

The Generalized No-Chaos Theorem

Bounded Complexity Growth at Any Fixed Coarse-Graining

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Abstract

Within the Quantum Gravity Consistency Framework (QGCF) - any finite-dimensional, closed, unitarily evolving system satisfying (QGCF1)-(QGCF4) - three logically distinct results are established. First, ontic chaos is impossible in three senses: spectral quasiperiodicity, Poincaré recurrence, and exact vanishing of the asymptotic quantum Lyapunov exponent λ_L (§5). Second, dynamical (Lyapunov/0-1-test) chaos at any FIXED coarse-graining scale ℓ is also impossible, unconditionally - a direct corollary of the same almost-periodicity that gives $\lambda_L=0$ (§7); the one established quantum-chaos diagnostic that survives this obstruction is level (spectral) statistics, checked preliminarily against a non-integrable/integrable benchmark pair (§7.3). Third, and logically independent of the first two: no observer realized as a proper subsystem can simultaneously reconstruct the complete global state from its own accessible data and evolve under an autonomous unitary law, once genuinely (entangling-ly) coupled to the rest of the system (§8) - a consequence of purification non-uniqueness (Hughston-Jozsa-Wooters) and the non-unitary character of open marginal dynamics, requiring no assumption beyond a generic (non-fine-tuned) boundary coupling. The two failures named in that theorem arise from the same physical mechanism - entanglement across the observer's own boundary - but are shown not to be strictly the same fact, only generically co-occurring. Thermalization of coarse blocks (Calabrese-Cardy growth to a Page-value plateau) is real, established physics, distinct from all of the above, and is presented (§9) together with the timescale $\tau^*(\ell)$ it requires, and its role as the only available justification for treating the system's effective Hamiltonian $\hat{H}_{\text{eff}}$ as possessing a meaningful temperature T_c , given that the axioms admit no literal external bath.

A note on generality. QGCF does not require a literal discrete graph. (QGCF3) is stated as a disjunction: a discrete, graph-based realization (QGCF3a), giving the sharper results of §8.2 and §8.6.2 exactly; or a continuum realization with only a locality/metric structure (QGCF3b), giving the standard, approximately-causal Lieb-Robinson bound instead. Everything else in this document §5-§7, §8.3-§8.6.1, §9 - depends only on finite-dimensionality and unitarity (QGCF1-QGCF2), or on (QGCF4)'s boundedness/genericity, and holds identically under either branch of (QGCF3).

1. Key Distinctions: Three Independent Questions

Three logically independent questions are addressed in this document, and are never conflated:

Question	Answer	Where
Is the exact microdynamics chaotic ($\lambda_L > 0$)?	No - exactly, unconditionally	§5
Does coarse-graining make the dynamics LOOK chaotic (Lyapunov/0-1-test sense)?	No - impossible at any fixed ℓ , structurally	§7.2; surviving diagnostic: level statistics, §7.3
Can an embedded observer fully reconstruct the global state/evolve autonomously?	No -generically, once genuinely coupled	§8 (Embedded-Observer No-Go)
Do coarse blocks look thermal after enough time?	Yes -this is real (it is thermalization, not chaos)	§9

Definition 1.1 (Ontic Chaos). Chaos at the level of the exact microdynamics: $\lambda_L > 0$.

Definition 1.2 (Coarse-grained Spectral Chaos). Since dynamical chaos at the coarse-grained level is proven impossible for any fixed ℓ (§7.2), chaos within the QGCF framework is defined at the level of the underlying Hamiltonian's spectrum: $\hat{H}$ (i.e. $\hat{H}_{\text{eff}}$ of QGCF4) exhibits quantum chaos if its level statistics match the GOE (Wigner-Dyson) prediction rather than Poisson (§7.3). This is a property of $\hat{H}$ itself, not of any observer's resolution ℓ .

2. Axiomatics

(QGCF1) Finiteness. H is finite-dimensional, $\dim H = D < \infty$.

(QGCF2) Closure/unitarity. The system evolves under a single global unitary $\hat{U}$, with nothing external to it.

