

Pre-string Organizational Channels from Spectral Locality

Emergent finite-graph filaments, core-mediated interaction, and partial exchange-law regularity in Universal Modular Dynamics

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Abstract

We investigate whether string-like organizational sectors can emerge from spectral locality without postulating fundamental strings, a background spacetime, or a prior geometric substrate. Within the framework of Universal Modular Dynamics, normalized spectral states are represented by modular coordinates and assembled into symmetrized nearest-neighbor organizational graphs. We then audit whether such graphs contain recurrent, effectively one-dimensional, dynamically persistent channels that can be interpreted as pre-string organizational filaments.

Across a frozen ensemble of 24 independent seeds and the finite scale window $k = 3, \dots, 6$, the construction yields 15056 baseline pre-string channels. These channels recur across the ensemble, exhibit effective one-dimensionality under intrinsic path-growth and finite-path dimension tests, and form dynamically renewing filament networks under controlled spectral flow. The resulting filaments are not rigid permanent objects: they undergo endpoint persistence, reconnection, decay, recovery, and leaky transport. Nevertheless, they support stable finite-graph mechanical observables, including spectral length, action, tension proxy, stiffness density, and mode-spectrum proxies.

We further find that the filaments are not isolated path-like structures. They are embedded in a higher-dimensional organizational core and interact through shared core neighborhoods and weighted bridge couplings. Reconnection events, when present, are core-mediated and mechanically finite. However, the audit does not establish a full reconnection law or a universal effective interaction law. Refined pre-flow susceptibility, edge-margin, core-response, and mechanical observables provide only a partial law-like signal for exchange and high-exchange behavior, while reconnection-law discrimination remains below threshold.

The resulting picture is therefore neither fundamental string theory nor a derivation of physical string dynamics. Rather, it supports a conservative pre-string conclusion: spectral locality can generate recurrent, effectively one-dimensional, dynamically renewing finite-graph filament sectors with emergent mechanics, embedded core-mediated interaction, and partial exchange-law regularity.

Keywords: Universal Modular Dynamics; spectral locality; pre-string channels; emergent geometry; finite-graph mechanics; filament-core coupling; core-mediated interaction; exchange-law discrimination.

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1 Introduction

1.1 Motivation

A central question in any pre-geometric approach to fundamental structure is how extended, quasi-one-dimensional objects can arise without being postulated as fundamental. In conventional geometric thinking, one begins with a background space, defines paths or curves within it, and then studies the dynamics of extended objects placed on that background. Such a construction is not available in a genuinely pre-geometric setting. If spacetime geometry itself is emergent, then any string-like sector must also be emergent from a more primitive layer of organization.

Universal Modular Dynamics provides a framework in which the primitive data are not points in spacetime, but operationally distinguishable spectral states. The relevant structure is encoded in the modular spectrum of a state and in the relational organization induced by spectral proximity. This motivates a different question: can spectral locality alone generate persistent, effectively one-dimensional organizational sectors before any background geometry or fundamental string degree of freedom is assumed?

This paper investigates that question in a finite, reproducible computational setting. We construct organizational graphs from modular spectral data and audit whether such graphs contain recurrent low-clustering extended channels. These channels are called *pre-string organizational channels*: not because they are physical strings, but because they exhibit a set of structural properties usually associated with string-like sectors, including effective one-dimensionality, persistence under flow, finite mechanical proxies, and interaction through an ambient organizational core.

The guiding idea is therefore conservative. We do not assume that strings are fundamental. We also do not assume a worldsheet, a Nambu–Goto action, a background metric, a scattering law, or a gauge interaction. Instead, we ask whether a weaker but more primitive object appears: an emergent finite-graph filament generated by spectral locality.

1.2 Main question

The main question addressed in this work is:

Can spectral locality generate recurrent, effectively one-dimensional, dynamically renewing, mechanically structured pre-string filament sectors, and if so, do these sectors exhibit embedded interaction through a higher-dimensional organizational core?

This question has several nested parts. First, one must establish that the organizational graph contains detectable extended channels rather than arbitrary graph paths. Second, these channels must recur across an ensemble and across a finite scale window. Third, they must behave as effectively one-dimensional sectors rather than as projections of generic high-dimensional graph structure. Fourth, they must persist in a controlled dynamical setting. Fifth, they must support finite mechanical observables. Finally, if they are to play the role of pre-string sectors, they should not remain isolated: they should couple to an ambient organizational core and interact with one another through that core.

The computational audit developed here follows precisely this sequence. The analysis proceeds through frozen stages GEO-02 through GEO-08. These stages test baseline channel detection, ensemble recurrence, effective one-dimensionality, dynamical persistence, finite-graph mechanics, filament-core coupling, and effective interaction-law discrimination.

The logical chain can be summarized as

operational distinguishability $\longrightarrow$ modular spectrum $\longrightarrow$ spectral locality $\longrightarrow$ organizational graph $\longrightarrow$ p

The purpose of this paper is to determine how far this chain can be pushed without importing ordinary spacetime geometry or fundamental string dynamics.

1.3 Main result

The main result is that spectral locality does generate a robust finite-graph pre-string sector in the tested ensemble. Across a frozen ensemble of 24 independent seeds and the finite scale window $k = 3, \dots, 6$, the construction yields 15056 baseline pre-string channels. These channels recur across the ensemble, exhibit effective one-dimensionality, form dynamically renewing filament networks under controlled spectral flow, and support finite mechanical observables.

The first part of the result is structural. The detected channels are not isolated artifacts of a single graph realization. They recur across seeds and scales, admit motif-level recurrence, and survive a finite scale-window audit. Their intrinsic path-growth and finite-path dimension profiles support the interpretation that they are effective one-dimensional sectors embedded in a larger organizational graph.

The second part of the result is dynamical. Under controlled spectral flow, the channels do not behave as permanently fixed rigid strings. Instead, they form a renewing filament network. Individual representatives may persist by endpoints, reconnect, decay, or recover. Transport along these structures is present but leaky. This is important: the correct object is not a rigid fundamental string, but a dynamical organizational filament.

The third part of the result is mechanical. The filaments support stable finite-graph mechanical observables, including spectral length, spectral action, a tension proxy, stiffness density, and mode-spectrum proxies. These quantities should not be confused with physical string tension or a continuum worldsheet action. They are effective finite-graph observables derived from the spectral geometry of the channel.

The fourth part of the result concerns interaction. The filament sectors are not isolated paths. They are embedded in a higher-dimensional organizational core. They exhibit strong filament-core interface, boundary leakage, shared-core neighborhoods, and core-mediated filament-filament interaction. Reconnection events, when present, are core-mediated and mechanically finite.

The final part of the result is deliberately limited. The audit finds partial law-like regularity in exchange observables, but it does not establish a full reconnection law or a universal effective interaction law. Refined pre-flow susceptibility, edge-margin, core-response, and mechanical observables improve exchange-related discrimination, but reconnection-law discrimination remains below threshold. Thus the final claim is not that a complete interaction law has been derived, but that the embedded filament-core system exhibits partial exchange-law regularity.

Main conclusion

Spectral locality can generate recurrent, effectively one-dimensional, dynamically renewing pre-string filament sectors with finite-graph mechanics and embedded core-mediated interaction. The present construction supports partial exchange-law regularity, but not a full reconnection law or universal physical interaction law.

1.4 Conservative scope of the claim

The terminology “pre-string” is used in a restricted sense. It denotes an emergent organizational filament that appears before the introduction of background spacetime geometry or fundamental string degrees of freedom. It does not denote a fundamental string in the sense of conventional string theory.

Several exclusions are essential to the interpretation of the results. The construction does not derive a Nambu–Goto action, a Polyakov action, a physical string tension, a worldsheet conformal field theory, a scattering amplitude, a gauge interaction, or a conserved physical charge. It also does not prove a continuum limit or a universal analytic interaction theorem. All claims are finite-graph, ensemble-based, and tied to the frozen protocol described in the computational appendix.

The strongest established claim is therefore organizational rather than ontological. The audit supports the emergence of a finite-graph pre-string sector from spectral locality. This sector has recurrent structure, effective one-dimensionality, controlled-flow dynamics, mechanical proxies, and embedded interaction through a surrounding core. These properties are sufficient to justify the term “pre-string organizational channel,” but not sufficient to identify the resulting object with a physical fundamental string.

This distinction is central. If string-like behavior can appear as a derived organizational sector, then string-like structures need not be inserted at the fundamental level. They may instead be stable emergent channels of modular spectral organization. The present paper demonstrates this possibility in a controlled finite setting and identifies the precise limitations that remain before any stronger physical interpretation could be justified.

1.5 Structure of the paper

The paper is organized as follows.

[Section 2](#) introduces the conceptual and mathematical foundations of the construction. It defines spectral states, modular coordinates, spectral locality, organizational neighborhoods, the symmetrized organizational graph, and the notion of a pre-string organizational channel.

[Section 3](#) presents the emergence of organizational geometry. It summarizes the baseline detection of pre-string channels, ensemble recurrence, motif recurrence, the finite scale window, and the effective one-dimensionality audit.

[Section 4](#) studies the behavior of the detected channels under controlled spectral flow. It shows that the channels form a dynamically renewing filament network with endpoint persistence, reconnection, decay, recovery, and leaky transport.

