

Deductive Formalism of Electrodynamics

Part 1

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Abstract: The article develops the deductive formalism of electrodynamics based on the four-dimensional pseudo-Euclidean geometry proposed by H. Poincaré and H. Minkowski. Utilizing this formalism, the description of the electromagnetic field is extended, and its equations of motion are refined. The paper addresses and resolves issues concerning the violation of Newton's third law in electrodynamics, the magnetic dynamo phenomenon, and the infinite velocity problem in Gauss's law for the electric field. A new additional wave equation for electromagnetic field propagation is derived, which is tentatively interpreted as the photon equation. Furthermore, a wave equation for scalar longitudinal tension/compression waves of the electromagnetic field is obtained.

Keywords: axiomatic deductive formalism, electrodynamics, pseudo-Euclidean geometry, Maxwell's theory, photon equation, longitudinal waves, electromagnetic field.

1 Introduction

The theory of the electromagnetic field (EMF) and the interaction of electric charges and currents, created by James Clerk Maxwell (1861-1865) used the inductive method based on the generalization of experimental facts accumulated during that historical period. When it was created, all known electromagnetic phenomena had already been satisfactorily explained by famous physicists Faraday, Ampere, Gauss, Weber and other researchers, and therefore Maxwell's theory was accepted indifferently. The theory's heyday came twenty years later, with the discovery of electromagnetic waves predicted by Maxwell by Heinrich Hertz in 1888, which have since become of great practical importance to humanity. The theoretical discovery of electromagnetic waves can be considered the main new result of Maxwell's theory, confirming the remarkable effectiveness of mathematics, but there was another equally fundamental discovery. After Hertz's experiments, many leading physicists and mathematicians of the time joined in the development of Maxwell's theory. The study of the properties of Maxwell's equations showed that they do not retain their form under Galilean transformations, and it becomes possible to distinguish between different moving inertial coordinate systems using electromagnetic or optical measurements. However, the experiments conducted by A. Michelson and others yielded negative results. To address this issue, in 1892, H. Lorentz modified the Galilean transformations to describe the transition from one inertial system to another. Analyzing Maxwell's theory and the works of Lorentz in 1905, A. Poincaré in his works [1, 2] made a fundamental discovery of the new geometry of our world – four-dimensional pseudo-Euclidean space-time. It is in this space that all physical processes of our world take place, and the law of conservation of energy-momentum and the wave equations of

electromagnetic fields are a direct consequence of this geometry. The theory of pseudo-Euclidean space-time was further developed by H. Minkowski in [3, 4], and it became known as Minkowski space. Poincaré and Minkowski were mathematicians, and it was natural for them to make the transition to four-dimensional physical quantities. To describe the electromagnetic field and its sources, Poincaré introduced the four-dimensional electromagnetic potential $A_\mu(\frac{1}{c}\varphi, i\mathbf{A})$ and the four-dimensional current density $J_\mu(c\rho, i\mathbf{J})$ $\mu=0,1,2,3$, which laid the foundation for the deductive formalism of electrodynamics.

H. Minkowski continued this work and developed a description of the EMF in the form of a four-dimensional antisymmetric second-rank tensor for the EMF intensities $\mathbf{E}$ and $\mathbf{B}$. This description is historically established, considered classical, and is widely used in educational literature. The description of the EMF introduced by Poincaré in the form of an electromagnetic potential is more fundamental, as these EMF characteristics are energetic in nature, whereas the force characteristics $\mathbf{E}$ and $\mathbf{B}$ are derived from them as their derivatives. Therefore, quantum electrodynamics and field theory employ electromagnetic potentials related to energy. The use of the electromagnetic potential significantly simplifies Maxwell's equations, turning some of them into mathematical identities, and also eliminates the ambiguity in the electric field intensity, which can be both potential and solenoidal, having different descriptions in terms of potentials. Regarding the introduction of the formalism of the four-dimensional electromagnetic potential, A. Sommerfeld commented: "The true mathematical nature of this formation will only now be able to emerge in all its beauty, just as a mountain landscape emerges when the fog clears." These words of A. Sommerfeld can serve as an epigraph to this work. Since all physical processes, including the fundamental laws of mechanics and electrodynamics, are a direct consequence of the pseudo-Euclidean geometry of our world, the possibility of deductively deducing these laws arises. The axiomatic deductive method is one of the fundamental scientific methods of theoretical study of nature, allowing for the construction of a consistent and coherent theory. One of the main advantages of axiomatic deductive formalism is that most statements in such a theory are analytical truths that do not require empirical justification, as their truth is guaranteed by their logical and mathematical derivability from the axioms of the theory. A theory constructed by this method possesses the important cognitive property of predicting and synthesizing new scientific knowledge.