(QGCF3) Locality -a disjunction, not a single requirement. At least one of the following two branches holds; QGCF does not require both, and does not require a literal discrete graph in general - only some notion of locality. This mirrors §4.1/§4.2 of the companion No-Go paper (Materov, 2026) exactly.

(QGCF3a) Discrete local circuit (sufficient for an exact causal order). The elementary subsystems carry a graph or lattice structure of bounded degree, and $\hat{U}$ is realized as a discrete local circuit: a product $\hat{U} = \hat{U}_K \circ \dots \circ \hat{U}_1$ over $K \leq z+1$ colour classes of pairwise-disjoint edges (Vizing's theorem guarantees such a partition exists for any bounded-degree graph), each $\hat{U}_k$ a product of gates acting only on the endpoints of its colour class.
Used for: §8.2's exact causal order (Theorem 8.2) and the discrete numerical verification of §8.6.2.

(QGCF3b) Continuous local Hamiltonian (sufficient for an approximate causal order). The elementary subsystems are indexed by points of a continuous manifold, or more generally any metric space, with a notion of distance, and $\hat{U}$ is generated by a Hamiltonian $\hat{H} = \sum_i h_i$, each h_i acting on a bounded neighbourhood of point i .
Used for: the standard Lieb-Robinson bound (Lieb & Robinson, 1972) sufficient for every result in §8 that does not depend on the exact, zero-outside-the-cone form of Theorem 8.2 specifically (see Corollary 8.2b).

Discrete, graph-based instances of QGCF -causal sets, causal dynamical triangulations, spin networks, tensor networks, Quantumograph -satisfy (QGCF3a) and hence the sharper results below. Continuum instances -quantum field theory on a fixed or dynamical background, or the algebraic/crossed-product formulation of the companion paper “Toward a Principle of Significance” (§9 there) -satisfy only (QGCF3b), and correspondingly only the weaker, Lieb-Robinson-type causal order of Corollary 8.2b below.

(QGCF4) Bounded local interaction. The system's effective Hamiltonian $\hat{H}_{\text{eff}}$ (however $\hat{U}$ is generated) is a sum of bounded local terms, $\|\hat{H}_{\text{eff}}\| < \infty$, and is generic (non-integrable) in the sense needed for §9's ETH-type argument.
Used for: the MSS bound context in Lemma 4.4(V)/Corollary 6.3, and §9's thermalization/ T_c construction. No specific microscopic coupling-constant formula is assumed -only boundedness and genericity.

3. Definitions of Chaos

Out-of-time-order correlator (OTOC). For Hermitian operators $\hat{W}(t)$, $\hat{V}$ (Heisenberg picture), $F(t) := \langle [\hat{W}(t), \hat{V}]^\dagger [\hat{W}(t), \hat{V}] \rangle$. Exponential growth $F(t) \approx F_0 e^{2\lambda_L t}$ for an interval of t , with $\lambda_L > 0$, is the standard operational signature of quantum chaos (scrambling).

Quantum Lyapunov exponent. λ_L as defined by the above growth rate, when it exists. The Maldacena-Shenker-Stanford (MSS) bound states $\lambda_L \leq \frac{2\pi T_c}{h}$ for any system admitting a meaningful temperature T_c (Maldacena, Shenker & Stanford, 2016).

4. Auxiliary Lemmas

Lemma 4.1 (Almost-periodicity). For finite D (QGCF1) and unitary $\hat{U}$ (QGCF2), the spectrum of $\hat{U}$ is discrete: $\hat{U} = \sum_{k=1}^D e^{i\theta_k} |\varphi_k\rangle\langle\varphi_k|$. Any matrix element $\langle\psi|\hat{U}^n|\chi\rangle$ is therefore a finite sum of D complex exponentials in n -an almost-periodic function of n , in the sense of Bohr.

Lemma 4.2 (Quantum Poincaré recurrence). For any $\varepsilon > 0$ and any initial state $|\Psi(0)\rangle$, there exist infinitely many n such that $|\langle\hat{U}^n|\Psi(0)\rangle - \langle\Psi(0)|\rangle| < \varepsilon$ (standard consequence of almost-periodicity in finite dimension).

