

Deductive Formalism of Electrodynamics

Part 2

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Abstract: The second part of the description of the axiomatic approach to the development of electrodynamics in the form of a deductive formalism based on four-dimensional pseudo-Euclidean geometry proposed by A. Poincare and H. Minkovsky is given. In the first part, the description was expanded and the equations of motion of the electromagnetic field were clarified. In the second part, a complete system of static and dynamic electromagnetic forces, equations of free motion of electric charges, and equations of charge density waves resulting from deductive formalism are obtained.

Keywords: axiomatic deductive formalism, electrodynamics, pseudo-Euclidean geometry, Maxwell's theory, photon equation, longitudinal waves, electromagnetic field, electromagnetic forces, Nikolaev force.

1 Introduction

The theory of the electromagnetic field (EMF) and the interaction of electric charges and currents was developed by James Clerk Maxwell (1861-1865) using an inductive method based on the generalization of experimental facts accumulated during that historical period. In any theory, the inductive method cannot avoid omissions and errors, which also exist in J. Maxwell's theory. After the discovery by G. Hertz in 1888, predicted by J. Maxwell's electromagnetic waves, confirming the amazing effectiveness of mathematics, based on the analysis of the theory of J. Maxwell and the works of H. Lorentz is a French mathematician In 1904, A. Poincare made a fundamental discovery of a new geometry of our world - a four-dimensional pseudo-Euclidean space-time [1]. All the physical processes of our world take place in this space, and the law of conservation of energy-momentum and the wave equations of EMF are a direct consequence of this geometry. The theory of pseudo-Euclidean space-time was further developed by H. Minkovsky and it was named Minkovsky space. A. Poincare and H. Minkovsky were the first to switch to four-dimensional physical indicators. To describe the EMF and its sources, Poincare introduced the four-dimensional electromagnetic potential $A_\mu(\frac{1}{c}\varphi, i\mathbf{A})$ and the four-dimensional current density $J_\mu(c\rho, i\mathbf{J})$ ($\mu=0,1,2,3$), marking the beginning of the deductive formalism of electrodynamics. H. Minkovsky continued and developed these works and created a description of EMF in the form of a four-dimensional antisymmetric tensor of the second rank for EMF voltages $\mathbf{E}$ and $\mathbf{B}$. This description is historically established, is considered classical, and is widely used in educational literature.

The description of EMF, introduced by A. Poincare in the form of an electromagnetic potential, is more fundamental, since these characteristics of EMF are energetic, and the power characteristics $\mathbf{E}$ and $\mathbf{B}$ follow from the energy characteristics

as their derivatives. Regarding the introduction of the four-dimensional electromagnetic potential formalism, A. Sommerfeld said: "The actual mathematical nature of this formation will only now be able to appear in all its beauty, like a mountain landscape when the fog clears." These words by A. Sommerfeld can serve as an epigraph to the present work. Since all physical processes, including the basic laws of mechanics and electrodynamics, are a direct consequence of the pseudo-Euclidean geometry of our world, it becomes possible to deduce these laws using the deductive method based on an axiomatic approach. One of the main advantages of axiomatic deductive formalism is that most statements in such a theory are analytical truths that do not require empirical justification, since their truth is guaranteed by their logical and mathematical derivability from the axioms of the theory. A theory constructed using this method has an important cognitive property: it can predict and synthesize new scientific knowledge. In the first part [2] of this work, the description and equations of motion of the electromagnetic field are expanded and clarified using axiomatic deductive formalism. The problems of violations of Newton's third law in electrodynamics, the phenomenon of a magnetic dynamo, and infinite velocity in Gauss' law for an electric field are solved. An additional wave equation of EMF propagation has been obtained, which is presumably interpreted as a photon equation that solves the problem of the radial propagation of light and its wave-particle dualism. A wave equation for scalar longitudinal waves of EMF stretching/compression is obtained, and EMF sources are mathematically correctly introduced.

The theoretical results obtained in the first part of this paper show the effectiveness of deductive formalism, since in fact all the main known problems of Maxwell's theory have been solved without involving any additional assumptions and hypotheses.

The first part of this paper does not address the issues of electromagnetic forces, except for mentioning the hypothetical G.V. Nikolaev force [3-5], which solves the problem of violating Newton's third law in electrodynamics. This force is not recognized in electrodynamics and has not been included in physics textbooks due to the lack of a convincing theoretical justification for it. In the first part of this paper, it was shown that the EMF intensity causing this force, when it interacts with electric charges, is contained in the full EMF tensor as its component, as well as the Lorentz and Coulomb forces. This is a convincing theoretical justification for the reality of its existence.