The topic of this article is the construction of an axiomatic deductive formalism of electrodynamics, the derivation of the basic laws of electrodynamics and their comparison with the laws obtained by Maxwell using the inductive method.

2 Deductive Formalism of Electrodynamics

The fundamental axiom of the formalism is the existence of a four-dimensional pseudo-Euclidean space-time of our world. The second axiom is the vectors of the four-dimensional electromagnetic potential $A_\alpha(\frac{1}{c}\varphi, i\mathbf{A})$ and the current density $J_\alpha(c\rho, i\mathbf{J})$ $\mu=0,1,2,3$, introduced by A. Poincaré. The scheme of the deductive formalism for

obtaining the basic laws of electrodynamics is shown in Figure 1. According to this formalism, the deductive method is divided into two branches. In the first branch, we obtain the tensor of EMF strengths in Cartesian coordinates as a four-dimensional derivative $\partial_\alpha(\partial_t/c, -i \cdot \nabla)$ of the electromagnetic potential $\mathbf{F}_{\alpha\beta} = \partial_\alpha \mathbf{A}_\beta$. From which we further obtain the EMF equations and the EMF propagation equations in space, the energy-momentum tensor, and the energy-momentum conservation equations. In the second branch, we obtain the energy-momentum tensor of the electromagnetic potential $A_\alpha(\frac{1}{c}\varphi, i\mathbf{A})$ interacting with the current density $J_\alpha(c\rho, i\mathbf{J})$. From this, we further obtain the energy-momentum conservation equations and all types of electromagnetic forces between the EMF and its sources.

It should be emphasized that all theoretical results obtained in this work are determined solely by the four-dimensional pseudo-Euclidean geometry of space-time and the initial points of the formalism in the form of two four-dimensional vectors of electromagnetic potential and current density, proposed by A. Poincaré. No external additional elements, assumptions, suppositions, or hypotheses are introduced into the formalism. The assumptions concern only the physical interpretation of the obtained theoretical results, which require experimental verification. Since the theory constructed by this method possesses the property of predicting and synthesizing new scientific knowledge, and at the same time, most of its conclusions are analytical truths that do not require empirical substantiation, since their truth is guaranteed by their logical and mathematical derivability, the analysis and experimental verification of such conclusions is of great practical importance. An example of this is the obtained equations of vector transverse-longitudinal waves and scalar longitudinal waves. Vector transverse-longitudinal waves are presumably identified as optical radiation of the EMF. Therefore, the task is to theoretically study the equation of these waves for subsequent practical use, since this radiation must have the property of ray propagation in space.

In the first part of this work, the main focus is on expanding the description of the EMF, obtaining the EMF motion equations, and the wave equations. The second branch of the deductive formalism describing the electromagnetic forces will be discussed in the second part of this work.