Lemma 4.3 (OTOC boundedness). For finite D , $F(t)$ is a finite sum of bounded almost-periodic terms and is therefore itself bounded for all t : $F(t) \leq F_{\text{max}} < \infty$ uniformly in t .

Lemma 4.4 (Pre-saturation growth is bounded by MSS). (I)–(IV) Lemmas 4.1–4.3 jointly rule out unbounded exponential growth of $F(t)$ as $t \rightarrow \infty$: an almost-periodic, uniformly bounded function cannot sustain $F(t) \approx F_0 e^{2\lambda_L t}$ with $\lambda_L > 0$ for all t . (V) On the finite pre-saturation interval where approximately exponential growth can and

typically does occur, its rate is bounded by the MSS inequality: an effective temperature T_c , whose existence for a sufficiently large coarse block is established generically via the eigenstate thermalization hypothesis in §9.3, given (QGCF4) -no specific microscopic coupling formula is required.

5. Impossibility of Ontic Chaos

Theorem 5.1

Let H be a finite-dimensional Hilbert space evolving under (QGCF1)-(QGCF2); for part (V), additionally (QGCF4). Then:

(I) The spectrum of $\hat{U}$ is discrete (Lemma 4.1).

(II) The system exhibits quantum Poincaré recurrence (Lemma 4.2).

(III) The OTOC $F(t)$ is bounded uniformly in t (Lemma 4.3).

(IV) Consequently, $\lambda_L = 0$ exactly, in the asymptotic ($t \rightarrow \infty$) sense: no sustained exponential OTOC growth is possible for all t .

(V) On any finite pre-saturation interval, the *rate* of approximately exponential growth, where such growth occurs, is bounded by the MSS inequality $\lambda_L \leq \frac{2\pi T_c}{h}$.

Status. (I)–(IV) are unconditional given only (QGCF1)–(QGCF2): *no* closed, finite-dimensional, unitarily evolving system can exhibit ontic chaos in the strict, all- t asymptotic sense. This is not a dynamical claim requiring fine-tuning - it follows from finite-dimensional linear algebra alone.

6. Corollaries

Corollary 6.1. Any apparent chaotic growth observed in a finite QGCF system is necessarily transient - confined to a pre-recurrence window - never asymptotic.

Corollary 6.2. The absence of ontic chaos does not by itself imply the absence of complex, unpredictable-looking dynamics on physically relevant timescales; it constrains only the strict $t \rightarrow \infty$ behaviour.

Corollary 6.3 (MSS as the only surviving rate bound). Once asymptotic $\lambda_L > 0$ is excluded by Theorem 5.1(IV), the MSS bound (V) is the sole remaining quantitative constraint on how fast, not whether, scrambling-like growth can proceed on the finite pre-saturation window.

7. No Chaos at Any Fixed Coarse-Graining

7.1 Lemma 7.1 (Coarse-graining as a fixed projection)

Let $\Pi_\ell = P$ be an orthogonal projector onto a fixed coarse-grained subspace (resolution ℓ , fixed once and for all, independent of t or n). The coarse-grained observable at step n is $P\hat{U}^n \rho \hat{U}^{-n} P$ (or the analogous Heisenberg-picture coarse-grained operator). Since $\hat{U}^n$ is almost-periodic (Lemma 4.1) and P is fixed and bounded, the coarse-grained observable is itself almost-periodic in n , for the same reason: a fixed bounded linear map applied to an almost-periodic operator-valued sequence yields an almost-periodic sequence.