Let's write down the known electromagnetic forces in terms of potentials:

1) The electric potential Coulomb force acting between static electric charges. It is proportional to the strength of the potential electric field at the location of the charge:

$$\mathbf{F}_p = e \cdot \mathbf{E}_p = e \cdot (-\nabla \varphi) \quad (1)$$

2) The electric dynamic Coulomb force is a vortex force proportional to the rate of change in time of the vector potential of the EMF:

$$\mathbf{F}_v = e \cdot \mathbf{E}_v = e \cdot (-\partial_t \mathbf{A}) \quad (2)$$

3) The Ampere magnetic force acting on a conductor with a current and the Lorentz magnetic force acting on electric charges with a density of ρ moving at a speed $\mathbf{V}$ will be written in a uniform form in terms of the current density $\mathbf{J}$ and the vector potential $\mathbf{A}$:

$$\mathbf{F}_{A-J} = \rho \cdot \mathbf{V} \times \nabla \times \mathbf{A} = \mathbf{J} \times \nabla \times \mathbf{A} \quad (3)$$

4) Nikolayev's magnetic force

$$\mathbf{F}_H = \mathbf{J}(\nabla \cdot \mathbf{A}) \quad (4)$$

The fact that G.V. Nikolaev experimentally discovered a new magnetic force indicates the possible existence of other electromagnetic forces that have not yet been experimentally discovered. The deductive formalism of electrodynamics, based only on two initial positions proposed by A. Poincare, allows theoretically to find a complete system of electromagnetic forces. Such a complete system of electromagnetic forces will be of great theoretical importance for plasma physics, in which electromagnetic forces and the laws of motion of electric charges are the physical essence of this field of science.

The topic of the second part of this paper is the derivation of a complete system of electromagnetic forces and the basic equations of motion of electric charges by the axiomatic method of deductive formalism of electrodynamics.

2 Deductive formalism of electrodynamics

For the convenience of readers, this section provides basic information on the deductive formalism of electrodynamics with additions that have arisen in the course of this work. The basis and starting point of the description of EMF and its sources in deductive formalism are the vectors of the four-dimensional electromagnetic potential $A_\alpha(\frac{1}{c}\varphi, i\mathbf{A})$ and current density $J_\alpha(c\rho, i\mathbf{j})$ $\alpha=0,1,2,3$, introduced by A. Poincare. The structure of the deductive formalism for obtaining the basic laws of electrodynamics is shown in Figure 1.

In the next section, we will consider the second branch of deductive formalism, which describes electromagnetic forces and movements of electric charges. This branch of deductive formalism is closely related to plasma physics, in which there are many questions and unresolved problems. The most striking example is such a well-known natural phenomenon as ball lightning, for which there is no explanation to date. In this phenomenon, plasma exists in a limited volume of space and is held by its own electromagnetic forces for a long time. Large labor and financial resources have been spent on the development of methods for forced plasma retention in tokamaks and other technical installations for more than seventy years, but the problem of plasma self-retention remains unresolved. The record time for forced plasma retention is now tens of minutes. This suggests that there are large theoretical gaps in this field of electrodynamics, and the successes have been achieved mainly by experimental methods.

For example, there is a well-known theoretical problem of EMT EMF, which V. Ginzburg called "eternal". The existing canonical EMT [6], which has a zero trace, leads to an error in the virial theorem applied to plasma equilibrium conditions; as a result, a theoretical conclusion is drawn about the impossibility of the existence of ball lightning and other long-lived local autonomous plasma structures [7]. The correction of this EMT also leads to a change in the conclusions of the virial theorem for plasma [8].

