

Recalibration of the UAT/UCP Acoustic Horizon: From the Empirical Approximation $r_d \approx 141$ Mpc to the Exact Value $r_d = 128.15$ Mpc

Miguel Ángel Percudani

Independent Researcher, Puan, Buenos Aires, Argentina

miguel_percudani@yahoo.com.ar

ORCID: 0009-0007-1748-3212

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IMPORTANT

The upload of this new update on the corresponding platform
supersedes Note 7 in the predecessor upload.

Abstract

This technical note documents the recalibration of the sound horizon used in the UAT/UCP framework. Earlier work reported $r_d \approx 141$ Mpc as a validated result. That value was not derived from the exact sound-horizon integral; it was obtained from an empirical scaling relation calibrated against an external reference. The present work replaces that approximation with a direct integration of the sound-horizon equation from $z_{\text{rec}} = 1089$ to $z_{\text{max}} = 20000$, using BBN-derived Ω_b , CMB-derived Ω_γ , and the stable scalar-field branch $\phi(z_{\text{rec}}) \approx 0$. The corrected value is $r_d = 128.15$ Mpc. This is not a new free parameter. It is a numerical correction of the same physical integral that the earlier approximation avoided. All calculations are performed in pure Python, without CLASS, CAMB, or Λ CDM libraries.

1 Introduction

The sound horizon at recombination is one of the most important scales in observational cosmology. It sets the size of the acoustic peaks in the CMB and provides the standard ruler for BAO measurements.

In the UAT/UCP framework, earlier publications used

$$r_d \approx 141.80 \text{ Mpc.} \tag{1}$$

This value appeared in several validation tables and was used in BAO comparisons.

The present note explains that this value was an empirical approximation, not the result of the exact sound-horizon integral. The corrected value is

$$\boxed{r_d = 128.15 \text{ Mpc.}} \quad (2)$$

2 Exact Definition of the Sound Horizon

The sound horizon is defined as

$$r_d = \int_{z_{\text{rec}}}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (3)$$

where the sound speed is

$$c_s^2(z) = \frac{1}{3(1 + R_b(z))}, \quad R_b(z) = \frac{3\Omega_b}{4\Omega_\gamma(1+z)}. \quad (4)$$

The integral runs from the recombination redshift to arbitrarily high redshift. If the upper limit is truncated too early, the result is underestimated.

3 The Earlier Approximation

The earlier value $r_d \approx 141 \text{ Mpc}$ was not obtained by evaluating Eq. (3) in full.

Instead, the historical implementation used a scaling relation:

$$r_d \propto \frac{1}{H_0} \sqrt{k_{\text{early}}}, \quad (5)$$

with an external calibration baseline of approximately 142.33 Mpc.

The original technical note stated:

“Direct numerical integration of Equation (4) produced unstable results, with the sound horizon r_d showing high sensitivity to integration limits and step sizes.

*Solution: Employed a calibrated approach using the physical scaling relation
...*”

This was an honest workaround for numerical instability, but it was not the full integral.

4 Why the Earlier Value Was an Approximation

Three limitations were present in the earlier approach.

1. **Truncated integration range.** The exact integral requires $z \rightarrow \infty$. The earlier method avoided this range because of numerical sensitivity.
2. **External calibration baseline.** The value 142.33 Mpc was calibrated from an external reference, not from UAT-derived densities.

3. **Scalar-field branch not distinguished.** The unstable branch $\phi_* = 0.348$ was still present in some calculations. The stable branch $\phi(z_{\text{rec}}) \approx 0$ had not yet been isolated.

For these reasons, the earlier value was not a direct prediction. It was an effective parameter.

5 The Corrected Calculation

The corrected calculation uses the exact integral, Eq. (3), with the following inputs.

5.1 Numerical inputs

- $z_{\text{rec}} = 1089$;
- $z_{\text{max}} = 20000$;
- $\Omega_b = 0.04182$ from BBN;
- $\Omega_\gamma = 5.38 \times 10^{-5}$ from COBE/FIRAS;
- $H_0 = 73.04$ km/s/Mpc from SH0ES.

5.2 Scalar-field branch

The stable branch is

$$\phi(z_{\text{rec}}) \approx 0, \quad (6)$$

which gives

$$k_{\text{eff}}(z_{\text{rec}}) = 1 + \xi\phi^2 \approx 1. \quad (7)$$

As a result,

$$H_{\text{UAT}}(z_{\text{rec}}) \approx H_{\Lambda\text{CDM}}(z_{\text{rec}}). \quad (8)$$

The early expansion history is therefore essentially unchanged at recombination.

5.3 Field equations used

The scalar field obeys

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \xi R\phi = 0, \quad (9)$$

with

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 + V_0, \quad (10)$$

and

$$\eta = \kappa_{\text{crit}} = 4.978, \quad \xi = -0.2810, \quad \lambda \approx 3.08 \times 10^{-112}. \quad (11)$$

The effective gravitational coupling is

$$G_{\text{eff}} = \frac{G}{1 + \xi\phi^2}, \quad (12)$$

and the effective expansion factor is

$$k_{\text{eff}}(z) = 1 + \xi\phi^2(z). \quad (13)$$

For the stable branch,

$$k_{\text{eff}}(z) \approx 1 \quad \text{for } z \gg 1. \quad (14)$$

5.4 Integration result

Pure Python integration of Eq. (3) gives

$$\frac{r_d H_0}{c} \approx 0.03122, \quad (15)$$

and therefore

$$r_d = 128.15 \text{ Mpc} \quad \text{for } H_0 = 73.04. \quad (16)$$

6 Comparison of Old and New Values

Table 1 summarises the difference.

Table 1: Sound horizon values before and after recalibration.

Source	r_d [Mpc]	Status
Earlier empirical calibration	141.80	superseded
Earlier scaling baseline	142.33	superseded
This work: exact integration	128.15	corrected

7 Consequences for Previous Analyses

All BAO fits that used $r_d = 141$ Mpc must be recomputed.

The corrected product is

$$r_d H_0 = 128.15 \times 73.04 \approx 9359 \text{ Mpc km/s/Mpc}. \quad (17)$$

H_0 remains adopted from SH0ES. It is not derived from the UAT/UCP action.

8 Supplementary Material

The three earlier technical documents that reported $r_d \approx 141$ Mpc are archived as Supplementary 2. They are provided for historical reference and to show the origin of the previous approximation.

9 Limitations

The following limitations remain:

1. The scalar-field trajectory has been solved using a fixed Λ CDM background for the Ricci scalar, not fully self-consistently.
2. The late-time transition that would produce $G_{\text{eff}}(0) \approx 1.062G$ has not been derived from the scalar-field equation.
3. No new BAO or CMB fit has yet been performed with the corrected r_d .
4. A full UAT/UCP Boltzmann implementation is still absent.

10 Conclusion

The UAT/UCP acoustic horizon has been recalibrated from the empirical approximation

$$r_d \approx 141 \text{ Mpc} \quad (18)$$

to the exact value

$$r_d = 128.15 \text{ Mpc}. \quad (19)$$

This correction follows from:

- the exact sound-horizon integral;
- the full redshift range;
- the stable scalar-field branch;
- BBN- and CMB-derived densities;
- pure Python integration without Λ CDM libraries.

It is a numerical correction, not a new free parameter.

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