7.2 Theorem 7.2 (No chaos at any fixed ℓ)

Let $\mathcal{E}$ be a finite-dimensional Hilbert space evolving unitarily under a time-independent Hamiltonian (or discrete-step unitary), and let ℓ be *any* fixed coarse-graining scale (any fixed projector P , or any fixed finite partition into coarse cells). Then the coarse-grained dynamics restricted to resolution ℓ cannot exhibit sustained exponential (Lyapunov-positive) divergence, and cannot pass a 0-1 chaos test asymptotically, for exactly the same reason as Theorem 5.1: almost-periodicity of the full unitary evolution is inherited by any fixed bounded coarse-graining of it (Lemma 7.1). This holds for every fixed ℓ simultaneously - chaos cannot be “hiding” at some particular coarse-graining scale while being absent at the microscopic level, because the coarse-grained observable's own time series is, itself, almost-periodic.

Status. This result requires nothing beyond (QGCF1)-(QGCF2) and is stated with no reference to any graph, lattice, or locality structure -it holds identically under either branch of (QGCF3), or under no locality assumption at all.

7.3 Level Statistics: the Surviving Diagnostic

Since dynamical (Lyapunov/0-1-test) chaos is excluded at every scale by Theorem 7.2, the only quantum-chaos diagnostic not ruled out by the framework's own finiteness is a *static* one: the level-spacing statistics of $\hat{H}_{\text{eff}}$ (Definition 1.2). Nearest-neighbour spacing statistics following the Gaussian Orthogonal Ensemble (GOE, Wigner–Dyson) rather than Poisson statistics is the standard signature of quantum chaos in this sense (Bohigas, Giannoni & Schmit, 1984), and is a property of the spectrum alone -unaffected by Theorem 5.1/7.2, which concern time-domain growth, not the static spectrum.

Preliminary numerical check: a generic (non-integrable) mixed-field Ising chain benchmark shows GOE-consistent level spacing, while an integrable free-fermion benchmark shows Poisson statistics, as expected -confirming the diagnostic behaves as anticipated on known reference cases before being applied to a specific QGCF instance's own $\hat{H}_{\text{eff}}$.

8. The Embedded-Observer No-Go Theorem, Instantiated for QGCF

This section instantiates Theorem 3.1 of the companion paper (Materov, 2026, “The Embedded-Observer No-Go Theorem”) for a QGCF system, and adds one new, general strengthening (§8.7) not present in that paper.

8.1 Setup

Fix a nonempty, proper subsystem O of the QGCF system, with $H = H_O \otimes H_R$, R the nonempty complement. This bipartition is a direct consequence of O being a definite, proper part of the whole system -no modelling choice is involved (H2 of the companion No-Go paper).

8.2 Causal Order

Lemma 8.1 (exact support growth, under QGCF3a). Let $\hat{A}$ have support X_0 , and let $\hat{A}(n) := \hat{U}^{-n} \hat{A} \hat{U}^n$. Under (QGCF3a), $\text{supp}(\hat{A}(n)) \subseteq \{v : \text{dist}_G(v, X_0) \leq nK\}$, where K is the number of colour classes and dist_G is graph distance. (Proof: each colour-class sublayer extends the support by at most one graph-distance unit, since gates within one colour class act on pairwise disjoint edges and hence commute with any operator not touching their endpoints.)

Theorem 8.2 (exact light cone, under QGCF3a). If $\text{dist}_G(X_0, Y) > nK$, then $[\hat{A}(n), \mathcal{B}] = 0$ *exactly* and not merely approximately for $\mathcal{B}$ supported on Y . This is the sharpest available causal order for a QGCF instance satisfying (QGCF3a).

Corollary 8.2b (approximate causal order, under QGCF3b alone). If only (QGCF3b) holds then a continuum realization with a local Hamiltonian $\hat{H} = \sum_i h_i$ but no discrete graph structure -the exact result of Theorem 8.2 does not apply, and the standard Lieb-Robinson bound holds instead: for operators $\hat{A}, \mathcal{B}$ separated by distance d ,

$$\|[\hat{A}(t), \mathcal{B}]\| \leq C \|\hat{A}\| \|\mathcal{B}\| \exp\left[-\frac{d - v_{LR} t}{\xi}\right],$$

an exponentially small, not exactly zero, commutator for $t < \frac{d}{v_{LR}}$ (Lieb & Robinson, 1972). Every result in §8.3–§8.7 below that requires only *some* causal order -not the exact vanishing of Theorem 8.2 specifically -goes through unchanged under this weaker bound; only the numerical verification of §8.6.2 requires (QGCF3a) specifically.

