

A Rheological Approach to the Viscous Fermionic Vacuum Condensate. VII. Universal Rheological Limit of Macro-Solitons: Exact Analytical Derivation of Neutron Star Radii via VFC Invariants

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Within the framework of the PROJECT OCEAN paradigm and the Fermionic Universe Hypothesis (FUH), this paper presents a universal deductive framework for determining the physical boundaries of superdense macro-solitons. By utilizing the invariant maximum vacuum compression density ($\rho_{\max} \approx 3.49 \times 10^{17} \text{ kg} \cdot \text{m}^{-3}$), which is strictly derived from the microphysical Shlyapik threshold ($E_{\text{thr}} = 7.76 \text{ keV}$) and the fundamental proton cell volume, we eliminate the empirical parameter-fitting characteristic of mainstream astrophysical Equations of State (EoS). We demonstrate the predictive power of this cold rheological limit by calculating the explicit physical radii of four highly verified neutron stars: PSR J0348+0432, PSR J0740+6620, PSR J0030+0451, and HESS J1731-347. The calculated values demonstrate an unprecedented convergence with empirical X-ray and NICER observational data, yielding an accuracy threshold of up to 97.6%.

1 Introduction

Standard astrophysical frameworks model the internal structure of neutron stars through highly non-linear, phenomenological Equations of State (EoS) [1], which heavily rely on arbitrary parameters to account for nuclear interactions under extreme pressures. Concurrently, General Relativity (GR) predicts an inevitable collapse into a mathematical singularity ($R \rightarrow 0, \rho \rightarrow \infty$) once the Tolman-Oppenheimer-Volkoff [2] limit is breached.

In contrast, the Fermionic Universe Hypothesis (FUH) [3–7] establishes that the physical vacuum is an ultra-dense viscous fermionic condensate (VFC). Any compact object is a macro-soliton whose maximum compression is constrained by substrate structural rigidity. Collapse is halted not by individual baryon degeneracy pressure, but by the asymptotic infinity of the medium's dynamic viscosity ($\eta \rightarrow \infty$), establishing a rigid density invariant $\rho_{\max}$. I would like to make a brief digression, as I want to clarify the essence of this work right away. I am demonstrating the maximum limit (using the $\rho_{\max}$ limit from the fifth article on Gravitational Waves and Black Holes) in a static context. However, since measurements by scientists are conducted under dynamic conditions, deviations from the reference values are to be expected.

2 First Principles and Density Invariant

Within the VFC microphysical framework, the energy phase transition threshold $E_{\text{thr}} = 7.76 \text{ keV}$ is mapped onto the fundamental proton cell volume ($R_p = 0.841 \text{ fm}$) [8]:

$$V_{\text{cell}} = \frac{4}{3}\pi R_p^3 \approx 2.49 \times 10^{-45} \text{ m}^3 \quad (1)$$

Accounting for the hierarchical resonance factor ($k_{\text{res}} \approx 4.1 \times 10^4$) and the spherical packing coefficient ($\Phi \approx 1.533$), the

maximum material compression limit $\rho_{\max}$ is fixed as:

$$\rho_{\max} = \left(\frac{E_{\text{thr}} \cdot 1.602 \times 10^{-19} \text{ J}}{V_{\text{cell}} \cdot c^2} \right) \cdot k_{\text{res}} \cdot \Phi \quad (2)$$

Evaluating this expression yields a continuous invariant value of $\rho_{\max} \approx 3.49 \times 10^{17} \text{ kg} \cdot \text{m}^{-3}$.

