beta A_\beta S_\beta}. \quad (181)$$

A.7 Transition Between Configurations

$$T_{\alpha\beta} = \frac{\sum_{i,j} p_i \chi_\alpha(i) T_{ij} \chi_\beta(j)}{\sum_i p_i \chi_\alpha(i)}. \quad (182)$$

A.8 Information-Theoretic Distance

$$d_{ij} = -\log T_{ij}. \quad (183)$$

In the continuum:

$$d(x, y) = -\log T(x, y). \quad (184)$$

A.9 Metric

$$g_{\mu\nu}(x) = \partial_\mu \partial_\nu (-\log T(x, y))|_{y=x}. \quad (185)$$

A.10 Curvature

Discrete curvature:

$$\mathcal{K}_{\alpha\beta\gamma} = \log \frac{T_{\alpha\gamma}}{T_{\alpha\beta} T_{\beta\gamma}}. \quad (186)$$

Continuum limit:

$$\mathcal{K} \rightarrow R_{\mu\nu\rho\sigma}. \quad (187)$$

A.11 Spectral Structure

$$T v_k = \lambda_k v_k, \quad (188)$$

with:

$$1 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \dots \quad (189)$$

A.12 Dynamic Drift

$$\Delta P = P^{(\infty)} - P^{\text{struct}}. \quad (190)$$

$$\|\Delta P\| \sim \frac{1}{1 - \lambda_2}. \quad (191)$$

A.13 Field Equations

$$\mathcal{G}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \quad (192)$$

$$\mathcal{G}_{\mu\nu} = \kappa T_{\mu\nu}. \quad (193)$$

A.14 Summary

The core structure of the theory is:

$$\boxed{T \rightarrow d \rightarrow g_{\mu\nu} \rightarrow R_{\mu\nu} \rightarrow \mathcal{G}_{\mu\nu}} \quad (194)$$

B Additional Derivations

B.1 Derivation of the Emergent Metric

We start from the definition:

$$d(x, y) = -\log T(x, y). \quad (195)$$

For nearby points:

$$y = x + \delta x, \quad (196)$$

we assume smoothness:

$$T(x, y) = 1 - \frac{1}{2} A_{\mu\nu}(x) \delta x^\mu \delta x^\nu + o(|\delta x|^2). \quad (197)$$

Taking the logarithm:

$$-\log T(x, y) \approx \frac{1}{2} A_{\mu\nu}(x) \delta x^\mu \delta x^\nu. \quad (198)$$

Thus:

$$\boxed{g_{\mu\nu}(x) = A_{\mu\nu}(x)} \quad (199)$$

B.2 Spectral Control of Dynamic Drift

Let the transition matrix admit spectral decomposition:

$$T = \sum_k \lambda_k v_k w_k^T. \quad (200)$$

The evolution of probability is:

$$P^{(t)} = \sum_k c_k \lambda_k^t v_k. \quad (201)$$

The stationary state corresponds to $\lambda_1 = 1$. Deviations are dominated by λ_2 :

$$P^{(t)} - P^{(\infty)} \sim \lambda_2^t. \quad (202)$$

Thus, the relaxation time is:

$$\tau \sim \frac{1}{1 - \lambda_2}. \quad (203)$$

Since drift accumulates over relaxation:

$$\boxed{\|\Delta P\| \sim \frac{1}{1 - \lambda_2}} \quad (204)$$

B.3 Derivation of the Diffusion Equation

Starting from:

$$P^{(t+1)} = T^T P^{(t)}, \quad (205)$$

we expand:

$$P(t + \Delta t) - P(t) \approx (T^T - I)P. \quad (206)$$

Assuming locality:

$$(T^T - I)P \sim \nabla_\mu (D^{\mu\nu} \nabla_\nu P), \quad (207)$$

we obtain:

$$\boxed{\partial_t \rho = \nabla_\mu (D^{\mu\nu} \nabla_\nu \rho)} \quad (208)$$

B.4 Variational Derivation of Field Equation

Consider the action:

$$S = \int \rho R \sqrt{g} - \lambda \int \rho \log \rho \sqrt{g}. \quad (209)$$

Varying with respect to ρ :

$$\delta S = \int \delta \rho (R - \lambda (\log \rho + 1)) \sqrt{g}. \quad (210)$$

Setting $\delta S = 0$ yields:

$$\boxed{R = \lambda \log \rho} \quad (211)$$

B.5 Tensor Structure Consistency

We verify that:

$$\nabla^\mu \mathcal{G}_{\mu\nu} = 0 \quad (212)$$

implies:

$$\nabla^\mu T_{\mu\nu} = 0, \quad (213)$$

ensuring consistency with conservation laws.

B.6 Summary of Derivations

The key derived relations are:

$$\boxed{g_{\mu\nu} \sim \partial_\mu \partial_\nu (-\log T)} \quad (214)$$

$$\boxed{\|\Delta P\| \sim \frac{1}{1 - \lambda_2}} \quad (215)$$

$$\boxed{\partial_t \rho = \nabla_\mu (D^{\mu\nu} \nabla_\nu \rho)} \quad (216)$$

$$\boxed{\mathcal{G}_{\mu\nu} = \kappa T_{\mu\nu}} \quad (217)$$

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