Section 5 develops the finite-graph mechanical interpretation. It introduces spectral length, action, a tension proxy, stiffness density, and mode-spectrum proxies, and summarizes their stability under controlled flow.

Section 6 analyzes filament-core coupling and interaction. It shows that the filaments are embedded in a higher-dimensional organizational core, interact through shared core neighborhoods, and undergo core-mediated reconnection when reconnection occurs.

Section 7 tests whether the observed interaction structure supports an effective interaction law. It distinguishes simple candidate-law scores from refined pre-flow susceptibility and core-response scores. The section establishes partial exchange-law regularity while showing that full reconnection-law discrimination is not achieved.

Section 8 explains why the result should be interpreted as pre-string organizational emergence rather than fundamental string theory. It discusses the distinction between finite-graph mechanics and continuum string dynamics, and between embedded interaction and physical force laws.

Section 9 summarizes the frozen theorem chain, states the conservative final result, and identifies the next steps required for stronger analytic and continuum-limit developments.

The appendices provide the computational protocol, frozen numerical anchors, stage registry, validation logic, and reproducibility structure underlying the audit.

2 Pre-String Foundations

The purpose of this section is to define the pre-geometric objects used throughout the paper. The construction begins not with a spacetime manifold, a metric background, or fundamental extended objects, but with a finite ensemble of operationally distinguishable spectral states. From these states we build modular coordinates, spectral distances, organizational neighborhoods, and a symmetrized organizational graph. Pre-string channels are then defined as extended low-clustering organizational paths in this graph.

The definitions introduced here are deliberately minimal. They are designed to test whether string-like organization can arise from spectral locality alone. No continuum string, worldsheet, background metric, Nambu–Goto action, Polyakov action, gauge field, physical force law, or scattering amplitude is assumed.

The frozen computational protocol used in the subsequent sections is based on

$$N = 64, \quad d = 16, \quad k \in \{3, 4, 5, 6\}, \quad 24 \text{ independent seeds.}$$

Thus the full ensemble contains

$$24 \times 4 = 96$$

seed-scale cells. Across this frozen ensemble, later audits identify

$$15056$$

baseline pre-string channels. These numerical anchors are not assumptions of the definitions below; rather, they are the frozen empirical outcome of applying these definitions and tests to the spectral ensemble.

2.1 Operational distinguishability before geometry

The first premise is that distinguishability is more primitive than geometry. A geometric distance presupposes a space in which objects are already located. In a pre-geometric framework this assumption is too strong. The primitive relation must instead be operational: two states are distinct insofar as they are distinguishable by the information encoded in their spectral structure.

In Universal Modular Dynamics, a state is represented by a positive normalized density object. In the finite setting considered here, each state is represented by a normalized spectral vector

$$\lambda_i = (\lambda_{i1}, \dots, \lambda_{id}), \quad \lambda_{ia} > 0, \quad \sum_{a=1}^d \lambda_{ia} = 1,$$

where $i = 1, \dots, N$ labels the state and $a = 1, \dots, d$ labels the spectral coordinate.

The key point is that no coordinate x^μ , manifold point, metric tensor, or ambient Euclidean embedding is assigned to the state. The only primitive information is the spectral distribution λ_i . Any geometry appearing later must therefore be reconstructed from relations between such spectral states.

Definition 2.1 (Finite spectral state). A finite spectral state is a normalized positive vector

$$\lambda_i \in \mathbb{R}_{>0}^d, \quad \sum_{a=1}^d \lambda_{ia} = 1.$$

In the frozen protocol of this paper, $d = 16$ and each realization contains $N = 64$ such states.

Lemma 2.1 (Distinguishability precedes geometry). *Let $\{\lambda_i\}_{i=1}^N$ be a finite collection of normalized spectral states. Any relational structure built solely from comparisons among the λ_i is definable without reference to a background spacetime geometry.*

Proof. The data entering the construction are the normalized spectra λ_i . Operations such as logarithmic spectral transformation, pairwise comparison, neighborhood selection, and graph construction can be performed directly on these spectra. None of these operations requires a spacetime point, a metric tensor, a differentiable manifold, or a coordinate chart. Therefore the resulting relational structure is pre-geometric in the restricted operational sense used here. $\square$

This lemma does not claim that physical spacetime is absent in nature. It states only that the specific structures audited in this paper can be generated before any spacetime geometry is introduced into the formal construction.

2.2 Spectral states and modular coordinates

Given a spectral state λ_i , its modular coordinates are defined by the logarithmic transformation

$$k_{ia} = -\log \lambda_{ia}.$$

Equivalently, if the spectral state is regarded as the spectrum of a density operator ρ_i , then the corresponding modular operator is formally

$$K_i = -\log \rho_i,$$

and k_{ia} are its eigenvalue coordinates.

The logarithmic form is essential. It converts multiplicative spectral contrasts into additive modular distances. Small spectral probabilities correspond to large modular coordinates, while large spectral probabilities correspond to smaller modular coordinates. Thus modular coordinates encode the distinguishability structure of the spectral state.

Definition 2.2 (Modular spectral coordinates). For a finite spectral state λ_i , the modular spectral coordinate vector is

$$k_i = (k_{i1}, \dots, k_{id}), \quad k_{ia} = -\log \lambda_{ia}.$$

The vector k_i is called the modular coordinate representation of the state.

In the frozen computational protocol, the spectra are generated as normalized positive random vectors:

$$x_i \sim \text{Uniform}(0, 1)^d, \quad \lambda_i = \frac{x_i}{\sum_{a=1}^d x_{ia}}, \quad k_i = -\log \lambda_i.$$

The exact code-level implementation uses

$$d = 16, \quad N = 64, \quad \epsilon = 10^{-15}$$

or an equivalent numerical cutoff to avoid logarithmic singularities. Since all generated entries are positive before normalization, the cutoff is a numerical safeguard rather than a structural modification.

Definition 2.3 (Spectral ensemble). A spectral ensemble is a finite collection

$$\mathcal{S} = \{\sigma_1, \dots, \sigma_N\}$$

where each σ_i is represented by a normalized spectrum λ_i and modular coordinate vector $k_i = -\log \lambda_i$.

Frozen ensemble protocol

The frozen ensemble used in the audits consists of 24 independent seeds. Each seed generates $N = 64$ spectral states of dimension $d = 16$. The primary finite scale window is

$$k \in \{3, 4, 5, 6\},$$

giving 96 seed-scale cells.

2.3 Spectral locality

Once modular coordinate vectors have been constructed, locality can be defined spectrally. This is not locality in a pre-existing spacetime; it is locality in modular spectral organization.

For two states σ_i and σ_j , define the spectral distance

$$d_{\text{spec}}(\sigma_i, \sigma_j) = \|k_i - k_j\|_2 = \left(\sum_{a=1}^d (k_{ia} - k_{ja})^2 \right)^{1/2}.$$

This distance measures proximity in modular coordinate space. States are spectrally local if their modular coordinate vectors are close.

Definition 2.4 (Spectral distance). The spectral distance between two spectral states σ_i and σ_j is

$$d_{\text{spec}}(\sigma_i, \sigma_j) = \left(\sum_{a=1}^d (-\log \lambda_{ia} + \log \lambda_{ja})^2 \right)^{1/2}.$$

Definition 2.5 (Spectral locality). Two states σ_i and σ_j are spectrally local relative to a finite ensemble if $d_{\text{spec}}(\sigma_i, \sigma_j)$ is small compared with the ensemble's pairwise spectral distances.

This definition is relational. It does not assign absolute positions to states. It only orders pairs of states by modular spectral proximity. Locality is therefore induced by distinguishability, not by a background space.

Lemma 2.2 (Geometry requires spectral organization). *Within the present construction, any emergent organizational geometry must be a function of the pairwise spectral relations among modular coordinate vectors.*

Proof. The construction contains no primitive geometric data other than the spectral state ensemble. Thus any graph, path, channel, or metric-like structure that appears must be computed from the modular coordinates k_i and their pairwise relations. Therefore emergent geometry, in this restricted setting, is necessarily a function of spectral organization. $\square$

2.4 Organizational neighborhoods

Spectral locality becomes combinatorial through nearest-neighbor selection. For each state σ_i , we define its organizational neighborhood as the set of k spectrally nearest states.

Definition 2.6 (Organizational neighborhood). Let $\mathcal{S} = \{\sigma_1, \dots, \sigma_N\}$ be a spectral ensemble and let k be a positive integer with $1 \leq k < N$. The organizational neighborhood of σ_i is

$$\mathcal{N}_k(\sigma_i) = \{\sigma_j \neq \sigma_i : \sigma_j \text{ is among the } k \text{ nearest states to } \sigma_i \text{ under } d_{\text{spec}}\}.$$

The parameter k is a finite organizational resolution scale. Small k emphasizes sparse local structure; larger k increases graph connectivity and may absorb extended channels into a dense core. The frozen audit identifies the most stable pre-string scale window as

$$k = 3, 4, 5, 6.$$

This window is important because the later scale audit shows recurrence of the channel signature across this range, while larger k -values tend toward overconnection and loss of filament resolution.

Frozen scale-window anchor

The frozen GEO-03 scale audit identifies the finite signature window

$$k = 3, \dots, 6.$$

Within this window, pre-string signatures recur across the ensemble. At higher k -values, overconnection increasingly absorbs the filamentary structure into the organizational core.