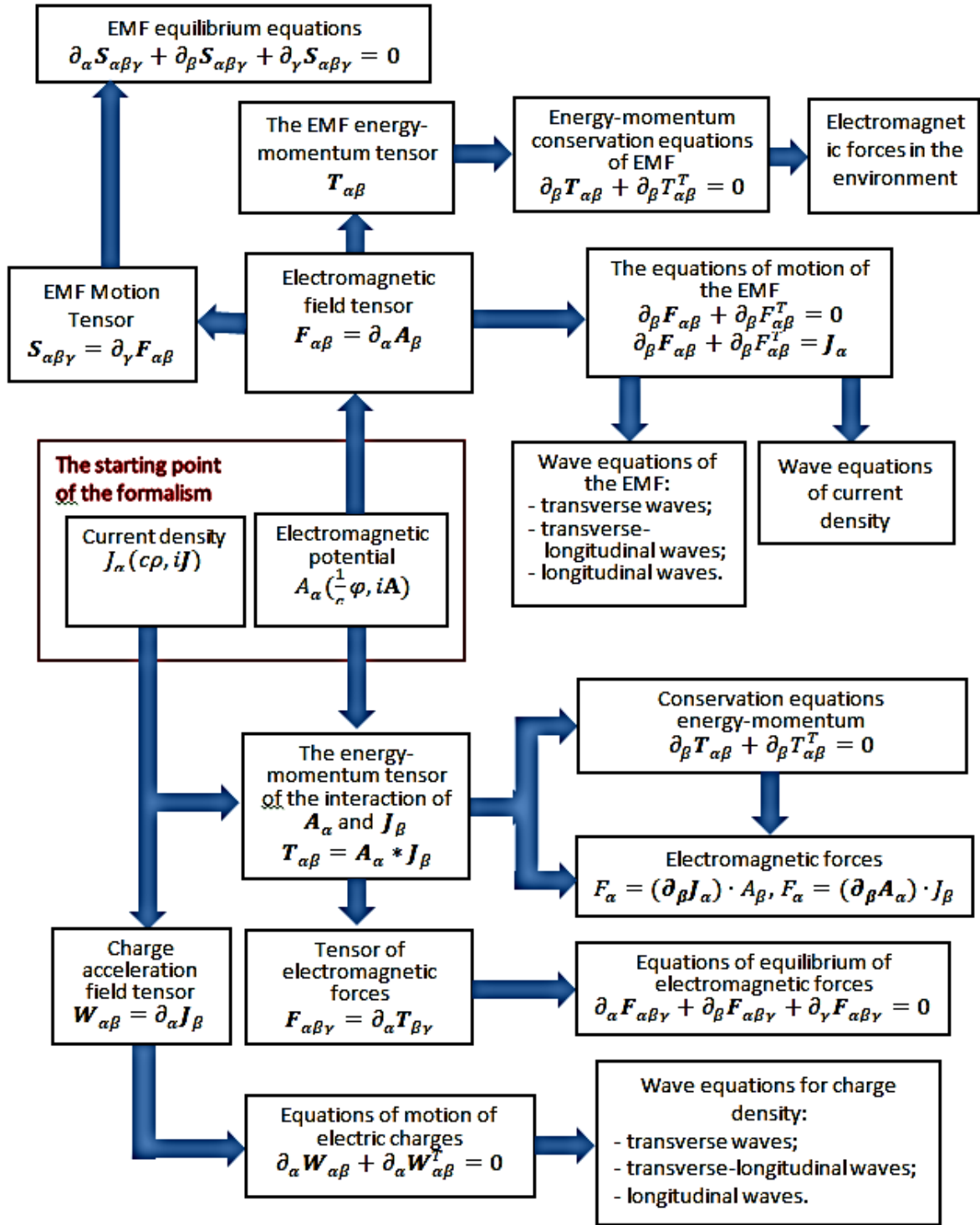


Fig. 1 Axiomatic deductive formalism of electrodynamics

Nikolayev's magnetic force has not had a convincing theoretical basis to date and is therefore not accepted in plasma physics. The fact that not all electromagnetic forces are known in electrodynamics is indicated by a simple calculation of the components of electromagnetic forces. The four known electromagnetic vector forces (1-4) contain 12 scalar components in total. Electromagnetic forces are derived from the energy of electromagnetic interaction. The EMT of electromagnetic interaction contains 16 components, and the four-dimensional derivative of this EMT leads to a third-rank tensor, the components of which are the scalar components of all electromagnetic forces. This

electromagnetic force tensor (EFT) contains 64 components. Thus, the electromagnetic forces unknown in electrodynamics comprise 52 scalar components. It should be recalled here that the EMF is a tensor field of the second rank, and its division into separate three-dimensional fields $\mathbf{E}$ and $\mathbf{B}$ is conditional. Similarly, electromagnetic forces are a single tensor field of the third rank, and their division into separate electromagnetic forces (Coulomb, Lorentz, Nikolaev, etc.) is conditional. The components of all electromagnetic forces are interconnected by differential equations arising from thermal power plants. Using these coupling equations and well-known vector identities, various three-dimensional variants (combinations) of the representation of electromagnetic forces can be formed from the components of thermal power plants. For example, the electromagnetic force determined by the gradient of the scalar product of the vector potential $\mathbf{A}$ by the current density vector $\mathbf{J}$ gives an identical expression that includes a number of electromagnetic forces, including the Lorentz (Ampere) $\mathbf{J} \times \nabla \times \mathbf{A}$ and Nikolaev $\mathbf{J}(\nabla \cdot \mathbf{A})$ forces:

$$\nabla(\mathbf{A} \cdot \mathbf{J}) = 2(\mathbf{A} \cdot \nabla)\mathbf{J} + \nabla \times (\mathbf{A} \times \mathbf{J}) + \mathbf{J} \times \nabla \times \mathbf{A} + \mathbf{A} \times \nabla \times \mathbf{J} + \mathbf{J}(\nabla \cdot \mathbf{A}) - \mathbf{A}(\nabla \cdot \mathbf{J}) \quad (5)$$

It also follows from this expression that the Lorentz force (Ampere) can be identically represented as a combination of six other electromagnetic forces unknown in electrodynamics. Thus, in a four-dimensional pseudo-Euclidean space-time, the electromagnetic force is described by a single four-dimensional tensor of the third rank having 64 components.