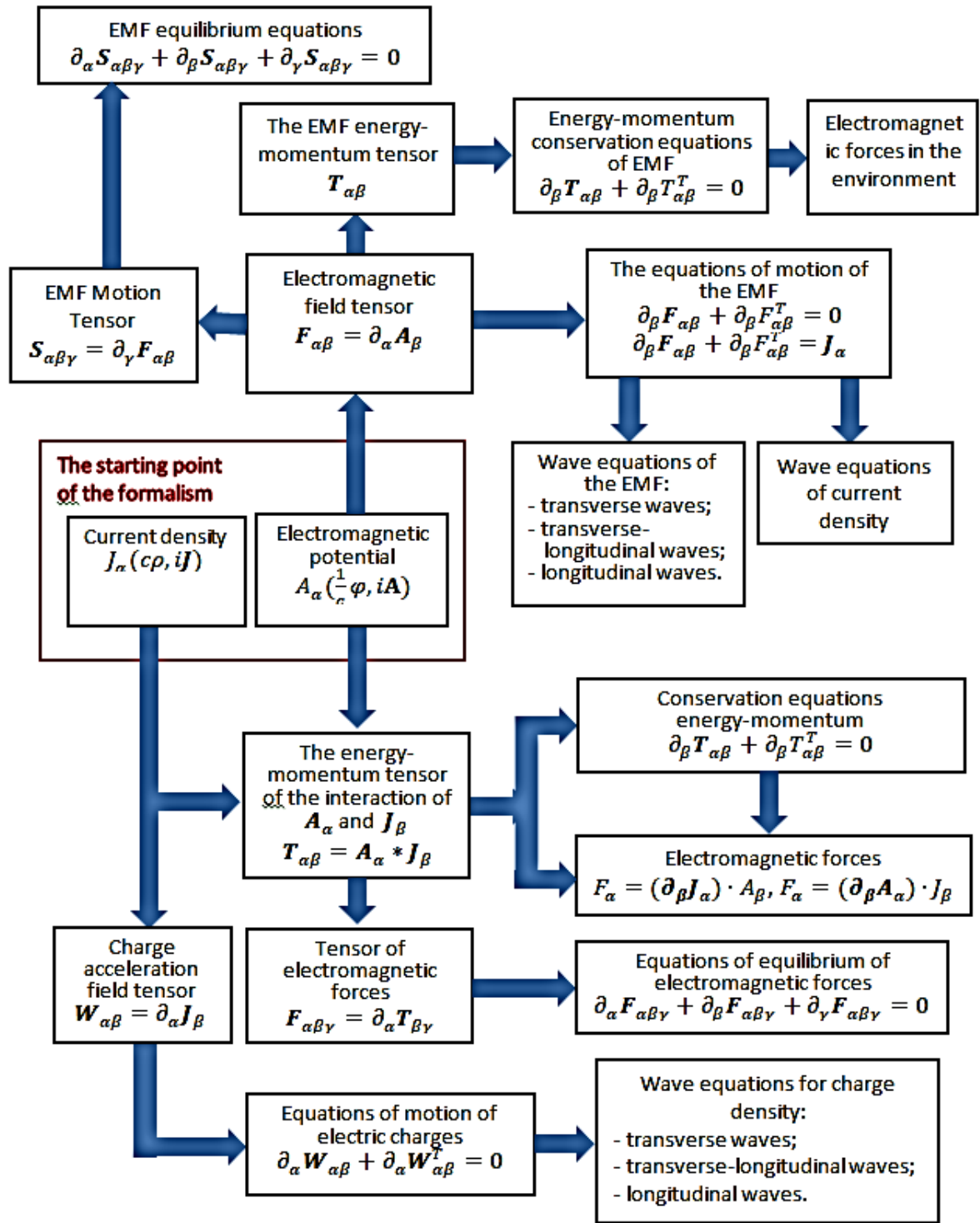


Fig. 1 Axiomatic deductive formalism of electrodynamics

3 Description of EMF in deductive electrodynamics

The field strength tensor $F_{\alpha\beta} = \partial_\alpha A_\beta$ is non-symmetric and can be uniquely decomposed into a symmetric the field strength tensor $F_{(\alpha\beta)}$ and an antisymmetric the field strength tensor $F_{[\alpha\beta]}$:

$$\mathbf{F}_{\alpha\beta} = \mathbf{F}_{(\alpha\beta)} + \mathbf{F}_{[\alpha\beta]} = \frac{1}{2}(\partial_\alpha \mathbf{A}_\beta + \partial_\beta \mathbf{A}_\alpha) + \frac{1}{2}(\partial_\alpha \mathbf{A}_\beta - \partial_\beta \mathbf{A}_\alpha) \quad (1)$$

Based on (1), we will refer to the non-symmetric EMF tensor $\mathbf{F}_{\alpha\beta} = \partial_\alpha \mathbf{A}_\beta$ as the complete EMF tensor. Minkowski introduced the antisymmetric EMF tensor $\mathbf{F}_{[\alpha\beta]}$ into electrodynamics to provide a four-dimensional description of EMF strengths and to derive Maxwell's equations. From the decomposition in (1), it can be concluded that the derivative of the electromagnetic potential $\mathbf{F}_{\alpha\beta} = \partial_\alpha \mathbf{A}_\beta$ provides a more general description of EMF than Minkowski's approach. The physical implications of the antisymmetric EMF tensor are well-known in electrodynamics textbooks.