2.5 The symmetrized organizational graph

The neighborhood relation $\sigma_j \in \mathcal{N}_k(\sigma_i)$ is directed: σ_j may be among the nearest neighbors of σ_i even if σ_i is not among the nearest neighbors of σ_j . To obtain an undirected organizational graph, we symmetrize this relation.

Definition 2.7 (Organizational graph). For a spectral ensemble $\mathcal{S} = \{\sigma_1, \dots, \sigma_N\}$ and neighborhood size k , the symmetrized organizational graph is

$$\mathcal{G}_k(\mathcal{S}) = (\mathcal{V}, \mathcal{E}_k),$$

where

$$\mathcal{V} = \mathcal{S},$$

and an undirected edge $\{\sigma_i, \sigma_j\} \in \mathcal{E}_k$ is present if and only if

$$\sigma_j \in \mathcal{N}_k(\sigma_i) \quad \text{or} \quad \sigma_i \in \mathcal{N}_k(\sigma_j).$$

This is the central graph object used throughout the paper. It is determined entirely by the modular spectra and the scale parameter k . It is not embedded in a spacetime manifold.

Lemma 2.3 (Organizational graph is spectrally determined). *For fixed spectral ensemble $\mathcal{S}$ and fixed k , the graph $\mathcal{G}_k(\mathcal{S})$ is fully determined by the pairwise spectral distances $d_{\text{spec}}(\sigma_i, \sigma_j)$.*

Proof. The vertex set is $\mathcal{S}$. For each vertex σ_i , the neighborhood $\mathcal{N}_k(\sigma_i)$ is determined by sorting the pairwise distances $d_{\text{spec}}(\sigma_i, \sigma_j)$ for $j \neq i$. The edge set is then obtained by symmetrizing these nearest-neighbor relations. Therefore no additional data enter the construction. $\square$

Proposition 2.1 (Pre-geometric locality-to-graph map). *The map*

$$\{\lambda_i\}_{i=1}^N \longmapsto \{k_i\}_{i=1}^N \longmapsto \{d_{\text{spec}}(\sigma_i, \sigma_j)\}_{i,j} \longmapsto \mathcal{G}_k(\mathcal{S})$$

defines a pre-geometric organizational graph from spectral data alone.

Proof. The first map is the modular logarithmic transformation. The second computes pairwise spectral distances. The third applies the finite nearest-neighbor and symmetrization rule of [Definition 2.7](#). None of the maps requires spacetime coordinates, a metric background, or fundamental extended objects. $\square$

2.6 Pre-string organizational channels

A pre-string channel is an extended organizational path in $\mathcal{G}_k$. However, not every graph path should be interpreted as pre-string-like. The relevant candidates must be long enough to represent an extended sector, sparse enough not to be merely a dense core artifact, and aligned with collective graph separation structure.

Let

$$P = (v_0, v_1, \dots, v_m)$$

be a simple path in $\mathcal{G}_k$. Its hop length is m . The path is extended if $m \geq m_{\min}$, where in the frozen protocol

$$m_{\min} = 3.$$

The path is low-clustering if its internal vertices have sufficiently low local clustering. If $C(v)$ denotes the local clustering coefficient of vertex v , then a typical criterion is

$$\frac{1}{m-1} \sum_{\ell=1}^{m-1} C(v_\ell) < C_{\text{thr}},$$

with frozen threshold

$$C_{\text{thr}} = 0.25.$$

Definition 2.8 (Pre-string organizational channel). Let $\mathcal{G}_k = (\mathcal{V}, \mathcal{E}_k)$ be a symmetrized organizational graph. A pre-string organizational channel is an extended low-clustering path

$$P = (v_0, v_1, \dots, v_m)$$

such that:

1. P is a simple path in $\mathcal{G}_k$;
2. $m \geq m_{\min}$, with $m_{\min} = 3$ in the frozen protocol;
3. the internal clustering of P is below the frozen low-clustering threshold $C_{\text{thr}} = 0.25$;
4. the path contributes to the ensemble-level recurrent channel family under the frozen detection and recurrence tests.

The last condition is important. A pre-string channel is not defined merely by one isolated path in one graph. The term is reserved for path-like organizational sectors that survive the audit logic: detection, recurrence, scale-window stability, effective one-dimensionality, dynamical behavior, and mechanical testing.

Remark 2.1 (Why “pre-string?”). The prefix “pre” indicates that the object is prior to physical string interpretation. A pre-string channel is not a fundamental string. It has no postulated worldsheet, physical tension, conformal symmetry, or target-space embedding. It is a finite-graph organizational filament generated from spectral locality.

2.7 Frozen detection logic

The foundational definitions become scientifically meaningful only after they pass a frozen detection protocol. The protocol checks whether the organizational graph contains extended low-clustering channels that recur across seeds and scales.

At the baseline level, the audit verifies that a graph built from spectral locality contains pre-string candidates with nontrivial score. In the frozen GEO-02 detection audit, the representative baseline run detects

100

candidate channels with score

0.825694,

passing all

7/7

detection gates. The result is

PRE_STRING_CANDIDATES_DETECTED.

GEO-02 foundational confirmation

The baseline pre-string detection audit confirms that spectral locality generates detectable extended low-clustering organizational channels. The frozen representative detection yields 100 candidates, score 0.825694, and passes 7/7 detection gates.

The ensemble audit then asks whether this is a stable phenomenon rather than a single-seed artifact. Across 24 seeds, GEO-03 finds

23/24

detected cases, 1/24 partial case, and no full non-detection. The detection fraction is therefore

$$\frac{23}{24} = 0.958333,$$

while the detected-or-partial fraction is

1.000000.

All 24 seeds pass the structural gate set.

GEO-03 ensemble confirmation

Across 24 independent seeds, the ensemble audit yields 23 detected cases, 1 partial case, and 0 non-detections. The detection fraction is 0.958333, and the detected-or-partial fraction is 1.000000. This supports the interpretation that the channel phenomenon is recurrent rather than accidental.

The scale-window audit further confirms that the signature is not tied to a single k -value.

The frozen finite signature window is

$$k = 3, 4, 5, 6.$$

Within this window, the channels form the baseline family used in later sections:

$$15056$$

channels across the full 24-seed ensemble.

Foundational numerical anchor

The definitions in this section lead, under the frozen audit protocol, to a stable finite ensemble of

$$15056$$

baseline pre-string channels across 24 seeds and $k = 3, \dots, 6$. This number becomes the baseline population for the dynamical, mechanical, coupling, and interaction-law audits in later sections.

2.8 From organizational channels to filament sectors

A pre-string channel becomes a filament sector when it satisfies further structural and dynamical tests. The present section defines the channel; later sections audit its dimensionality, dynamics, mechanics, and interaction.

The distinction is useful:

channel $\longrightarrow$ effective one-dimensional sector $\longrightarrow$ dynamical filament $\longrightarrow$ core-coupled filament.

A graph path alone is not yet a filament. It becomes filament-like only when it exhibits effective one-dimensionality, persists or renews under flow, and supports finite mechanical observables.

This motivates the following theorem-like statement, which summarizes the foundational level of the construction.

Theorem 2.1 (Spectral-locality channel emergence). *In the frozen $N = 64$, $d = 16$, 24-seed spectral ensemble with scale window $k = 3, \dots, 6$, the map from modular spectra to symmetrized organizational graphs generates a recurrent population of pre-string organizational channels. The frozen baseline population contains 15056 channels across 96 seed-scale cells.*

Proof. By [Definitions 2.1](#), [2.2](#), [2.4](#) and [2.7](#), the graph construction is fully determined by spectral data and k . Applying the frozen detection protocol to the resulting graphs yields baseline detection in GEO-02 and ensemble recurrence in GEO-03. The full finite scale window $k = 3, \dots, 6$ over 24 seeds gives 96 seed-scale cells and a frozen baseline population of 15056 channels. Thus the channel population is generated by spectral locality and recurs across the audited ensemble. $\square$

2.9 What is not assumed

The construction deliberately avoids several assumptions that would make the result less informative. In particular, the following are not assumed:

1. a spacetime manifold;
2. a background metric;
3. a continuum worldsheet;
4. fundamental string degrees of freedom;
5. a Nambu–Goto or Polyakov action;
6. physical string tension;
7. target-space embedding;
8. gauge charges or gauge fields;
9. a physical scattering law;
10. a universal analytic interaction law.

Instead, all structure is built from the sequence

$$\lambda_i \mapsto k_i = -\log \lambda_i \mapsto d_{\text{spec}}(\sigma_i, \sigma_j) \mapsto \mathcal{N}_k(\sigma_i) \mapsto \mathcal{G}_k \mapsto \mathcal{C}.$$

Here $\mathcal{C}$ denotes the resulting pre-string channel population.

Limitation 2.1 (Finite-graph foundation). The foundational construction defines pre-string organizational channels only as finite-graph objects. It does not by itself establish continuum strings, physical string dynamics, or an analytic interaction law.

2.10 Foundational summary

The foundational layer of the paper can be summarized in four steps.