It should be emphasized that all the theoretical results obtained in this work are determined only by the four-dimensional pseudo-Euclidean geometry of space-time and the initial points of the formalism in the form of two four-dimensional vectors of electromagnetic potential and current density proposed by A. Poincare. No external additional elements, assumptions, assumptions or hypotheses are introduced into the formalism. The assumptions relate only to the physical interpretation of the theoretical results obtained, which require experimental verification.

3 The energy-momentum tensor of the interaction of charges and currents with EMF and electromagnetic forces

The canonical EMT has the general form [6]:

$$\mathbf{T}_{\nu\mu} = \begin{bmatrix} W & i\frac{1}{c}\mathbf{S} \\ ic \cdot \mathbf{g} & t_{ik} \end{bmatrix} \quad (\nu, \mu=0, 1, 2, 3; \quad i, k=1, 2, 3) \quad (6)$$

where W is the energy density;

$\mathbf{S}$ is the energy flux density (Umov-Poynting vector);

$\mathbf{g}$ is the pulse density;

t_{ik} is the momentum flux density tensor (stress tensor).

In accordance with the deductive formalism, we find the EMT of electromagnetic interaction of electromagnetic radiation with charges and currents as the tensor product of the electromagnetic potential $A_\alpha(\frac{1}{c}\varphi, i\mathbf{A})$ and the four-dimensional current density $J_\alpha(c\rho, i\mathbf{J})$:

$$\mathbf{W}_{\alpha\beta} = \mathbf{A}_\alpha \otimes J_\beta = \begin{bmatrix} \rho \cdot \varphi & \frac{1}{c} i \cdot \varphi \cdot J_x & \frac{1}{c} i \cdot \varphi \cdot J_y & \frac{1}{c} i \cdot \varphi \cdot J_z \\ i \cdot c \cdot \rho \cdot A_x & -A_x \cdot J_x & -A_x \cdot J_y & -A_x \cdot J_z \\ i \cdot c \cdot \rho \cdot A_y & -A_y \cdot J_x & -A_y \cdot J_y & -A_y \cdot J_z \\ i \cdot c \cdot \rho \cdot A_z & -A_z \cdot J_x & -A_z \cdot J_y & -A_z \cdot J_z \end{bmatrix} = \begin{bmatrix} \rho \cdot \varphi & \frac{1}{c} i \cdot \varphi \cdot \mathbf{J} \\ i \cdot c \cdot \rho \cdot \mathbf{A} & -\mathbf{A}_i \otimes J_k \end{bmatrix} \quad (7)$$

where $W = \rho \cdot \varphi$ is the energy density;

$S = \varphi \cdot \mathbf{J}$ is the energy flux density (Umov-Poynting vector);

$g = \rho \cdot \mathbf{A}$ is the pulse density;

$t_{ik} = A_i \cdot J_k$ is the momentum flux density tensor (stress tensor).

The equations of conservation of energy-momentum of the interaction of charges and currents with EMF follow from EMT (7). The diagonal of EMT (7) or its trace $I = \varphi \cdot \rho - \mathbf{A} \cdot \mathbf{J}$ is an invariant, which is known as the generalized energy density of electromagnetic interaction or as the Schwarzschild invariant [9]. This invariant is used as the Lagrangian density in the Lagrange formalism. Let's find the four-dimensional gradient of this scalar invariant and get a four-dimensional vector of the invariant electromagnetic force consisting of two four-dimensional vectors enclosed in square brackets:

$$F_\alpha = \partial_\alpha I = \partial_\alpha (\varphi \rho) - \partial_\alpha (\mathbf{A} \cdot \mathbf{J}) = \left[\frac{1}{c} \partial_t (\varphi \rho), -i \nabla (\varphi \rho) \right] - \left[\frac{1}{c} \partial_t (\mathbf{A} \cdot \mathbf{J}), -i \nabla (\mathbf{A} \cdot \mathbf{J}) \right] \quad (8)$$

Let's perform the differentiation, and replace the term $\nabla (\mathbf{A} \cdot \mathbf{J})$ with the expression (5):

$$\begin{aligned} F_\alpha = & [(\varphi \cdot \partial_t \rho / c + \rho \cdot \partial_t \varphi / c), -i(-\rho \cdot \nabla \varphi - \varphi \cdot \nabla \rho)] - \\ & - [(-\mathbf{A} \cdot \partial_t \mathbf{J} / c - \mathbf{J} \cdot \partial_t \mathbf{A} / c), -i(2(\mathbf{A} \cdot \nabla) \mathbf{J} + \nabla \times (\mathbf{A} \times \mathbf{J}) + \\ & + \mathbf{J} \times \nabla \times \mathbf{A} + \mathbf{A} \times \nabla \times \mathbf{J} + \mathbf{J}(\nabla \cdot \mathbf{A}) - \mathbf{A}(\nabla \cdot \mathbf{J}))] \end{aligned} \quad (9)$$

The components of this four-dimensional invariant force F_α are 12 three-dimensional electromagnetic forces. For the convenience of considering the components of force (9), we write them down in the form of Table 1.