Let us consider what physical conclusions the symmetric tensor of EMF leads to and compare them with the conclusions made from the antisymmetric tensor of EMF, obtained in the process of development of electrodynamics. If we draw an analogy with the mechanics of a continuous medium, then we can assume that the antisymmetric tensor of EMF describes small four-dimensional rotations of EMF, since it is a four-dimensional rotor, and the symmetric tensor of EMF describes a four-dimensional deformation of EMF. Since the mathematical operations for obtaining equations from the EMF tensors are uniform (we find the divergence of the tensor in Cartesian coordinates for each of the indices), we can obtain the electrodynamics equations separately a) from the original non-symmetric EMF tensor $\mathbf{F}_{\alpha\beta} = \partial_\alpha \mathbf{A}_\beta$, b) from the symmetric EMF tensor $\mathbf{F}_{(\alpha\beta)}$, and c) from the antisymmetric EMF tensor $\mathbf{F}_{[\alpha\beta]}$, and compare the results. It is evident that the sum of the equations obtained on the right-hand side of expansion (1) must be equal to the equations obtained on its left-hand side. Since the electromagnetic field tensor is of the second rank, one can find its two divergences with respect to each of its indices; in the general case, these results will be different. Consequently, in the general case, the total divergence of a second-rank tensor is equal to the sum of its divergences with respect to each of its indices. For a symmetric second-rank tensor, its divergences with respect to each index are equal, and the total divergence is equal to twice the divergence with respect to any one index. For an antisymmetric tensor, its divergences with respect to each index are equal in magnitude but opposite in sign, and the total divergence is zero. This is natural, as an antisymmetric second-rank tensor is a four-dimensional curl, which has no sources. Let us verify the above by deriving the total divergences of the asymmetric electromagnetic field tensor $\mathbf{F}_{\alpha\beta}$ and the symmetric electromagnetic field tensor $\mathbf{F}_{(\alpha\beta)}$. They must be equal. Let's write the symmetric EMF tensor $\mathbf{F}_{(\alpha\beta)}$ in matrix form:

$$\mathbf{F}_{(\alpha\beta)} = \begin{pmatrix} \frac{2}{c^2} \partial_t \varphi & \frac{1}{c} i \cdot (\partial_t A_x - \partial_x \varphi) & \frac{1}{c} i \cdot (\partial_t A_y - \partial_y \varphi) & \frac{1}{c} i \cdot (\partial_t A_z - \partial_z \varphi) \\ \frac{1}{c} i \cdot (\partial_t A_x - \partial_x \varphi) & 2\partial_x A_x & (\partial_x A_y + \partial_y A_x) & (\partial_x A_z + \partial_z A_x) \\ \frac{1}{c} i \cdot (\partial_t A_y - \partial_y \varphi) & (\partial_x A_y + \partial_y A_x) & 2\partial_y A_y & (\partial_y A_z + \partial_z A_y) \\ \frac{1}{c} i \cdot (\partial_t A_z - \partial_z \varphi) & (\partial_x A_z + \partial_z A_x) & (\partial_y A_z + \partial_z A_y) & 2\partial_z A_z \end{pmatrix} \quad (2)$$

We'll find its divergence for the EMF in a vacuum without field sources. Since the divergences of a symmetric tensor for each index are equal, the result must be multiplied by 2, but in the decomposition (1) before the symmetric tensor there is a

coefficient $\frac{1}{2}$. Therefore, we write the divergence for one index and group the terms of the resulting equations as follows:

$$2\frac{1}{c^2}\partial_{tt}\varphi + \partial_t \nabla \cdot \mathbf{A} - \Delta\varphi = 0 \quad \text{or}$$

$$\left(\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi\right) + \partial_t\left(\frac{1}{c^2}\partial_t\varphi + \nabla \cdot \mathbf{A}\right) = 0 \quad (3)$$

$$\frac{1}{c^2}\partial_{tt}\mathbf{A} - \frac{1}{c^2}\partial_t \nabla \varphi - \nabla(\nabla \cdot \mathbf{A}) - \Delta\mathbf{A} = 0 \quad \text{or}$$

$$\left(\frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A}\right) - \nabla\left(\frac{1}{c^2}\partial_t\varphi + \nabla \cdot \mathbf{A}\right) = 0 \quad (4)$$

The full non-symmetric EMF tensor in matrix form:

$$\mathbf{F}_{\alpha\beta} = \partial_\alpha \mathbf{A}_\beta = \begin{pmatrix} \frac{1}{c^2}\partial_t\varphi & \frac{1}{c}i\cdot\partial_t A_x & \frac{1}{c}i\cdot\partial_t A_y & \frac{1}{c}i\cdot\partial_t A_z \\ -\frac{1}{c}i\cdot\partial_x\varphi & \partial_x A_x & \partial_x A_y & \partial_x A_z \\ -\frac{1}{c}i\cdot\partial_y\varphi & \partial_y A_x & \partial_y A_y & \partial_y A_z \\ -\frac{1}{c}i\cdot\partial_z\varphi & \partial_z A_x & \partial_z A_y & \partial_z A_z \end{pmatrix} \quad (5)$$