8.3 Geometrically Induced (Not Chosen) Reduction

The information accessible to an observer confined to O is the expectation value of every operator in $A_O := \mathcal{B}(H_O) \otimes \hat{I}_R$. The unique state on A_O reproducing these is $\rho_O(n) := \text{Tr}_R[|\Psi(n)\rangle\langle\Psi(n)|]$. No choice is involved: this is forced entirely by $H=H_O \otimes H_R$ and nothing else.

8.4 The No-Go Theorem

Call the O–R coupling ACTIVE at step n if some boundary interaction is updated by an entangling (non-product) gate -the generic case for any evolution not specifically fine-tuned to avoid it.

Lemma 8.3 (no complete self-description once entangled; Hughston-Jozsa-Wootters). If $S(\rho_O(n)) > 0$, there exist infinitely many distinct global pure states - related by arbitrary unitaries on H_R alone - all reducing to the same $\rho_O(n)$. Consequently $\rho_O(n)$ does not determine $|\Psi(n)\rangle$ uniquely. *Proof:* for $\rho_O(n) = \sum_k p_k |k\rangle\langle k|$ and any orthonormal $|f_k\rangle \subset H_R$, $|\Phi\rangle := \sum_k \sqrt{p_k} |k\rangle \otimes |f_k\rangle$ reduces to $\rho_O(n)$ exactly; distinct $|f_k\rangle$ give distinct $|\Phi\rangle$ whenever $\text{rank}(\rho_O(n)) \geq 2$.

Lemma 8.4 (no autonomous unitary law once entangled). If the O–R coupling is active at step n , no fixed $\hat{W}$ on H_O alone satisfies $\rho_O(n+1) = \hat{W} \rho_O(n) \hat{W}^\dagger$ for every state of R .

Theorem 8.5 (Embedded-Observer No-Go). Let O be a subsystem with $H = H_O \otimes H_R$ (H_2 , Embeddedness), R the complementary subsystem, coupled by at least one active boundary interaction at some step n . Then the observer realized on O cannot simultaneously: (a) reconstruct $|\Psi(n)\rangle$ from $\rho_O(n)$ alone; (b) evolve under a fixed, autonomous unitary law independent of R ; while (c) remaining a genuine part of the same connected, interacting system.

8.5 Precise Relation Between (a) and (b)

(a) depends only on the *current* mixedness $S(\rho_O(n)) > 0$, accumulated from the entire history of O–R coupling; (b) depends on the coupling being active specifically at the transition $n \rightarrow n+1$. These are logically distinct: a boundary interaction that happens to be an exact product at one particular step could leave (b) satisfied at that transition even while (a) already fails, if $S(\rho_O(n)) > 0$ from earlier activity. (a) and (b) arise from the same physical mechanism - entanglement generated across O 's own boundary - and generically co-occur, but are not one fact restated twice.

Corollary 8.5a (genericity, not universality). The hypothesis fails only if every boundary interaction ever applied is an exact product unitary - measure-zero among all such unitaries, and a condition that would also make O physically decoupled in every meaningful sense. For any generic evolution and any O genuinely part of an interacting system, the theorem's conclusion is unavoidable.

No choice of description available to an observer confined to a proper subsystem of an interacting system can be simultaneously complete and autonomously unitary, once genuine interaction has occurred across its boundary.

8.6 Relation to §5–§7 and §9 (Why This Is Not Chaos, and Not (Only) Thermalization)

Theorem 8.5 is not a chaos statement: it says nothing about growth rates, Lyapunov exponents, or spectral statistics, and requires no time asymptotics at all - it can hold at the very first active step. It is also not merely a restatement of thermalization (§9): §9's Calabrese-Cardy/Page content describes *how much* entanglement accumulates and how fast, over a timescale $\tau^*(\ell)$; §8 establishes that *any* nonzero entanglement - however small, however quickly or slowly accumulated - already suffices to break both complete self-description and autonomous unitarity. §8 is thus the logically weaker-hypothesis, structurally prior fact; §9 is the quantitative refinement of how bad (a) and (b) become, and on what timescale, once the mechanism is active.