First, normalized spectra define modular coordinates:

$$\lambda_i \mapsto k_i = -\log \lambda_i.$$

Second, modular coordinates define spectral distances:

$$d_{\text{spec}}(\sigma_i, \sigma_j) = \|k_i - k_j\|_2.$$

Third, spectral distances define organizational neighborhoods and symmetrized graphs:

$$d_{\text{spec}} \mapsto \mathcal{N}_k \mapsto \mathcal{G}_k.$$

Fourth, extended low-clustering recurrent paths in these graphs define pre-string organizational channels:

$$\mathcal{G}_k \mapsto \mathcal{C}.$$

The frozen audits confirm that this construction is nontrivial. It yields a recurring population of 15056 baseline pre-string channels across 24 independent seeds and $k = 3, \dots, 6$. These channels provide the objects studied in the remaining sections: their geometry in [Section 3](#), dynamics in [Section 4](#), mechanics in [Section 5](#), core-mediated interaction in [Section 6](#), and interaction-law discrimination in [Section 7](#).

3 Emergent Organizational Geometry

The preceding section defined pre-string organizational channels as extended low-clustering recurrent paths in a symmetrized spectral-neighborhood graph. The present section establishes that these channels are not isolated artifacts of a single graph realization. Rather, they form a reproducible emergent organizational geometry: a connected, distributed, moderately extended, clustered-core graph with low-clustering channels aligned with collective spectral modes.

The term *emergent organizational geometry* is used in a restricted finite-graph sense. It does not refer to a continuum manifold, a differentiable metric, or a background spacetime. It refers to the graph-theoretic geometry induced by spectral locality:

$$\{\lambda_i\} \mapsto \{k_i = -\log \lambda_i\} \mapsto d_{\text{spec}} \mapsto \mathcal{G}_k \mapsto \mathcal{C}.$$

The main claim of this section is that the resulting graph geometry contains recurrent pre-string channels and that these channels behave as effective one-dimensional sectors embedded in a higher-dimensional organizational core.

The frozen audits supporting this claim are:

$$\text{GEO-02} \quad \text{GEO-03} \quad \text{GEO-04}.$$

GEO-02 establishes baseline detection in a representative frozen graph. GEO-03 establishes recurrence across the 24-seed ensemble and across the finite scale window $k = 3, \dots, 6$. GEO-04 establishes effective one-dimensionality.

Section-level claim

Spectral locality induces a finite organizational geometry containing recurrent pre-string channels. Across the frozen 24-seed ensemble and $k = 3, \dots, 6$, the audit identifies 15056 baseline channels. These channels recur across seeds and scales, exhibit motif-level stability, and behave as effective one-dimensional sectors relative to the surrounding organizational core.

3.1 Baseline channel detection

The baseline geometry audit begins with the organizational graph

$$\mathcal{G}_k = (\mathcal{V}, \mathcal{E}_k)$$

constructed from modular spectral locality. In the representative frozen configuration,

$$N = 64, \quad d = 16, \quad k = 6.$$

The resulting graph has

$$|\mathcal{V}| = 64, \quad |\mathcal{E}| = 308,$$

with mean degree

$$\langle d \rangle = 9.625,$$

degree variance

$$\text{Var}(d) = 18.203125,$$

and degree coefficient of variation

$$\text{CV}(d) = 0.443274.$$

The graph is therefore neither a sparse chain nor a complete dense object. It is a moderately heterogeneous organizational network induced by spectral locality.

Table 1: Frozen GEO-02 baseline graph statistics.

Quantity	Frozen value
Vertices	64
Edges	308
Mean degree	9.625
Degree variance	18.203125
Degree coefficient of variation	0.443274
Connected components	1
Largest component fraction	1.000000
Fragmentation	0.000000

The connectedness audit confirms that the graph is a single organizational component:

$$c(\mathcal{G}) = 1, \quad \frac{|\mathcal{V}_{\max}|}{|\mathcal{V}|} = 1.000000, \quad F_{\text{frag}} = 0.$$

This is important. A disconnected graph would indicate fragmented spectral organization, while a trivially complete graph would destroy extended channels. The observed graph lies between these extremes: connected enough to support global organization, but not so dense as to erase channel structure.

Lemma 3.1 (Connected organizational support). *In the frozen GEO-02 baseline graph, all 64 spectral states belong to a single connected organizational component.*

Proof. The connected-component audit gives exactly one connected component, largest component fraction 1.000000, and fragmentation 0. Therefore every vertex belongs to the same connected organizational component. $\square$

The backbone audit further shows that the graph is not held together by fragile cut structures. It has

$$0$$

articulation points and

$$0$$

bridges. The maximum core number is

$$6,$$

and the maximum core includes all 64 vertices:

$$|\mathcal{K}_{\max}| = 64, \quad \frac{|\mathcal{K}_{\max}|}{|\mathcal{V}|} = 1.000000.$$

Thus the graph has a distributed core backbone rather than a tree-like or bottleneck-dominated skeleton.

Table 2: Frozen GEO-02 organizational backbone statistics.

Quantity	Frozen value
Articulation points	0
Bridges	0
Maximum core number	6
Maximum core size	64
Maximum core fraction	1.000000
Backbone mode	distributed core backbone

Proposition 3.1 (Distributed core support). *The frozen baseline organizational graph supports a distributed core backbone rather than a fragile chain-like backbone.*

Proof. A fragile chain-like backbone would typically be characterized by articulation points, bridges, or a small high-core component. The frozen audit finds no articulation points, no bridges, and a maximum core of size 64, equal to the full vertex set. Therefore the graph’s global support is distributed across the full organizational component. $\square$

The path-geometry audit then establishes that the graph has moderately extended geodesic structure. The average shortest-path length is

$$\langle \ell \rangle = 2.124008,$$

with diameter

$$D = 4,$$

radius

$$R = 2,$$

and 5 diameter-realizing vertex pairs. The audit identifies

$$553$$

long-path pairs and mean tortuosity

$$1.836805.$$

Thus the graph is compact but not trivial. It contains enough extension to support long channels, while remaining globally connected through the organizational core.

Table 3: Frozen GEO-02 path geometry.

Quantity	Frozen value
Average shortest path	2.124008
Diameter	4
Radius	2
Diameter pair count	5
Long path pairs	553
Mean tortuosity	1.836805
Path mode	moderately extended geodesic structure

The clustering audit identifies a clustered organizational core with lower-clustering extended channels. The graph contains

$$331$$

triangles. The average local clustering coefficient is

$$\overline{C} = 0.376995,$$

and the transitivity is

$$T = 0.306576.$$

Among the 553 long-path pairs, the audit identifies

$$132$$

low-clustering long paths. The resulting structural mode is:

clustered core with linear channels.

Table 4: Frozen GEO-02 clustering structure.

Quantity	Frozen value
Triangle count	331
Average local clustering	0.376995
Transitivity	0.306576
Long path pairs	553
Low-clustering long paths	132
Clustering mode	clustered core with linear channels

This split is essential. The organizational graph is not globally one-dimensional. It has a clustered core. The pre-string candidates arise as lower-clustering extended sectors within that core-supported graph. This is the first appearance of the basic architecture that later sections develop:

$$\text{clustered core} + \text{extended low-clustering channels.}$$

The spectral graph audit further confirms that these channels are aligned with a collective mode of the graph Laplacian. The Laplacian has zero-eigenvalue multiplicity

$$1,$$

consistent with connectedness. The second Laplacian eigenvalue is

$$\lambda_2 = 2.342034849059,$$

and the Fiedler partition divides the graph into

$$35/29$$

vertices. The associated conductance is

$$\Phi = 0.266187.$$

The fraction of low-clustering channels crossing the Fiedler partition is

$$0.757576.$$

The frozen spectral mode is therefore:

$$\text{collective mode with low-cluster channels.}$$

Table 5: Frozen GEO-02 spectral graph audit.

Quantity	Frozen value
Zero eigenvalue multiplicity	1
Second Laplacian eigenvalue λ_2	2.342034849059
Fiedler partition	35/29
Conductance	0.266187
Low-cluster cross fraction	0.757576
Spectral mode	collective mode with low-cluster channels

Lemma 3.2 (Collective-mode alignment). *In the frozen baseline graph, low-clustering long channels are preferentially aligned with the collective Fiedler separation of the graph.*

Proof. The graph is connected, since the zero Laplacian eigenvalue has multiplicity 1. The Fiedler partition divides the graph into 35/29 vertices. The audit finds that 0.757576 of low-clustering long channels cross this partition. Hence the low-clustering channels are not arbitrary local paths; they align with a collective graph separation mode. $\square$

Finally, the baseline pre-string detection gate reports

100

detected candidates with detection score

0.825694,

passing

7/7

frozen gates. The verdict is

PRE_STRING_CANDIDATES_DETECTED.

GEO-02 frozen result

The frozen baseline organizational geometry audit detects pre-string candidates in the representative graph. The graph has 64 vertices and 308 edges, is connected, has no articulation points or bridges, contains a full distributed 6-core, exhibits clustered-core structure with low-clustering extended channels, and has low-clustering channels aligned with the Fiedler collective mode. The final pre-string detection stage identifies 100 candidates, score 0.825694, and passes 7/7 gates:

PRE_STRING_CANDIDATES_DETECTED.

3.2 Robustness of the baseline geometry

A baseline detection result would be weak if it depended on an unstable perturbation-sensitive graph. GEO-02 therefore includes robustness tests under spectral noise and edge dropout.

The frozen robustness audit confirms stability under spectral noise up to

$$\sigma = 0.100$$

and under edge dropout up to

$$p = 0.100.$$

The scale-selective robustness audit identifies a robust-to-moderate window

$$k = 4, \dots, 7,$$

while $k = 8$ becomes fragile. This is consistent with the later ensemble scale-window result: the pre-string signal is not a single-scale accident, but it is also not invariant under arbitrary overconnection.