Let's find its divergences for each of its indices and record their sum:

$$\left(\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi\right) + \partial_t\left(\frac{1}{c^2}\partial_t\varphi + \nabla \cdot \mathbf{A}\right) = 0 \quad (6)$$

$$\left(\frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A}\right) - \nabla\left(\frac{1}{c^2}\partial_t\varphi + \nabla \cdot \mathbf{A}\right) = 0 \quad (7)$$

As a result, we have a system of equations that is completely identical to the system of equations (3) and (4). Thus, the sum of the divergences of the full EMF tensor (5) over two indices is equal to the divergence of the symmetric tensor (2) over one of the indices. It also follows from this that the divergence of the antisymmetric tensor is identically equal to zero, since the decomposition (1) holds. Since the divergence of the symmetric tensor may not be equal to zero, we introduce field sources into equations (6) and (7):

$$\left(\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi\right) + \partial_t\left(\frac{1}{c^2}\partial_t\varphi + \nabla \cdot \mathbf{A}\right) = \rho / \varepsilon_0 \quad (8)$$

$$\left(\frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A}\right) - \nabla\left(\frac{1}{c^2}\partial_t\varphi + \nabla \cdot \mathbf{A}\right) = \mu_0 \mathbf{J} \quad (9)$$

Let's compare the obtained EMF motion equations (8) and (9) with Maxwell's equations, considering that the representation of $\mathbf{E}$ and $\mathbf{B}$ EMF intensities in potentials is $\mathbf{E} = -\nabla\varphi - \partial_t\mathbf{A}$, $\mathbf{B} = \nabla \times \mathbf{A}$. Let's write down Maxwell's equations in potentials for an isotropic homogeneous medium without dispersion by taking $\varepsilon=\mu=1$:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\partial_t \mathbf{E} \quad \text{in potentials} \quad \nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{J} - \frac{1}{c^2}\partial_t \nabla \varphi - \frac{1}{c^2}\partial_{tt}\mathbf{A} \quad (10)$$

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \quad \text{in potentials} \quad \nabla \cdot (-\nabla \varphi - \frac{1}{c^2}\partial_t \mathbf{A}) = \rho / \varepsilon_0 \quad (11)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \quad \text{in potentials} \quad -\partial_t (\nabla \times \mathbf{A}) = -\partial_t (\nabla \times \mathbf{A}) \quad (12)$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{in potentials} \quad \nabla \cdot (\nabla \times \mathbf{A}) = 0 \quad (13)$$

Since equations (12) and (13) in the potentials are identities, only equations (10) and (11) remain for comparison. Maxwell's equation (10) corresponds to the equation of deductive formalism (9). For comparison, we will bring the equation (9) to the form as close as possible to the equation (10) by opening the brackets and using the known vector identity, we will obtain:

$$\nabla \times \nabla \times \mathbf{A} - 2\nabla(\nabla \cdot \mathbf{A}) = \mu_0 \mathbf{J} + \frac{1}{c^2} \partial_t \nabla \varphi - \frac{1}{c^2} \partial_{tt} \mathbf{A} \quad (14)$$

Maxwell's equation (11) corresponds to the scalar equation of deductive formalism (8). Let's transform equation (8) into a form that is as close as possible to equation (11) and obtain:

$$2\frac{1}{c^2} \partial_{tt} \varphi + \nabla(-\nabla \varphi + \partial_t \mathbf{A}) = \rho / \varepsilon_0 \quad (15)$$

Let's note the features of the obtained EMF equations of motion (8) and (9). They are very symmetrical. Each of them has a wave part (the first brackets), indicating the speed of propagation of the electromagnetic disturbance. In Maxwell's equation (11), there is no such wave part, and therefore some critics say that Maxwell's equation (11) indicates an infinite speed of propagation of the electric field. Each of the equations, in addition to the wave part, contains an expression $\partial_t \varphi / c^2 + \nabla \cdot \mathbf{A}$ corresponding to the Lorenz gauge condition. This expression is an invariant of the full EMF tensor (5), as it represents its trace. In the case of the Lorenz gauge $\partial_t \varphi / c^2 + \nabla \cdot \mathbf{A} = 0$, the EMF motion equations (8) and (9) are transformed into the canonical wave equations of Maxwell's theory for the electromagnetic potentials φ and $\mathbf{A}$. Thus, the Lorenz gauge condition takes on a physical meaning of longitudinal deformation of the EMF. The physical meaning of the Lorenz gauge is to eliminate the longitudinal four-dimensional deformation of the EMF. In statics, equation (8), like Maxwell's equation (11), reduces to the Gauss equation for the scalar potential φ .