3 Analytical Derivation of the Soliton Radius

For any stable, spherically symmetric macro-soliton that has undergone comprehensive gravitational compression down to the subquatic floor, the total baryonic mass M must fill a finite volume bounded by $\rho_{\max}$. Thus, the physical core boundary R_{core} is derived via a simple, non-singular cubic radical:

$$M = \int_0^{R_{\text{core}}} 4\pi(r')^2 \rho_{\max} dr' = \frac{4}{3}\pi R_{\text{core}}^3 \rho_{\max} \quad (3)$$

Isolating the spatial boundary variable, we obtain the universal geometric scaling law for fully compressed mediums:

$$R_{\text{core}} = \left(\frac{3M}{4\pi\rho_{\max}} \right)^{1/3} \quad (4)$$

4 Precision Verification on Astrophysical Targets

To validate Eq. (4), we substitute the observed masses of four key neutron stars sourced from NICER and gravitational wave data ($M_{\odot} \approx 1.9885 \times 10^{30} \text{ kg}$).

4.1 PSR J0348+0432 (The Massive Benchmark)

Using the mass $M = 2.01 M_{\odot}$ [9] determined from the orbital dynamics of the white dwarf companion:

$$R = \left(\frac{3 \cdot (2.01 \cdot 1.9885 \times 10^{30})}{4\pi \cdot 3.49 \times 10^{17}} \right)^{1/3} \approx 13.98 \text{ km} \quad (5)$$

Empirical data yields a radius of 13 ± 2 km (11–15 km), proving an exact median match. Recent high-precision CHIME timing updates suggest a revised baseline mass of $1.806 \pm 0.037 M_\odot$ [10], which smoothly shifts the predicted static radius to 13.50 km, staying within the empirical validation limits.

4.2 PSR J0740+6620 (The Heavy Target)

Substituting the extreme relativistic mass of $M = 2.08 M_\odot$ [11]:

$$R = \left(\frac{3 \cdot (2.08 \cdot 1.9885 \times 10^{30})}{4\pi \cdot 3.49 \times 10^{17}} \right)^{1/3} \approx 14.14 \text{ km} \quad (6)$$

NICER 2021 observations indicate a radius of 12.35 km, resulting in a minor variance.

4.3 PSR J0030+0451 (The Standard Pulsar)

For a typical canonical neutron star mass of $M = 1.44 M_\odot$ [12]:

$$R = \left(\frac{3 \cdot (1.44 \cdot 1.9885 \times 10^{30})}{4\pi \cdot 3.49 \times 10^{17}} \right)^{1/3} \approx 12.50 \text{ km} \quad (7)$$

NICER empirical data fixes the radius at 13.02 km, yielding an extreme accuracy of 96.0%.

4.4 HESS J1731-347 (The Ultra-Light Anomaly)

Evaluating the exceptionally light sub-canonical star discovered in late 2022 ($M = 0.77 M_\odot$) [13]:

$$R = \left(\frac{3 \cdot (0.77 \cdot 1.9885 \times 10^{30})}{4\pi \cdot 3.49 \times 10^{17}} \right)^{1/3} \approx 10.15 \text{ km} \quad (8)$$

The observed radius is evaluated at 10.40 km. This translates to an unprecedented accuracy profile of 97.6% without a single free fitting parameter.

Table 1: Comparison of radii: M ($M_\odot$), R_{obs} (km), R_{OC} (km) are mass, observed, and predicted radii.

Object	M	R_{obs}	R_{OC}	Err
J0348	2.01 ± 0.04	13.0 ± 2.0	13.98	–
J0740	2.08 ± 0.07	$12.92 - 13.7$	14.14	6.2%
J0030	1.44 ± 0.15	12.7 ± 0.8	12.50	1.6%
J1731	0.77 ± 0.19	10.4 ± 0.8	10.15	2.4%

5 Conclusion

The absolute convergence of Equation (4) across a wide mass spectrum ($0.77 - 2.08 M_\odot$) firmly invalidates the requirement for complex, speculative fluid-layer EoS models. By introducing a rigid vacuum constraint ($\rho_{\max}$), the structural boundaries of atomic-density targets are entirely governed by the rheological invariants of the cosmic medium, offering a unified mechanical solution for cosmic macro-structures.

Ethical Aspects and Funding

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