8.7 Local Ignorance Theorem: a Quantitative Sharpening of Lemma 8.3

Definition 8.6 (ignorance rank). For subsystem O with complement R and state $|\Psi(n)\rangle$, let $\rho_O(n) = \text{Tr}_R[|\Psi(n)\rangle\langle\Psi(n)|]$. Define $r(O,n) := \text{rank } \rho_O(n)$, the number of nonzero eigenvalues of $\rho_O(n)$.

Lemma 8.6a (parametrization of the equivalence class). The set $\{|\Phi\rangle \in H : \text{Tr}_R|\Phi\rangle\langle\Phi| = \rho_O(n)\}$ is parametrized, up to phase, by $U(r)/U(1)^r$ with $r = r(O,n)$: fixing the Schmidt basis and coefficients of $\rho_O(n)$ on O , every choice of orthonormal $\{|f_k\rangle\}_{k=1}^r$ on the complementary subspace of H_R gives a distinct globally compatible pure state, and every compatible pure state arises this way. $\dim_R = r^2 - r$.

Remark 8.6a (spectral determinism, standard). For any finite-dimensional unitary evolution $\hat{U}$ with known spectral decomposition $\hat{U} = \sum_k e^{i\omega_k} |\varphi_k\rangle\langle\varphi_k|$, exact knowledge of $|\Psi(t_0)\rangle$ and $\{\omega_k\}$ determines $|\Psi(T)\rangle$ for every T , via $|\Psi(T)\rangle = \sum_k \langle\varphi_k|\Psi(t_0)\rangle e^{i\omega_k(T-t_0)} |\varphi_k\rangle$. This is the ordinary spectral theorem, not a new result; it is stated here only to make explicit what Corollary 8.6a's contrast with Theorem 8.6 requires - global reducibility needs exactly the data (exact $|\Psi(t_0)\rangle$) that Theorem 8.6 shows no embedded observer O can supply from its own accessible data alone.

Theorem 8.6 (Local Ignorance Theorem). Let O be a proper subsystem with complement R , and suppose $r(O,n) \geq 2$ (nontrivial boundary entanglement at step n -by Corollary 8.5a the generic case for any active boundary coupling). Then:

(I) No measurement confined to O can distinguish among the $(r(O,n)^2 - r(O,n))$ -real-dimensional family of globally distinct states consistent with $\rho_O(n)$.

(II) The hypothesis of Remark 8.6a (exact knowledge of $|\Psi(t_0)\rangle$) is satisfiable by an observer confined to O if and only if $r(O,n)=1$. I.e., if and only if O falls under the Corollary 8.5a measure-zero case, decoupled from R .

(III) The relevant measure of local ignorance is rank, not entropy: two states ρ_O, ρ_O' with $S(\rho_O)=S(\rho_O')$ but $\text{rank}(\rho_O) \neq \text{rank}(\rho_O')$ admit indistinguishable-state families of different dimension by (I). Entropy weights the Schmidt spectrum; rank measures its support -and it is the support, not the weighting, that fixes the size of the family in (I).

Corollary 8.6a. Global reducibility (Remark 8.6a) and local ignorance (Theorem 8.6) do not conflict. The former's input requirement (exact $|\Psi(t_0)\rangle$) is, by (II), unavailable to any O with $r(O,n) \geq 2$ - which by Corollary 8.5a is the generic case for any O genuinely embedded in an interacting system. Remark 8.6a's $O(2^N)$ -scale reconstruction is available only to a global observer, never to a proper subsystem: global determinism and local unpredictability coexist because the latter's object (O) structurally cannot supply the former's premise.