Let's consider Maxwell's equation (10) in statics and eliminate the current density from it. For this case, it becomes $\nabla \times \nabla \times \mathbf{A} = 0$. This indicates that in the static case, when there is no conduction current or displacement current, the magnetic field rotor is zero. For this case, equation (9) becomes $\nabla \times \nabla \times \mathbf{A} = 2\nabla(\nabla \cdot \mathbf{A})$ or $\nabla \times \nabla \times \mathbf{A} = -2\Delta \mathbf{A}$.

This equation shows that the magnetic field rotor can exist in a static mode even in the absence of conduction and displacement currents. The right-hand side of this equation can be considered as a virtual current. This magnetic phenomenon is observed in astrophysics as the cosmic magnetic dynamo, where there are no displacement or conduction currents but the magnetic field rotor still exists. This magnetic phenomenon has not been fully explained in Maxwell's theory, and there are several hypotheses to explain it. However, in deductive electrodynamics, there is a direct explanation for this phenomenon. Another difference between equation (9) and Maxwell's equation (10) is the sign of the time derivative of the potential electric field $+\partial_t \nabla \varphi / c^2$, which is opposite to the sign of the time derivative of the vortex electric field $-\partial_{tt} \mathbf{A} / c^2$.

In Maxwell's equation (10), the signs of the time derivatives of these electric fields are the same. Let's explore this fact in more detail. It is believed that the electric field derivative, or "displacement current," was introduced by Maxwell in the Ampere

equation to describe the flow of alternating electric current through a capacitor in a series electrical circuit [5]. In this case, the voltage across the capacitor is phase-shifted by $-\pi/2$ relative to the alternating current flowing through it. In addition to the capacitor, a series circuit can also include an inductor, where the voltage is phase-shifted by $+\pi/2$ relative to the current flowing through it. Thus, to describe electrical processes in a series circuit with a capacitor and an inductor, the Ampere equation must contain two terms with time derivatives of the potential electric field (for the capacitor) and the eddy electric field (for the inductor), but with opposite signs. This equation will also describe the phenomenon of series resonance, where the sum of the time derivatives of the potential and eddy electric fields is equal to zero. Equation (9) describes such cases, while Maxwell's equation (10) does not. Thus, equation (9) describes a wider range of electrodynamics issues than Maxwell's equation (10).

One of the well-known problems in electrodynamics is the violation of Newton's third law [5]. This long-standing problem is solved by equation (9) using the term $2\nabla(\nabla \cdot \mathbf{A})$. When an EMF interacts with charges and currents, this term corresponds to an electromagnetic force, which is known in the literature as the Nikolaev force [6-8]. This force has been observed in numerous experiments, but it has not been widely accepted in electrodynamics due to a lack of a convincing theoretical explanation. It is shown here that the Nikolaev force, as well as the Ampere force, follows from the full EMF tensor (5). Table 1 compares the descriptions of the EMF theory of Maxwell and its deductive extension. Table 1 introduces the notation $\mathbf{E}_p = -\nabla\varphi$, $\mathbf{E}_v = -\partial_t\mathbf{A}$.

Table 1. Comparison of EMF descriptions and its basic equations

	Maxwell's theory	Deductive electrodynamics
The EMF tensor	$F_{[\alpha\beta]} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$	$F_{\alpha\beta} = \partial_\alpha A_\beta = F_{(\alpha\beta)} + F_{[\alpha\beta]} = (\partial_\alpha A_\beta + \partial_\beta A_\alpha)/2 + (\partial_\alpha A_\beta - \partial_\beta A_\alpha)/2$
Static	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$	$\nabla \times \mathbf{B} - 2\nabla(\nabla \cdot \mathbf{A}) = \mu_0 \mathbf{J}$
	$\nabla \cdot \mathbf{E}_p = -\Delta\varphi = \rho/\varepsilon_0$	$\nabla \cdot \mathbf{E}_p = -\Delta\varphi = \rho/\varepsilon_0$
Dynamics	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \partial_t(\mathbf{E}_v + \mathbf{E}_p)$	$\nabla \times \mathbf{B} - 2\nabla(\nabla \cdot \mathbf{A}) = \mu_0 \mathbf{J} + \frac{1}{c^2} \partial_t(\mathbf{E}_v - \mathbf{E}_p)$
	$\nabla \cdot \mathbf{E} = \nabla \cdot (\mathbf{E}_p + \mathbf{E}_v) = \rho/\varepsilon_0$	$2\frac{1}{c^2} \partial_t\varphi + \nabla \cdot (\mathbf{E}_p - \mathbf{E}_v) = \rho/\varepsilon_0$

The descriptions that match are marked in green.