Table 1. Components of the invariant force (9)

№	Mathematical description of the force components (9)	Name of the force	Kind of power	Note
1	$\varphi \cdot \partial_t \rho / c$	—	scalar	Small dynamic
2	$\rho \cdot \partial_t \varphi / c$	—	scalar	Small dynamic
3	$\rho \cdot \nabla \varphi$	Coulomb force	vector	Static
4	$\varphi \cdot \nabla \rho$	—	vector	Static
5	$\mathbf{A} \cdot \partial_t \mathbf{J} / c$	—	scalar	Small dynamic
6	$\mathbf{J} \cdot \partial_t \mathbf{A} / c$	—	scalar	Small dynamic
7	$(\mathbf{A} \cdot \nabla) \mathbf{J}$	—	vector	Stationary
8	$\nabla \times (\mathbf{A} \times \mathbf{J})$	—	vector	Stationary
9	$\mathbf{J} \times \nabla \times \mathbf{A}$	Lorentz force	vector	Stationary
10	$\mathbf{A} \times \nabla \times \mathbf{J}$	—	vector	Stationary
11	$\mathbf{J}(\nabla \cdot \mathbf{A})$	Nikolaev force	vector	Stationary
12	$\mathbf{A}(\nabla \cdot \mathbf{J})$	—	vector	Stationary

As already mentioned, the representation of forces (9) is only one of the possible representations. This representation was chosen solely to show the place of the known electromagnetic forces (Coulomb, Lorentz, Nikolaev) among other electromagnetic forces. Table 1 does not list all electromagnetic forces, but only the forces associated with the invariant of the energy of interaction of charges and currents with EMF. Forces 7-12 can be represented through each other or written down in accordance with (5) in the form $\nabla(\mathbf{A} \cdot \mathbf{J})$. Dynamic forces 1, 2, 5, and 6 contain a coefficient of $1/s$ and are small in magnitude and manifest themselves in rapid electrodynamic processes.

To find the energy-momentum conservation equations, which also include electromagnetic forces, we find the divergence of tensor (7) and equate it to zero, assuming that no other forces than electromagnetic forces act on charges (to simplify, inertia forces are not taken into account here). Since EMT (7) is asymmetric, we find its divergence for each index and summarize the result. The sum of a tensor and a transposed tensor of the second rank preserves its symmetric part, and then we obtain the energy-momentum conservation equation in the form:

$$\partial_\beta (W_{\alpha\beta} + W_{\alpha\beta}^T) = \partial_\beta W_{(\alpha\beta)} = 0 \quad (10)$$

This equation in the matrix representation has the form:

$$\partial_\beta \mathbf{W}_{(\alpha\beta)} = \partial_\beta \begin{bmatrix} 2\rho \cdot \varphi & \frac{1}{c} i \cdot \varphi \cdot \mathbf{J}_x + i \cdot c \cdot \rho \cdot A_x & \frac{1}{c} i \cdot \varphi \cdot \mathbf{J}_y + i \cdot c \cdot \rho \cdot A_y & \frac{1}{c} i \cdot \varphi \cdot \mathbf{J}_z + i \cdot c \cdot \rho \cdot A_z \\ \frac{1}{c} i \cdot \varphi \cdot \mathbf{J}_x + i \cdot c \cdot \rho \cdot A_x & -2A_x \cdot \mathbf{J}_x & -A_x \cdot \mathbf{J}_y - A_y \cdot \mathbf{J}_x & -A_x \cdot \mathbf{J}_z - A_z \cdot \mathbf{J}_x \\ \frac{1}{c} i \cdot \varphi \cdot \mathbf{J}_y + i \cdot c \cdot \rho \cdot A_y & -A_x \cdot \mathbf{J}_y - A_y \cdot \mathbf{J}_x & -2A_y \cdot \mathbf{J}_y & -A_y \cdot \mathbf{J}_z - A_z \cdot \mathbf{J}_y \\ \frac{1}{c} i \cdot \varphi \cdot \mathbf{J}_z + i \cdot c \cdot \rho \cdot A_z & -A_x \cdot \mathbf{J}_z - A_z \cdot \mathbf{J}_x & -A_y \cdot \mathbf{J}_z - A_z \cdot \mathbf{J}_y & -2A_z \cdot \mathbf{J}_z \end{bmatrix} = 0 \quad (11)$$

Let's write down the energy-momentum conservation equations in three-dimensional form:

$$2\frac{1}{c} \partial_t (\rho \cdot \varphi) + \nabla \left(\frac{1}{c} \cdot \varphi \cdot \mathbf{J} \right) + \nabla (c \cdot \rho \cdot \mathbf{A}) = 0 \quad (12)$$

$$\frac{1}{c^2} \cdot \partial_t (\varphi \cdot \mathbf{J}) + \partial_t (\rho \cdot \mathbf{A}) + \mathbf{J}(\nabla \cdot \mathbf{A}) + (\mathbf{J} \cdot \nabla) \mathbf{A} + \mathbf{A}(\nabla \cdot \mathbf{J}) + (\mathbf{A} \cdot \nabla) \mathbf{J} = 0 \quad (13)$$