Robustness anchor

The baseline pre-string geometry is robust under spectral noise up to $\sigma = 0.100$ and edge dropout up to $p = 0.100$. The scale-selective audit supports a finite robustness window rather than unlimited k -invariance.

Remark 3.1 (Finite scale dependence). The existence of a scale window is not a defect. In a finite organizational graph, too small a neighborhood may fragment the graph, while too large a neighborhood may absorb extended channels into a dense core. The observed finite window is therefore structurally meaningful.

3.3 Ensemble recurrence

The next question is whether the baseline detection persists across independent spectral realizations. GEO-03 addresses this by auditing 24 independent seeds across the finite k -window. The seed set is fixed and frozen:

31700033, 31700034, 31700035, 31700036, 31700037,
 41170021, 41170022, 41170023, 41170024,
 73001991, 73001992, 73001993, 73001994,
 99173003, 99173004, 99173005, 99173006,
 12031007, 15042011, 18052017, 21062023,
 24072029, 27083031, 30093037.

The ensemble audit yields

$$23/24$$

detected seeds,

$$1/24$$

partial seed, and

$$0/24$$

non-detected seeds. Thus the detection fraction is

$$\frac{23}{24} = 0.958333,$$

and the detected-or-partial fraction is

$$1.000000.$$

All 24 seeds pass the five structural gates used in Stage 1 of the ensemble audit.

Table 6: Frozen GEO-03 ensemble detection summary.

Quantity	Frozen value
Seeds	24
Detected seeds	23
Partial seeds	1
Non-detected seeds	0
Detection fraction	0.958333
Detected-or-partial fraction	1.000000
Mean detection score	0.791310
Median detection score	0.804230
Mean candidate count	72.75
Mean low-cluster cross fraction	0.645653
Mean λ_2	2.355221
Verdict	ensemble pre-string signature pass

The score distribution audit confirms that the detection is not narrowly concentrated in a few seeds. The score statistics are

$$\mu_{\text{score}} = 0.791310, \quad \sigma_{\text{score}} = 0.071537,$$

with minimum

$$0.609882$$

at seed 41170023, and maximum

$$0.907466$$

at seed 73001993. The confidence distribution contains

high-confidence detections,

10

moderate-confidence detections, and

1

partial detection.

Table 7: Frozen GEO-03 detection score distribution.

Quantity	Frozen value
Mean score	0.791310
Score standard deviation	0.071537
Minimum score	0.609882
Minimum-score seed	41170023
Maximum score	0.907466
Maximum-score seed	73001993
High-confidence detections	13
Moderate-confidence detections	10
Partial detections	1
Verdict	detection score distribution pass

Proposition 3.2 (Ensemble recurrence of pre-string geometry). *The pre-string organizational geometry is recurrent across the frozen 24-seed ensemble.*

Proof. GEO-03 detects the pre-string signature in 23 of 24 seeds, with the remaining seed classified as partial rather than non-detected. The detected-or-partial fraction is 1.000000, and all 24 seeds pass the structural gate set. Therefore the pre-string geometry is not a single-seed artifact but an ensemble-recurrent structure. $\square$

3.4 Motif recurrence and scale window

A recurrent channel population is stronger if it exhibits a stable finite scale window and recurring motif structure. GEO-03 therefore audits the behavior of the pre-string signature across k -values and motif families.

The scale sweep yields the following frozen pattern:

k	status	detection fraction	mean score
3	SCALE PASS	0.958	0.917
4	PASS	1.000	0.892
5	PASS	1.000	0.846
6	PASS	0.958	0.791
7	FAIL / mixed	—	0.567
8	FAIL / mostly none	—	0.473
9	FAIL	—	0.466

Thus the finite signature window is

$$k = 3, 4, 5, 6.$$

Table 8: Frozen GEO-03 scale-window recurrence.

k	Frozen status	Detection fraction	Mean score
3	SCALE PASS	0.958	0.917
4	PASS	1.000	0.892
5	PASS	1.000	0.846
6	PASS	0.958	0.791
7	FAIL / mixed	–	0.567
8	FAIL / mostly none	–	0.473
9	FAIL	–	0.466

The decreasing score beyond the window has a natural structural interpretation. As k increases, the graph becomes more densely connected. Dense connectivity can absorb extended low-clustering channels into the core, reducing the contrast between filamentary sectors and the surrounding graph. Thus overconnection does not strengthen the pre-string signal; it weakens the ability to resolve it.

Lemma 3.3 (Finite scale-window resolution). *The pre-string signature is resolved in the finite scale window $k = 3, \dots, 6$, while larger k -values increasingly erase the filamentary contrast by overconnection.*

Proof. The frozen scale sweep passes for $k = 3, 4, 5, 6$. The mean score decreases from 0.917 at $k = 3$ to 0.791 at $k = 6$, but remains within the accepted signature window. At $k = 7, 8, 9$, the audit fails with scores 0.567, 0.473, and 0.466. The loss of the signature at larger k is consistent with overconnection absorbing low-clustering channels into a denser organizational core. $\square$

The motif recurrence audit gives a second, more structural confirmation. Across the full channel population, GEO-03 identifies

15056

motif-channel instances,

23

unique family motifs,

1902

detailed motif types, and

9

recurrent family motifs. Motif recurrence covers all

24/24

seeds and

$$88/96 = 0.916667$$

seed-scale cells. Full-window support is present in

$$16/24$$

seeds.

The normalized family motif entropy is

$$0.671245,$$

while the detailed motif entropy is

$$0.885623.$$

This indicates a useful balance: the geometry has recurring family-level structure but remains microscopically diverse. The dominant family motif is

$$\text{hop4|C_near_threshold|cross1|core_full_scale},$$

with seed support

$$1.000000$$

and seed-scale support

$$0.635417.$$

Table 9: Frozen GEO-03 motif recurrence audit.

Quantity	Frozen value
Motif-channel instances	15056
Unique family motifs	23
Detailed motif types	1902
Recurrent family motifs	9
Seed coverage	24/24
Seed-scale coverage	$88/96 = 0.916667$
Full-window seed support	16/24
Family entropy, normalized	0.671245
Detailed entropy	0.885623
Dominant family support	1.000000
Dominant family seed-scale support	0.635417
Verdict	channel motif recurrence pass

Proposition 3.3 (Motif-level recurrence with microscopic diversity). *The frozen pre-string channel population exhibits motif-level recurrence without collapsing to a single microscopic channel type.*

Proof. The motif audit identifies 23 family motifs and 1902 detailed motif types among 15056 channel instances. Nine family motifs are recurrent. Seed coverage is 24/24, and seed-scale coverage is $88/96 = 0.916667$. The family entropy 0.671245 indicates nontrivial family concentration, while detailed entropy 0.885623 indicates microscopic diversity. Therefore the motif structure is recurrent at the family level while remaining diverse at the detailed level. $\square$

GEO-03 closes with a final ensemble verdict:

ENSEMBLE_PRE_STRING_ORGANIZATIONAL_CHANNELS_PASS,

high confidence, with

$8/8$

final gates passed and score

0.863458.

GEO-03 frozen result

The ensemble universality audit confirms that pre-string organizational channels recur across the frozen spectral ensemble. GEO-03 yields detection in 23/24 seeds, partial detection in 1/24, no non-detection, a finite signature window $k = 3, \dots, 6$, motif recurrence across 24/24 seeds, seed-scale motif coverage $88/96 = 0.916667$, and final score 0.863458. The frozen verdict is

ENSEMBLE_PRE_STRING_ORGANIZATIONAL_CHANNELS_PASS.

3.5 Effective one-dimensionality

The existence of recurrent extended channels is not yet enough to justify the term “pre-string.” One must show that the channels behave as effective one-dimensional sectors rather than generic graph paths. GEO-04 performs this audit.

The first dimensional test uses intrinsic ball growth along the channel. The frozen result is decisive:

median intrinsic dimension = 1.0

for all $k = 3, 4, 5, 6$, with seed-scale coverage

$96/96$.

The total channel count is again

15056,

and the one-dimensional fraction is

1.000000.

The median ambient relative dimension is

$$3.0.$$

Thus the channel sector is effectively one-dimensional relative to a higher-dimensional organizational environment.

Table 10: Frozen GEO-04 intrinsic dimensionality audit.

Quantity	Frozen value
Channel count	15056
Seed-scale coverage	96/96
Median intrinsic channel dimension	1.0
One-dimensional channel fraction	1.000000
Median ambient relative dimension	3.0
Verdict	ball-growth intrinsic one-dimensionality pass

The channel-core contrast audit confirms that the channel sector is dimensionally lower than the surrounding organizational core. At the channel level, the contrast pass fraction is

$$0.983860.$$

The seed-scale pass coverage is

$$86/96 = 0.895833,$$

which is just below the strict 0.900000 threshold and therefore freezes as partial at that substage. However, a boundary-window correction identifies $k = 3$ as the lower-boundary limitation and confirms the stable window $k = 4, 5, 6$:

$$71/72 = 0.986111.$$

The boundary-window contrast audit passes

$$8/8$$

gates.

Table 11: Frozen GEO-04 channel-core dimensional contrast.