4 Equations of EMF propagation in deductive electrodynamics

Since the experimental discovery of electromagnetic waves, the problem of identifying light as an electromagnetic wave has emerged. The coincidence of the propagation speeds of light and electromagnetic waves led to the identification of light as electromagnetic waves with very short wavelengths. However, not all physical properties of light and electromagnetic waves matched. The main feature and distinction of light from the Maxwell-Hertz electromagnetic wave is its rectilinear propagation from the source and the absence of attenuation over vast distances. The discovery of the quantum properties of light intensified the debate regarding its nature. Since this issue

remained unresolved within the framework of Maxwellian electrodynamics, the concept of the dual nature of light, i.e. wave-particle duality-became established. Regarding this issue, A. Sommerfeld wrote: «There is no visible way to overcome the dualism between light wave and light quantum».

If we assume that the starting point of the deductive formalism proposed by Poincaré is correct, then the deductive formalism should lead to two descriptions of the possible propagation of EMF, one of which should describe Maxwell-Hertz electromagnetic waves, and the other should describe the propagation of photons.

Maxwell's equations lead to wave equations for the $\mathbf{E}$ and $\mathbf{B}$ fields and the potentials φ and $\mathbf{A}$. Let's consider the wave equations derived from equations (8) and (9). Take the gradient of both sides of equation (8) and the time derivative of both sides of equation (9):

$$\nabla\left(\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi\right) + \nabla\partial_t\left(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}\right) = \nabla\rho/\varepsilon_0 \quad (16)$$

$$\partial_t\left(\frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A}\right) - \partial_t\nabla\left(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}\right) = \mu_0\partial_t\mathbf{J} \quad (17)$$

These equations have the same dimensionality of their terms. Next, by finding their sum and difference, we obtain two vector equations for the propagation of EMF in the form:

$$\left(\frac{1}{c^2}\partial_{tt} - \Delta\right)(\nabla\varphi + \partial_t\mathbf{A}) = \nabla\rho/\varepsilon_0 + \mu_0\partial_t\mathbf{J} \quad (18)$$

$$\left(\frac{1}{c^2}\partial_{tt} - \Delta\right)(\nabla\varphi - \partial_t\mathbf{A}) + 2\partial_t\nabla(\partial_t\varphi/c^2 + \nabla\cdot\mathbf{A}) = \nabla\rho/\varepsilon_0 - \mu_0\partial_t\mathbf{J} \quad (19)$$

Rewrite equation (18) as follows:

$$\left(\Delta - \frac{1}{c^2}\partial_{tt}\right)(-\partial_t\mathbf{A} - \nabla\varphi) = \Delta\mathbf{E} - \frac{1}{c^2}\partial_{tt}\mathbf{E} = \nabla\rho/\varepsilon_0 + \mu_0\partial_t\mathbf{J} \quad (20)$$

Equation (20) is the canonical wave equation for the electric field, which describes Maxwell-Hertz electromagnetic waves. The theory and solutions of this equation are described in all textbooks on electrodynamics. To obtain the wave equation for the magnetic field, it is sufficient to find the rotor of both parts of equation (20).

The second equation (19), which also follows from the full EMF tensor (5), is much more complex and is not known in classical electrodynamics. However, since it is derived from equations (8) and (9) and by the same method as the canonical wave equation (20), equation (19) for electrodynamics should have an equivalent value. Thus, equations (8) and (9), as expected, lead to two equations for the main types of EMF propagation. This suggests that equation (19) may describe a photon. Then equations (18) and (19) describe the propagation of EMF in the form of classical waves and in the form of photons.