Remark 8.6b (why this is immediate, and not a restatement of §9). Numerically (§8.7.1), $r(O,n)$ reaches its ceiling $\dim H_O$ within one to two steps of the boundary becoming active, essentially independent of causal-graph diameter (in the QGCF3a numerical instances tested) - consistent with §8.6's claim that the mechanism is structurally immediate rather than a gradual accumulation like §9's entanglement entropy, which on the same systems has not approached its Page value after an order of magnitude more steps. This theorem does not assert, and must not be read as asserting, anything about $F(t)$ or λ_L : it is an epistemic statement about state indistinguishability at fixed n , not a dynamical stability claim, and so does not conflict with Theorem 7.2.

8.7.1 Numerical Verification (QGCF3a instances; see GitHub repository in References)

Exact state-vector simulation ($N \leq 16$ qubits) was run on four discrete, graph-based (QGCF3a) topologies - a ring ($N=16, K=2, \text{diam}=8$), a mixed torus ($8 \times 2, K=3, \text{diam}=5$), a square torus ($4 \times 4, K=4, \text{diam}=4$), and one concrete 4-torus realization ($K=4, \text{diam}=4$), chosen for its high symmetry and convenient automorphism group, taken at its minimal exactly-simulable period $L=2$, identical as a graph to the 4-cube Q_4 . The numerical method (causal-cone cross-check against $n_{\text{sat}}(O)$) applies to any graph satisfying (QGCF3a). Two checks were made on each:

Causal-cone cross-check (Lemma 8.1). The predicted first-activation step $\lceil \text{diam}/K \rceil$ for an end-to-end commutator was confirmed exactly on all four topologies, including a genuine pre-threshold zero regime (to machine precision) on the ring and the 8×2 torus, where the diam/K ratio (4.0 and 1.67 respectively) leaves room to distinguish “before” from “at” threshold.

$n_{\text{sat}}(O)$ (first step at which $\text{rank}(\rho_O(n))$ reaches $\dim H_O$). Found to be 1–2 steps across all four topologies and all tested $|O| \leq 8$, including on the ring - which rules out the concern that instant saturation is merely an artifact of the minimal, degenerate 4-torus instance (there $\text{diam}=K$ exactly). On the ring, causal information about a specific distant perturbation still takes up to $n=4$ steps to arrive, while $n_{\text{sat}}(O)$ for a local block is already 1–2 -the two quantities are governed by different structure (n_{sat} by O 's boundary degree, a local property; the causal cone by graph diameter, a global one), evidence for, not against, Remark 8.6b.

Computability caveat. Exact simulation costs $\dim(H)=2^N$ with $N=L^4$ for period L on the chosen realization; $L=2$ ($N=16$) is the largest exactly tractable instance. $L=3$ already requires a 2^{81} -dimensional Hilbert space, out of reach of exact methods on any presently foreseeable hardware. Code, raw results, and a topology-agnostic graph builder accompany this document as a public repository.

9. Thermalization of Coarse Blocks

Real, established physics, distinct from §7 (chaos, shown impossible at fixed ℓ) and from §8 (the No-Go theorem, which requires no timescale). Presented here for completeness and because it is the mechanism that legitimizes the T_c construction used elsewhere in this work (§9.3).

9.1 Reduced State and Purity Loss

For a block of ℓ contiguous elementary subsystems in a total system of size L , $\rho_\ell(t) := \text{Tr}_{L-\ell}(|\Psi(t)\rangle\langle\Psi(t)|)$, and $\varepsilon(\ell, t) := 1 - \text{Tr}(\rho_\ell(t)^2) = 1 - 2^{-S_2(\ell, t)}$.

9.2 A Two-Regime Growth Law

Calabrese & Cardy (2005): linear growth $S(\ell, t) \approx 2v_E s_{\text{eq}} t$ for $t \lesssim \ell/2v_E$, saturating to $S(\ell, t) \approx s_{\text{eq}} \ell$ for $t \gtrsim \frac{\ell}{2v_E}$, defining $\tau^*(\ell) \approx \frac{\ell}{2v_E}$. For $t \gtrsim \tau^*(\ell)$, $\ell \ll L - \ell$: $\varepsilon(\ell, t) \rightarrow 1 - 2^{-\ell}$ (Page, 1993; Lubkin, 1978) - the generic (typical-state) value.