Quantity	Frozen value
Channel-level contrast pass fraction	0.983860
Full-window seed-scale pass coverage	86/96 = 0.895833
Full-window substage verdict	partial pass
Stable-window contrast coverage	71/72 = 0.986111
Stable-window gate result	8/8 pass
Interpretation	lower-boundary $k = 3$ limitation resolved by stable window

The heat-kernel spectral-dimension audit provides an independent confirmation. The channel heat-kernel dimension agrees with a finite-path reference, lying approximately in the range

$$0.614\text{--}0.657,$$

while the core dimension is higher, approximately

$$1.75\text{--}2.70.$$

Thus the channel is not merely visually path-like. It has a finite-path diffusion profile distinct from the surrounding core.

Heat-kernel dimensional split

GEO-04 confirms that channel heat-kernel dimension matches the finite-path reference range 0.614–0.657, while the organizational core lies in the higher range 1.75–2.70. This supports the interpretation of the channels as effective one-dimensional sectors embedded in a higher-dimensional core.

Dimensional stability across seeds and k -values is then confirmed in GEO-04 Stage 4. All

$$24$$

seeds are stable. The audit passes

$$8/8$$

gates with score

$$0.998264.$$

The final GEO-04 verdict is

ONE_DIMENSIONAL_PRE_STRING_CHANNELS_PASS,

with final score

$$0.994293.$$

GEO-04 frozen result

The effective one-dimensionality audit confirms that the 15056 frozen pre-string channels behave as effective one-dimensional finite-graph sectors. The intrinsic ball-growth median dimension is 1.0, the one-dimensional fraction is 1.000000, the stable-window channel-core contrast coverage is $71/72 = 0.986111$, the dimensional stability score is 0.998264, and the final GEO-04 score is 0.994293. The frozen verdict is

ONE_DIMENSIONAL_PRE_STRING_CHANNELS_PASS.

Theorem 3.1 (Effective one-dimensionality of pre-string channels). *In the frozen $N = 64$, $d = 16$, 24-seed ensemble over $k = 3, \dots, 6$, the recurrent pre-string channel population forms an effective one-dimensional finite-graph sector relative to the higher-dimensional organizational core.*

Proof. The frozen channel population contains 15056 channels across 96 seed-scale cells. The intrinsic ball-growth audit gives median channel dimension 1.0 for all $k = 3, \dots, 6$, with one-dimensional channel fraction 1.000000. The median ambient relative dimension is 3.0. The channel-core contrast audit passes at the channel level with fraction 0.983860, and the stable-window correction passes $71/72 = 0.986111$ seed-scale cells for $k = 4, 5, 6$. The heat-kernel audit further separates the channel dimension range 0.614–0.657 from the higher core range 1.75–2.70. Dimensional stability across all 24 seeds passes 8/8 gates with score 0.998264, and the final GEO-04 score is 0.994293. Therefore the recurrent channels satisfy the frozen criteria for effective one-dimensionality. $\square$

3.6 Emergent geometry theorem

Combining GEO-02, GEO-03, and GEO-04 yields the first major theorem of the article. The theorem does not state that physical spacetime has been derived. It states that spectral locality generates a nontrivial finite organizational geometry with recurrent effective one-dimensional sectors.

Theorem 3.2 (Emergent organizational geometry from spectral locality). *For the frozen ensemble of 24 independent spectral realizations with $N = 64$, $d = 16$, and finite scale window $k = 3, \dots, 6$, spectral locality induces a symmetrized organizational graph family containing a recurrent population of effective one-dimensional pre-string organizational channels. The frozen population contains 15056 baseline channels across 96 seed-scale cells, with ensemble recurrence, motif recurrence, finite scale-window support, and effective one-dimensionality.*

Proof. By the construction of [Section 2](#), spectral states define modular coordinates, spectral distances, organizational neighborhoods, and symmetrized organizational graphs. GEO-02 verifies that the baseline graph is connected, distributed, moderately extended, clustered-core structured, and contains low-clustering channels aligned with the Fiedler collective mode. GEO-02 detects 100 baseline pre-string candidates with score 0.825694 and passes 7/7 gates.

GEO-03 extends this result to the ensemble. It detects the pre-string signature in 23/24 seeds, classifies the remaining seed as partial, gives detected-or-partial fraction 1.000000, identifies the

finite signature window $k = 3, \dots, 6$, and confirms motif recurrence with 15056 motif-channel instances, 23 family motifs, 1902 detailed motifs, 9 recurrent family motifs, seed coverage $24/24$, and seed-scale coverage $88/96 = 0.916667$. The final GEO-03 score is 0.863458 with $8/8$ gates passed.

GEO-04 then establishes effective one-dimensionality. The full channel population contains 15056 channels across 96 seed-scale cells, with median intrinsic dimension 1.0, one-dimensional fraction 1.000000, stable-window contrast coverage $71/72 = 0.986111$, heat-kernel channel dimension range 0.614–0.657, higher core dimension range 1.75–2.70, and final score 0.994293. Therefore the graph family induced by spectral locality contains recurrent effective one-dimensional pre-string channels. $\square$

3.7 What this geometry is, and what it is not

The geometry established here is organizational and finite-graph based. It is real within the audit protocol, but it should not be overinterpreted. The result establishes the following:

1. spectral locality generates nontrivial organizational graphs;
2. these graphs are connected and distributed rather than tree-fragile;
3. they contain clustered cores and low-clustering extended channels;
4. the channels recur across seeds and scales;
5. motif families recur while detailed channel morphology remains diverse;
6. the channel sector is effectively one-dimensional relative to the core.

It does not establish the following:

1. a continuum spacetime manifold;
2. a differentiable metric tensor;
3. physical spatial distance;
4. a worldsheet;
5. a fundamental string;
6. a Nambu–Goto or Polyakov action;
7. physical string tension;
8. a continuum limit.

Limitation 3.1 (Organizational geometry only). The emergent geometry established in this section is a finite organizational graph geometry induced by spectral locality. It is not yet a continuum spacetime geometry and should not be identified with physical string geometry.

3.8 Summary of the emergent geometry layer

The emergent geometry layer can be summarized as follows:

spectral locality $\Rightarrow$ connected organizational graph $\Rightarrow$ clustered core with low-clustering channels
 $\Rightarrow$ ensemble recurrence $\Rightarrow$ motif recurrence $\Rightarrow$ effective one-dimensionality.

The frozen numerical core of this section is:

$$\begin{aligned} |\mathcal{V}| &= 64, \\ |\mathcal{E}| &= 308, \\ \langle d \rangle &= 9.625, \\ \lambda_2 &= 2.342034849059, \\ \text{baseline candidates} &= 100, \\ \text{baseline detection score} &= 0.825694, \\ \text{ensemble detection fraction} &= 0.958333, \\ \text{detected-or-partial fraction} &= 1.000000, \\ \text{baseline channel population} &= 15056, \\ \text{seed-scale cells} &= 96, \\ \text{motif seed-scale coverage} &= 0.916667, \\ \text{one-dimensional channel fraction} &= 1.000000, \\ \text{final GEO-04 score} &= 0.994293. \end{aligned}$$

These results justify treating the objects defined in [Section 2](#) as genuine pre-string organizational channels. The next section studies whether these channels persist, reconnect, decay, recover, and transport under controlled spectral flow.

4 Dynamical Filament Network

The preceding section established that spectral locality generates recurrent effective one-dimensional pre-string channels. The present section asks whether these channels remain meaningful under controlled spectral evolution. A pre-string channel would be geometrically interesting but dynamically weak if it existed only as a static path in one frozen graph. To qualify as a filamentary organizational sector, it must persist, reconnect, decay, recover, or otherwise participate in a controlled dynamical process.

The central result of this section is that the frozen pre-string channels do not behave as rigid permanent strings. Instead, they form a *dynamically renewing filament network*. Individual channel representatives may persist by endpoints, reconnect through nearby organizational structure, decay, or recover. Transport along these structures is present but leaky. Thus the correct object is not a single immutable string-like curve, but an evolving finite-graph filament population.

The frozen dynamical audit is GEO-05. It starts from the same baseline population established in [Section 3](#):

$$|\mathcal{C}_0| = 15056,$$

where $\mathcal{C}_0$ is the frozen baseline pre-string channel set across 24 seeds and $k = 3, \dots, 6$. The controlled spectral flow is sampled at 6 flow steps, giving 5 noninitial transitions for each baseline channel. Hence the frozen transition count is

$$15056 \times 5 = 75280.$$

The audit finds

$$13612$$

active carriers and

$$49217$$

active episodes under flow. These numbers are reproduced again in later GEO-08 law-discrimination audits and therefore serve as frozen dynamical anchors.

Section-level dynamical claim

The frozen pre-string channel population forms a dynamically renewing filament network under controlled spectral flow. Out of 15056 baseline channels, 13612 are active at least once, giving active carrier coverage 0.904091. Across 75280 possible noninitial channel transitions, 49217 active episodes are observed, giving active episode fraction 0.653786. The final GEO-05 verdict is

DYNAMICAL_PRE_STRING_FILAMENT_NETWORK_PASS.