The left-hand side of equation (19) contains a wave term from the difference of the electric potential and the vortex field. This term can describe wave resonance. The second term is also dynamic in nature and describes the rate of change of the gradient of the scalar longitudinal part or trace of the EMF tensor (5), which is its invariant. Equation (19) without field sources can be written as:

$$\left(\frac{1}{c^2}\partial_{tt} - \Delta\right)(\nabla\varphi - \partial_t\mathbf{A}) = -2\partial_t\nabla(\partial_t\varphi/c^2 + \nabla\cdot\mathbf{A}) \quad (21)$$

Its right-hand part can be considered a virtual source for the left-hand wave part. As is known, one of the distinctive properties of light is its ray-like, i.e., longitudinal propagation. The right-hand part of equation (21) in brackets contains the invariant trace of the EMF tensor (5), which describes the longitudinal part of the EMF. This part of equation (21) may be responsible for the longitudinal ray-like propagation of light. Such a longitudinal part is absent in the wave equation (20). If we accept the resonance condition $\nabla\varphi = \partial_t \mathbf{A}$ for the left part, then we can set the right part equal to zero and consider it separately, keeping in mind the resonance condition for the left wave part:

$$0 = -2\partial_t \nabla(\partial_t \varphi / c^2 + \nabla \cdot \mathbf{A}) \quad (22)$$

Given the resonance condition, equation (22) can be reduced to:

$$0 = \partial_t(\partial_{tt} \mathbf{A} / c^2 + \nabla(\nabla \cdot \mathbf{A})) \quad \text{or} \quad 0 = \partial_t(\partial_{tt} \mathbf{A} / c^2 + \nabla \times \nabla \times \mathbf{A} + \Delta \mathbf{A}) \quad \text{or} \\ 0 = \partial_t(-\partial_t \mathbf{E}_V / c^2 + \nabla \times \mathbf{B} + \Delta \mathbf{A}) \quad (23)$$

From equation (23), we can conclude that the expression in brackets is a constant in time and space. Thus, equation (21) describes a complex resonant wave structure of the EM field, which is presumably a photon. Solutions for equation (21) have yet to be found.

Let's take the time derivative of both sides of equation (8) and the divergence of both sides of equation (9):

$$\varepsilon_0(2\frac{1}{c^2}\partial_t\partial_{tt}\varphi + \partial_t\partial_t\nabla \cdot \mathbf{A} - \Delta\partial_t\varphi) = \partial_t\rho \quad (24)$$

$$(\frac{1}{c^2}\partial_{tt}\nabla \cdot \mathbf{A} - \frac{1}{c^2}\partial_t\Delta\varphi - 2\Delta\nabla \cdot \mathbf{A}) / \mu_0 = \nabla \cdot \mathbf{J} \quad (25)$$

The terms of these equations have the same dimension. Add these equations and get the scalar wave equation:

$$2(\frac{1}{c^2}\partial_{tt} - \Delta)(\frac{1}{c^2}\partial_t\varphi + \nabla \cdot \mathbf{A}) / \mu_0 = \partial_t\rho + \nabla \cdot \mathbf{J} \quad (26)$$

This equation describes scalar longitudinal waves of the trace of the EMF tensor (5), i.e. EMF stretching/compressing waves. The right-hand side of this equation is the continuity equation of the current density. In Maxwell electrodynamics, the continuity equation does not have a wave field part. However, the continuity equation is obeyed by any electrodynamic processes involving charges and currents and therefore it must contain a field part.

5 Conclusion

The application and development of the axiomatic deductive formalism of electrodynamics based on four-dimensional pseudo-Euclidean geometry, proposed by A. Poincaré and H. Minkowski, made it possible to expand the description of the EMF. The description of the EMF proposed by H. Minkowski was incorporated into this description in the form of an antisymmetric part. The equations of motion obtained on the basis of a complete description of EMF include Maxwell's equations with additions that expand their capabilities in terms of describing electromagnetic processes (magnetic dynamo, description of resonance in a series circuit, fulfillment of Newton's

third law in electrodynamics), and also allowed solving some problems of J. Maxwell's theory (infinite velocity of propagation of an electric field in the equation). The equations of motion of the electromagnetic field are refined. In addition to the canonical D'Alembert wave equation for the electric and magnetic fields, a new wave equation for the propagation of EMF has been obtained, which is presumably interpreted as the photon equation. A wave equation for scalar longitudinal EMF tension/compression waves has also been derived. Thus, the axiomatic deductive formalism of electrodynamics leads to three forms of EMF propagation: transverse (radio waves), transverse-longitudinal optical waves (photons), and longitudinal waves. In the second part of the deductive formalism of electrodynamics, electromagnetic forces and other issues of deductive electrodynamics will be examined.

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