Numerical check: checked numerically (mixed-field Ising, $N=10$): purity loss flat across ℓ at short t , cleanly monotonic in ℓ once t exceeds a crossover consistent with $\tau^*(\ell) \sim \frac{\ell}{2v_E}$. A small-system check, not a proof for arbitrary N .

Genericity/ETH caveat. The Page-value asymptote assumes generic exploration of the energy shell (D'Alessio, Kafri, Polkovnikov & Rigol, 2016) -an explicitly flagged assumption, not proven from (QGCF1)-(QGCF4) alone.

9.3 Connection to T_c

This is the same mechanism that alone justifies treating $\hat{H}_{\text{eff}}$ (QGCF4) as possessing a meaningful temperature T_c . T_c is defined operationally as the temperature of the thermal ensemble that best fits the reduced state of a sufficiently large coarse block for $t \gtrsim \tau^*(\ell)$ -an ETH-type fit licensed by (QGCF4)'s genericity requirement rather than read off any particular microscopic coupling formula.

(QGCF1)-(QGCF4) describe a strictly closed, unitary system -there is no literal external bath. “ T ” is legitimate only insofar as a large-enough block, evolved for $t \gtrsim \tau^*(\ell)$, looks thermal with respect to the rest of the system via §9.2's ETH-type argument. No separate “noise” object is posited: $\hat{H}_{\text{eff}}$ is the single generic local Hamiltonian already required by (QGCF4), serving both as the generator of the local circuit (QGCF3) and as the object whose thermalization licenses T_c .

10. Comparison Table

Property	Classical Chaos	QGCF (ontic/global)	QGCF (coarse/embedded)
Lyapunov exponent	$\lambda > 0$ forever	$\lambda_L = 0$	$= 0$ for any fixed ℓ (§7.2, structural)
Level statistics	N/A	N/A (single trajectory)	GOE-like for non-integrable $\hat{H}_{\text{eff}}$; Poisson/sub-Poisson for integrable (§7.3)
Complete self-description	N/A	Global $ \Psi\rangle$ fully determined by construction	Impossible once entangled across boundary (§8, Lemma 8.3)
Autonomous evolution	N/A	$\hat{U}$ is exactly autonomous, globally	Impossible once boundary coupling active (§8, Lemma 8.4)
Local ignorance (family size)	N/A	N/A	$r(O, n)^2 - r(O, n)$ real dims of indistinguishable global states; saturates within 1-2 steps of active boundary coupling (§8.7)
Coarse-block thermality	N/A	N/A	Approaches Page value for $t \gtrsim \tau^*(\ell)$ (§9)
Recurrence	No	Yes (Theorem 5.1-II)	Not practically observable

11. Summary

$\lambda_L = 0$ (exact, unconditional, global). Dynamical chaos at any fixed coarse-graining ℓ : impossible, unconditionally (§7.2) - the same almost-periodicity argument as $\lambda_L = 0$. Level statistics (§7.3): the surviving chaos diagnostic. The Embedded-Observer No-Go Theorem (§8): a logically independent, better-grounded, timescale-free result - no

observer confined to a proper subsystem can both fully reconstruct the global state and evolve autonomously, once genuinely coupled to the rest of the system; its two component failures share a mechanism (boundary entanglement) but are not strictly one fact. A quantitative sharpening (§8.7, Local Ignorance Theorem) shows this failure is structurally immediate -saturating within 1-2 steps of active boundary coupling, regardless of graph diameter in the QGCF3a instances tested and cross-validates this numerically against §9's much slower entropy growth on the same systems. Thermalization (§9): real, distinct physics, quantifying how much and how fast a coarse block comes to look thermal, and the same mechanism that legitimizes T_c . Locality (§2, QGCF3) is stated as a disjunction so that none of §5-§7, §8.3-§8.6.1, or §9 depend on a literal discrete graph; only the sharpest form of §8.2's causal order and the numerical checks of §8.7.1 require the discrete branch (QGCF3a).

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