4.1 Controlled spectral flow

The dynamical test begins by deforming the spectral ensemble through a controlled flow. Let

$$\lambda_i(0)$$

denote the baseline normalized spectrum of state σ_i . A controlled spectral flow assigns to each state a one-parameter family

$$\lambda_i(s), \quad s \in [0, 1],$$

with normalization preserved:

$$\sum_{a=1}^d \lambda_{ia}(s) = 1, \quad \lambda_{ia}(s) > 0.$$

The corresponding modular coordinates are

$$k_i(s) = -\log \lambda_i(s),$$

and the spectral distance matrix becomes

$$d_{\text{spec}}^{(s)}(\sigma_i, \sigma_j) = \|k_i(s) - k_j(s)\|_2.$$

For each flow step s , the organizational graph is reconstructed:

$$\mathcal{G}_k(s) = \mathcal{G}_k(\{\lambda_i(s)\}_{i=1}^N).$$

The pre-string channel set at time s is denoted

$$\mathcal{C}_s.$$

Definition 4.1 (Controlled spectral flow). A controlled spectral flow is a finite family

$$\{\lambda_i(s_t)\}_{i=1, \dots, N}^{t=0, \dots, T-1}$$

such that each $\lambda_i(s_t)$ is positive and normalized, and such that organizational graphs $\mathcal{G}_k(s_t)$ and channel sets $\mathcal{C}_{s_t}$ are reconstructed at every sampled flow step.

In the frozen GEO-05 protocol, the flow amplitude is

$$0.12,$$

and the flow is sampled at

$$6$$

steps. The audit verifies that normalization is preserved with trace error bounded at approximately

$$4.44 \times 10^{-16}.$$

The baseline regression is exact:

$$|\mathcal{C}_0| = 15056.$$

The controlled-flow stability audit gives channel presence

$$0.998264,$$

passes

$$8/8$$

gates, and obtains score

$$0.898171.$$

Table 12: Frozen GEO-05 controlled spectral-flow audit.

Quantity	Frozen value
Baseline channel count	15056
Flow amplitude	0.12
Flow steps	6
Noninitial transitions per channel	5
Total transition count	75280
Trace/normalization error	$\approx 4.44 \times 10^{-16}$
Channel presence under flow	0.998264
Gates passed	8/8
Stage score	0.898171
Verdict	controlled spectral flow pass

Lemma 4.1 (Normalization-preserving dynamical reconstruction). *The frozen controlled spectral flow preserves the normalized spectral-state structure to numerical precision and allows reconstruction of organizational graphs at every sampled flow step.*

Proof. The frozen audit reports trace/normalization error approximately 4.44×10^{-16} , which is at machine-precision scale. Therefore the states remain normalized under the sampled flow. Since modular coordinates and spectral distances are computed from these normalized spectra at each step, the organizational graph and channel population can be reconstructed consistently along the flow. $\square$

This establishes that the dynamical audit is not comparing unrelated graphs. It compares organizational geometries reconstructed from a controlled deformation of the same spectral ensemble.

4.2 Endpoint persistence and filament identity

A baseline channel

$$C_0 = (v_0, \dots, v_m) \in \mathcal{C}_0$$

can be tracked under flow in several ways. The strictest notion is endpoint persistence: the channel remains active between the same endpoints at a later flow step. This captures rigid or quasi-rigid identity. However, a pre-string filament need not preserve its endpoints at every step. It may instead reconnect while retaining partial path support or core-mediated continuity.

Definition 4.2 (Endpoint-persistent channel). A baseline channel $C_0 \in \mathcal{C}_0$ is endpoint-persistent at flow step s_t if there exists a channel $C_t \in \mathcal{C}_{s_t}$ with the same endpoint pair as C_0 .

Definition 4.3 (Active carrier). A baseline channel $C_0 \in \mathcal{C}_0$ is an active carrier if it has at least one noninitial active representative under the controlled spectral flow, either by endpoint persistence or by reconnection.

The first persistence audit, GEO-05 Stage 2, gives a deliberately strict result. The endpoint-active fraction is

0.733329,

while the final-step endpoint persistence is

0.342256.

The stable endpoint fraction is

0.398512,

and the median endpoint life fraction is

0.400000.

When a channel persists by endpoints, the path Jaccard score is

1.000000,

meaning endpoint-persistent representatives are path-identical under the strict matching rule. However, the seed-scale pass fraction is only

0.666667,

and the stable coverage is

0.680556.

Thus the strict endpoint-persistence stage closes as

WEAK PARTIAL / FROZEN,

despite a high score of

0.885069.

Table 13: Frozen GEO-05 strict endpoint-persistence audit.

Quantity	Frozen value
Endpoint-active fraction	0.733329
Final-step endpoint persistence	0.342256
Stable endpoint fraction	0.398512
Median endpoint life fraction	0.400000
Path Jaccard when persisted	1.000000
Seed-scale pass fraction	0.666667
Stable coverage	0.680556
Gates passed	6/8
Stage score	0.885069
Verdict	weak partial endpoint persistence

The weak partial status is scientifically informative. It shows that pre-string filaments are not rigid permanent endpoint-defined strings. Strict endpoint identity captures only part of the dynamical phenomenon.

Proposition 4.1 (Non-rigid filament identity). *The frozen dynamical audit rules out a purely rigid endpoint-persistent interpretation of pre-string filaments.*

Proof. If the filament population were purely rigid and endpoint-persistent, endpoint persistence would remain high across the flow and would pass the strict endpoint-stability gates. Instead, GEO-05 Stage 2 gives final-step endpoint persistence 0.342256, stable endpoint fraction 0.398512, seed-scale pass fraction 0.666667, and only 6/8 gates passed. Therefore strict endpoint identity is insufficient to describe the observed filament dynamics. $\square$

4.3 Reconnection, decay, and recovery

The failure of strict endpoint persistence motivates a richer state decomposition. A baseline channel can be:

$$\text{endpoint-persistent,} \quad \text{reconnected,} \quad \text{decayed.}$$

It may also recover after decay, or rewire between different active representatives.

Definition 4.4 (Reconnected channel representative). A baseline channel C_0 is reconnected at flow step s_t if it no longer appears with the same endpoint pair, but admits an active channel representative $C_t \in \mathcal{C}_{s_t}$ with sufficient path overlap or organizational continuity under the frozen matching rule.

Definition 4.5 (Dynamical channel state). At each noninitial flow step, a baseline channel is classified as:

$$\text{endpoint,} \quad \text{reconnected,} \quad \text{decayed.}$$

The active state is the union

$$\text{active} = \text{endpoint} \cup \text{reconnected.}$$

GEO-05 Stage 2B decomposes the endpoint-persistence limitation and shows that the full dynamical identity is much stronger than strict endpoint persistence alone. The endpoint-active fraction remains

0.733329,

but the reconnected carrier fraction is

0.508967,

and the active carrier fraction rises to

0.904091.

The decayed/reconfigured fraction is

0.198658.

The rigid fraction is only

0.097635.

The stable endpoint fraction is

0.298951,

the intermittent endpoint fraction is

0.203507,

and the reconnected filament fraction is

0.362912.

The dynamical mode entropy is high:

0.974802.

The stage passes

8/8

gates with score

0.904847.

Table 14: Frozen GEO-05 persistence-mode decomposition.

Quantity	Frozen value
Endpoint-active fraction	0.733329
Reconnected carrier fraction	0.508967
Active carrier fraction	0.904091
Decayed/reconfigured fraction	0.198658
Rigid fraction	0.097635
Stable endpoint fraction	0.298951
Intermittent endpoint fraction	0.203507
Reconnected filament fraction	0.362912
Dynamical mode entropy	0.974802
Gates passed	8/8
Stage score	0.904847
Verdict	persistence mode decomposition pass

Lemma 4.2 (Renewing identity decomposition). *The frozen filament population has a renewing dynamical identity composed of endpoint persistence, reconnection, decay, and recovery modes.*

Proof. The strict endpoint audit gives only weak partial persistence, but the persistence-mode decomposition passes 8/8 gates. It identifies endpoint-active carriers, reconnected carriers, active carriers, decayed/reconfigured carriers, and a high dynamical mode entropy 0.974802. Since the active carrier fraction is 0.904091, while the rigid fraction is only 0.097635, the dominant identity mode is not rigid persistence but renewal through multiple dynamical modes. $\square$

The event-level transition audit gives a more detailed temporal picture. Across

75280

transitions, the active episode count is

49217.

The state fractions are:

$$f_{\text{endpoint}} = 0.421148, \quad f_{\text{reconnected}} = 0.232638, \quad f_{\text{decayed}} = 0.346214, \quad f_{\text{active}} = 0.653786.$$

The decay onset fraction is

0.144288,

the recovery fraction is

0.087673,

the reconnection fraction is

0.182147,

and the rewire fraction is

0.036318.

The always-active fraction is

0.373605,

the decay-recovery fraction is

0.308448,

and the persistent-decay fraction is

0.095909.

Table 15: Frozen GEO-05 decay, reconnection, and recovery event structure.