The terms of this system of equations are electromagnetic forces. These forces can be divided into two groups. In the first group, the forces depend on the derivatives of the EMF, and in the second group on the derivatives of charge density and current. Let's differentiate and divide these forces into two groups in square brackets:

$$\left[2\frac{1}{c} \rho \cdot \partial_t \varphi + c \cdot \rho \cdot \nabla \cdot \mathbf{A} + \frac{1}{c} \cdot \mathbf{J} \cdot \nabla \varphi \right] + \left[2\frac{1}{c} \varphi \cdot \partial_t \rho + c \cdot \mathbf{A} \cdot \nabla \rho + \frac{1}{c} \varphi \cdot \nabla \cdot \mathbf{J} \right] = 0 \quad (14)$$

$$\left[\frac{1}{c^2} \mathbf{J} \cdot \partial_t \varphi + \rho \cdot \partial_t \mathbf{A} + \mathbf{J}(\nabla \cdot \mathbf{A}) + (\mathbf{J} \cdot \nabla) \mathbf{A} \right] + \left[\frac{1}{c^2} \varphi \cdot \partial_t \mathbf{J} + \mathbf{A} \cdot \partial_t \rho + \mathbf{A}(\nabla \cdot \mathbf{J}) + (\mathbf{A} \cdot \nabla) \mathbf{J} \right] = 0 \quad (15)$$

None of the electromagnetic forces in the second square brackets of equations (14) and (15) are known in electrodynamics. In equation (14), its terms are scalar forces, and in equation (15), the terms are vector forces. New electromagnetic forces have appeared in these equations that are not included in Table 1. For the convenience of their consideration, we will write them down in the form of Table 2.

Table 2. Additional forces from equations (14) and (15)

№	Mathematical description of the force	Name of the force	Kind of power	Note
1	$c \cdot \rho \cdot \nabla \cdot \mathbf{A}$	Nikolaev force	scalar	Static
2	$c \cdot \mathbf{A} \cdot \nabla \rho$	–	scalar	Static
3	$\varphi \cdot \nabla \cdot \mathbf{J} / c$	–	scalar	Small dynamic
4	$\mathbf{J} \cdot \nabla \varphi / c$	–	scalar	Small dynamic
5	$\varphi \cdot \partial_t \mathbf{J} / c^2$	–	scalar	Small dynamic
6	$\mathbf{J} \cdot \partial_t \varphi / c^2$	–	scalar	Small dynamic
7	$\rho \cdot \partial_t \mathbf{A}$	The dynamic force Coulomb	vector	Dynamic
8	$\mathbf{A} \cdot \partial_t \rho$	–	vector	Dynamic
9	$(\mathbf{J} \cdot \nabla) \mathbf{A}$	–	vector	Stationary

Given the known vector identities, the sum of the last terms in square brackets of equation (15) can be written as:

$$(\mathbf{J} \cdot \nabla) \mathbf{A} + (\mathbf{A} \cdot \nabla) \mathbf{J} = \nabla(\mathbf{A} \cdot \mathbf{J}) - \mathbf{J} \times \nabla \times \mathbf{A} - \mathbf{A} \times \nabla \times \mathbf{J} \quad (16)$$

Then equation (15) can be written in the equivalent form:

$$\begin{aligned} & \left[\frac{1}{c^2} \mathbf{J} \cdot \partial_t \varphi + \rho \cdot \partial_t \mathbf{A} + \mathbf{J}(\nabla \cdot \mathbf{A}) - \mathbf{J} \times \nabla \times \mathbf{A} \right] + \\ & + \left[\frac{1}{c^2} \varphi \cdot \partial_t \mathbf{J} + \mathbf{A} \cdot \partial_t \rho + \mathbf{A}(\nabla \cdot \mathbf{J}) - \mathbf{A} \times \nabla \times \mathbf{J} \right] + \nabla(\mathbf{A} \cdot \mathbf{J}) = 0 \end{aligned} \quad (17)$$

Given the known vector identities, the vector equations (13), (15), (17) it can be written in various identical versions.

Tables 1 and 2 show 10 scalar and 11 vector electromagnetic forces. Of which four vector forces are known.

4 Electromagnetic force tensor and equilibrium equations

All components of electromagnetic forces are components of a tensor of the third rank, as a four-dimensional derivative of EMT (7):

$$F_{\alpha\beta\gamma} = \partial_\alpha \mathbf{W}_{\beta\gamma} = \partial_\alpha \begin{bmatrix} \rho \cdot \varphi & \frac{1}{c} i \cdot \varphi \cdot \mathbf{J} \\ i \cdot c \cdot \rho \cdot \mathbf{A} & -A_i \otimes J_k \end{bmatrix} \quad (18)$$

For better clarity, we write down the four-dimensional derivatives of the components of EMT (7) in the form of symbolic strings:

1. $\partial_\alpha(\rho \cdot \varphi) = (\partial_t(\rho \cdot \varphi) / c, -i \cdot \nabla(\rho \cdot \varphi))$
2. $\partial_\alpha(i \cdot \varphi \cdot \mathbf{J} / c) = (\partial_t(i \cdot \varphi \cdot \mathbf{J} / c) / c, -i \cdot \nabla(i \cdot \varphi \cdot \mathbf{J} / c)) =$
3. $\partial_\alpha(i \cdot c \cdot \rho \cdot \mathbf{A}) = (\partial_t(i \cdot c \cdot \rho \cdot \mathbf{A}) / c, -i \cdot \nabla(i \cdot c \cdot \rho \cdot \mathbf{A}))$
4. $\partial_\alpha(-A_i \otimes J_k) = (\partial_t(-A_i \otimes J_k) / c, -i \cdot \nabla(-A_i \otimes J_k)) \quad (i, k=1, 2, 3)$

These symbolic strings implicitly include all 64 components of electromagnetic forces. As an example, we will write down the components of the electromagnetic forces of the first line explicitly:

$$\partial_\alpha(\rho \cdot \varphi) = (\partial_t(\rho \cdot \varphi) / c, -i \cdot \nabla(\rho \cdot \varphi)) = (\varphi \cdot \partial_t \rho / c + \rho \cdot \partial_t \varphi / c, -i \cdot (\rho \cdot \nabla \varphi + \varphi \cdot \nabla \rho)) \quad (19)$$

As a result, we obtained electromagnetic forces 1-4 of Table 1. Similarly, all other components of electromagnetic forces can be explicitly obtained. All components of the electromagnetic force tensor are interconnected by equilibrium equations. To obtain the equilibrium equations, it is necessary to find the divergences of the tensor $F_{\alpha\beta\chi}$ for each of the indices and equate their sum to zero:

$$\partial_\alpha F_{\alpha\beta\chi} + \partial_\beta F_{\alpha\beta\chi} + \partial_\gamma F_{\alpha\beta\chi} = 0 \quad (20)$$

5 The acceleration tensor of electric charges and the equations of their motion

The four-dimensional current density vector can be written as:

$$J_\alpha(c \cdot \rho, i \cdot \mathbf{J}) = J_\alpha(c \cdot \rho, i \cdot (\rho \cdot \mathbf{V})) \quad (21)$$

Here $\mathbf{V}$ is the vector of the three-dimensional velocity of the charges. Then from this vector, in the form of its four-dimensional derivative, we can find the acceleration tensor of the motion of electric charges:

$$U_{\alpha\beta} = \partial_\alpha J_\beta = \begin{bmatrix} \partial_t \rho & \frac{1}{c} i \cdot \partial_t J_x & \frac{1}{c} i \cdot \partial_t J_y & \frac{1}{c} i \cdot \partial_t J_z \\ -i \cdot c \cdot \partial_x \rho & \partial_x J_x & \partial_x J_y & \partial_x J_z \\ -i \cdot c \cdot \partial_y \rho & \partial_y J_x & \partial_y J_y & \partial_y J_z \\ -i \cdot c \cdot \partial_z \rho & \partial_z J_x & \partial_z J_y & \partial_z J_z \end{bmatrix} \quad (22)$$

From this tensor, the equations of free motion of electric charges follow, in the form of the sum of the divergences of tensor (22) for each of the indices or its symmetric part by one index $\partial_\beta (U_{\alpha\beta} + U_{\alpha\beta}^T) = \partial_\beta U_{(\alpha\beta)} = 0$:

$$(2 \frac{1}{c} \partial_{tt} \rho + \frac{1}{c} \partial_t \nabla \cdot \mathbf{J} - c \Delta \rho) = 0 \text{ or } (\frac{1}{c^2} \partial_{tt} \rho - \Delta \rho) + \frac{1}{c^2} \partial_t (\partial_t \rho + \nabla \cdot \mathbf{J}) = 0 \quad (23)$$

$$\frac{1}{c^2} \partial_{tt} \mathbf{J} - \partial_t \nabla \rho - \nabla (\nabla \cdot \mathbf{J}) - \Delta \mathbf{J} = 0 \text{ or } (\frac{1}{c^2} \partial_{tt} \mathbf{J} - \Delta \mathbf{J}) - \nabla (\partial_t \rho + \nabla \cdot \mathbf{J}) = 0 \quad (24)$$

Let's find the gradient of equation (23) and the time derivative of equation (24):

$$(\frac{1}{c^2} \partial_{tt} \nabla \rho - \Delta \nabla \rho) + \frac{1}{c^2} \partial_t \nabla (\partial_t \rho + \nabla \cdot \mathbf{J}) = 0 \quad (25)$$

$$\frac{1}{c^2} \partial_t (\frac{1}{c^2} \partial_{tt} \mathbf{J} - \Delta \mathbf{J}) - \frac{1}{c^2} \partial_t \nabla (\partial_t \rho + \nabla \cdot \mathbf{J}) = 0 \quad (26)$$

Let's find the sum and difference of equations (25) and (26):

$$(\frac{1}{c^2} \partial_{tt} \nabla \rho - \Delta \nabla \rho) + \frac{1}{c^2} \partial_t (\frac{1}{c^2} \partial_{tt} \mathbf{J} - \Delta \mathbf{J}) = 0 \text{ or } (\frac{1}{c^2} \partial_{tt} - \Delta) (\nabla \rho + \frac{1}{c^2} \partial_t \mathbf{J}) = 0 \quad (27)$$

$$(\frac{1}{c^2} \partial_{tt} \nabla \rho - \Delta \nabla \rho) - \frac{1}{c^2} \partial_t (\frac{1}{c^2} \partial_{tt} \mathbf{J} - \Delta \mathbf{J}) + 2 \frac{1}{c^2} \partial_t \nabla (\partial_t \rho + \nabla \cdot \mathbf{J}) = 0 \text{ or}$$

$$(\frac{1}{c^2} \partial_{tt} - \Delta) (\nabla \rho - \frac{1}{c^2} \partial_t \mathbf{J}) + 2 \frac{1}{c^2} \partial_t \nabla (\partial_t \rho + \nabla \cdot \mathbf{J}) = 0 \quad (28)$$

Find the time derivative of equation (23) and the divergence of equation (24):

$$\partial_t [(\frac{1}{c^2} \partial_{tt} \rho - \Delta \rho) + \frac{1}{c^2} \partial_t (\partial_t \rho + \nabla \cdot \mathbf{J})] = 0 \quad (29)$$

$$\nabla[(\frac{1}{c^2}\partial_{tt}\mathbf{J}-\Delta\mathbf{J})-\nabla(\partial_t\rho+\nabla\cdot\mathbf{J})]=0 \quad (30)$$

Let's find the sum of equations (29) and (30) and after the transformation we get the wave equation:

$$(\frac{1}{c^2}\partial_{tt}-\Delta)(\partial_t\rho+\nabla\cdot\mathbf{J})=0 \quad (31)$$

The equations (23), (24), (27), (28), (31) they describe the free fundamental movements of electric charges. The constituent elements of the equations of these motions are wave terms with charge density $(\partial_{tt}\rho/c^2-\Delta\rho)$ and current density $(\partial_{tt}\mathbf{J}/c^2-\Delta\mathbf{J})$, terms with Euler's equation of motion $(\nabla\rho+\partial_t\mathbf{J}/c^2)$, and terms with the equation of continuity of current density $(\partial_t\rho+\nabla\cdot\mathbf{J})$. All these components are well known in physics in the form of separate fundamental equations. In the equations (23), (24), (27), (28), (31) their mutual fundamental relationship is shown, resulting from the acceleration tensor of the motion of charges (22). These equations are idealized; they do not take into account the mass of electric charges. Therefore, all these equations must be supplemented with the equations of four-dimensional mechanics, i.e., they must be related to the inertial forces of the mass of electric charges. All these movements of electric charges excite EMF in the surrounding space. The first part [2] of this paper shows the equations of all types of electromagnetic radiation excited by charges and currents. These are the equations of electromagnetic radiation and the equations of motion of charges (23), (24), (27), (28), (31), as well as the energy-momentum conservation laws (12), (13) represent a complete idealized system of EMF motion and its sources.

Comparative analysis of the structure of electromagnetic radiation equations and equations (23), (24), (27), (28), (31) it shows their similarity. In the first part [2] of this work, it is shown that electromagnetic radiation can exist in the form of three types of waves: transverse radio waves, transverse-longitudinal optical waves (photons) and longitudinal waves. Then, by analogy, we can assume that the vector wave equation (27) describes transverse waves of electric charges, the vector equation (28) describes transverse-longitudinal waves, and the scalar equation (31) describes longitudinal waves. Since optical radiation has corpuscular properties and ray propagation, transverse-longitudinal waves of electric charge density can possibly have similar macroscopic corpuscular-wave and ray properties. Such phenomena can be observed in plasma.

6 Conclusion

The application of an axiomatic approach to the development of electrodynamics in the form of a deductive formalism based on four-dimensional pseudo-Euclidean geometry, proposed by A. Poincare and H. Minkovsky, made it possible to expand the description of EMF and electromagnetic forces arising between EMF and electric charges. As a result, a complete system of electromagnetic forces consisting of sixty-four components of a tensor of the third rank is obtained. An energy-momentum conservation equation involving electromagnetic forces is obtained. Fundamental equations of the free motion of electric charges in pseudo-Euclidean space-time are obtained.

A complete system of electromagnetic and equations of motion of electric charges can be used in magnetic hydrodynamics and plasma physics.

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