Quantity	Frozen value
Transition count	75280
Active carrier count	13612
Active episode count	49217
Endpoint episode fraction	0.421148
Reconnected episode fraction	0.232638
Decayed episode fraction	0.346214
Active episode fraction	0.653786
Decay onset fraction	0.144288
Recovery fraction	0.087673
Reconnection fraction	0.182147
Rewire fraction	0.036318
Always-active fraction	0.373605
Decay-recovery fraction	0.308448
Persistent-decay fraction	0.095909
Gates passed	8/8
Stage score	0.604841
Verdict	decay/reconnection event audit pass

Proposition 4.2 (Dynamical renewal rather than permanent identity). *The frozen event structure supports a dynamically renewing filament network rather than a population of permanently identical filaments.*

Proof. The transition audit contains 75280 possible noninitial channel transitions. Of these, 49217 are active episodes, giving active fraction 0.653786. The episode population splits into endpoint, reconnected, and decayed states, with fractions 0.421148, 0.232638, and 0.346214, respectively. The audit also identifies decay onset, recovery, reconnection, and rewiring events. Therefore the dynamical identity is not permanent pointwise identity; it is renewal through persistence, reconnection, decay, and recovery. $\square$

4.4 Leaky transport along active filaments

A filament network should not only persist or reconnect; it should also support transport-like behavior. In the present finite-graph setting, transport is not a physical particle current. It is an organizational flow proxy along active channel representatives. The relevant question is whether active filaments retain transport mass along the channel or leak into the surrounding core.

Definition 4.6 (Active episode). An active episode is a noninitial flow-step occurrence in which a baseline channel has either an endpoint-persistent or reconnected representative.

Definition 4.7 (Leaky filament transport). Leaky filament transport is present when an active channel representative carries nonzero path-directed transport proxy while also exhibiting non-negligible leakage into the surrounding organizational core.

GEO-05 Stage 3 identifies

13612

active carriers, giving active-carrier coverage

0.904091.

It also identifies

49217

active episodes, giving active-episode fraction

0.653786.

Within active episodes, the endpoint episode fraction is

0.644168,

while the reconnected episode fraction is

0.355832.

The embedded target is

0.003013562337,

the path mass is

0.367741,

and leakage is

0.866972.

The control target is

0.004250955686.

The retention ratio is

0.997467,

and delivery is

0.716894.

The stage passes

7/8

gates with score

0.932256.

It freezes as partial because transport exists but is strongly leaky and not purely channel-confined.

Table 16: Frozen GEO-05 transport along persistent and reconnected channels.

Quantity	Frozen value
Active carriers	13612
Active-carrier coverage	0.904091
Active episodes	49217
Active-episode fraction	0.653786
Endpoint episode fraction among active	0.644168
Reconnected episode fraction among active	0.355832
Embedded target	0.003013562337
Path mass	0.367741
Leakage	0.866972
Control target	0.004250955686
Retention ratio	0.997467
Delivery	0.716894
Gates passed	7/8
Stage score	0.932256
Verdict	partial transport pass

The leakage decomposition resolves the apparent tension. GEO-05 Stage 3B shows that high leakage is not a failure of filament identity; it is a signature of embedded filament-core coupling. The high-leakage fraction is

0.639,

and the moderate-leakage fraction is

0.352.

Delivery is channel-superior in fraction

0.379

and near-channel in fraction

0.104.

Retention is channel-superior in fraction

0.488

and near-channel in fraction

0.203.

The leakage decomposition passes

8/8

gates with score

0.878383.

Table 17: Frozen GEO-05 transport contrast and leakage decomposition.

Quantity	Frozen value
High-leakage fraction	0.639
Moderate-leakage fraction	0.352
Delivery channel-superior fraction	0.379
Delivery near-channel fraction	0.104
Retention channel-superior fraction	0.488
Retention near-channel fraction	0.203
Gates passed	8/8
Stage score	0.878383
Verdict	leakage decomposition pass

Lemma 4.3 (Transport is filamentary but leaky). *The frozen transport audit supports filamentary transport proxies, but not isolated channel-confined transport.*

Proof. GEO-05 Stage 3 identifies 13612 active carriers and 49217 active episodes, with path mass 0.367741, retention ratio 0.997467, and delivery 0.716894. This supports nontrivial transport along active channel representatives. However, leakage is 0.866972, and the leakage decomposition identifies high leakage fraction 0.639. Therefore transport is filamentary in the finite-graph sense, but strongly coupled to and leaky into the surrounding organizational core. $\square$

4.5 Final GEO-05 dynamical verdict

The final GEO-05 synthesis integrates the controlled-flow stability, endpoint-persistence limitation, persistence-mode decomposition, transport audit, leakage decomposition, and event-transition structure. The final verdict is:

DYNAMICAL_PRE_STRING_FILAMENT_NETWORK_PASS.

The confidence class is high, with

8/8

final gates passed and final score

0.896787.

GEO-05 frozen result

The dynamical audit closes as

DYNAMICAL_PRE_STRING_FILAMENT_NETWORK_PASS.

The frozen baseline channel population is 15056. Under controlled spectral flow, 13612 channels are active at least once, giving active-carrier coverage 0.904091. Across 75280 possible noninitial transitions, 49217 active episodes occur, giving active-episode fraction 0.653786. The network includes endpoint persistence, reconnection, decay, recovery, and leaky transport. The final audit passes 8/8 gates with score 0.896787.

4.6 Dynamical theorem

We can now state the dynamical result in theorem form.

Theorem 4.1 (Dynamically renewing pre-string filament network). *In the frozen $N = 64$, $d = 16$, 24-seed ensemble over $k = 3, \dots, 6$, the recurrent pre-string channel population forms a dynamically renewing filament network under controlled spectral flow. The network exhibits endpoint persistence, reconnection, decay, recovery, and leaky transport, with 13612 active carriers and 49217 active episodes across 75280 noninitial transitions.*

Proof. The baseline channel population contains 15056 channels. The controlled spectral-flow audit preserves spectral normalization to numerical precision and reconstructs organizational graphs at 6 flow steps, giving $75280 = 15056 \times 5$ noninitial channel transitions. GEO-05 Stage 1 passes 8/8 gates with channel presence 0.998264, establishing controlled-flow stability.

Strict endpoint persistence alone freezes as weak partial: endpoint-active fraction is 0.733329, final-step endpoint persistence is 0.342256, stable endpoint fraction is 0.398512, and the strict endpoint stage passes only 6/8 gates. However, the persistence-mode decomposition passes 8/8 gates and finds active-carrier fraction 0.904091, reconnected carrier fraction 0.508967, rigid fraction only 0.097635, and dynamical mode entropy 0.974802. This establishes renewing rather than rigid identity.

The transition audit identifies 49217 active episodes, giving active fraction 0.653786. The episode fractions are endpoint 0.421148, reconnected 0.232638, and decayed 0.346214, with decay onset 0.144288, recovery 0.087673, reconnection 0.182147, and rewired 0.036318. The transport audit finds active carriers 13612, active episodes 49217, path mass 0.367741, retention ratio 0.997467, delivery 0.716894, and leakage 0.866972. The leakage decomposition passes 8/8 gates. The final GEO-05 synthesis passes 8/8 final gates with score 0.896787. Therefore the frozen channel population forms a dynamically renewing leaky filament network. $\square$

4.7 Interpretation: filaments, not rigid strings

The dynamical audit clarifies the meaning of the term “filament.” A filament is not a rigidly preserved graph path. It is an organizational sector with a dynamically tracked identity. This identity can be realized by endpoint persistence, reconnection, or recovery after decay. Its

transport is not isolated; it leaks into the organizational core. Its reconfiguration is therefore intrinsic to the structure.

The frozen numbers make this distinction unavoidable. The rigid fraction is only

$$0.097635,$$

whereas the active carrier fraction is

$$0.904091.$$

Thus most dynamically meaningful filaments are not rigid. They are renewing objects embedded in the evolving spectral graph.

This is precisely why the construction is called *pre-string* rather than string-theoretic. A conventional fundamental string is typically treated as a persistent extended object with a worldsheet history. Here there is no worldsheet. The object is a finite-graph organizational channel that survives dynamically through persistence, reconnection, decay, and recovery.

Limitation 4.1 (No rigid string dynamics). GEO-05 does not establish rigid physical string dynamics. It establishes a finite-graph renewing filament network under controlled spectral flow. The relevant object is an organizational filament, not a fundamental string with a worldsheet.

4.8 Summary of the dynamical layer

The dynamical layer can be summarized as

$$\begin{aligned} \text{static pre-string channels} &\longrightarrow \text{controlled spectral flow} \longrightarrow \text{active filament carriers} \\ &\longrightarrow \text{endpoint persistence} + \text{reconnection} + \text{decay} + \text{recovery} + \text{leaky transport}. \end{aligned}$$

The frozen numerical core of this section is:

$$\begin{aligned}
|\mathcal{C}_0| &= 15056, \\
\text{flow steps} &= 6, \\
\text{transition count} &= 75280, \\
\text{active carriers} &= 13612, \\
\text{active carrier fraction} &= 0.904091, \\
\text{active episodes} &= 49217, \\
\text{active episode fraction} &= 0.653786, \\
\text{endpoint episode fraction} &= 0.421148, \\
\text{reconnected episode fraction} &= 0.232638, \\
\text{decayed episode fraction} &= 0.346214, \\
\text{reconnected carrier fraction} &= 0.508967, \\
\text{rigid fraction} &= 0.097635, \\
\text{dynamical mode entropy} &= 0.974802, \\
\text{transport leakage} &= 0.866972, \\
\text{final GEO-05 score} &= 0.896787.
\end{aligned}$$

These results establish the pre-string channel population as a dynamical filament network. The next section asks whether these dynamically renewing filaments also support stable finite-graph mechanical observables.

5 Effective Finite-Graph Filament Mechanics

The preceding sections established that spectral locality generates recurrent effective one-dimensional pre-string channels and that these channels form a dynamically renewing filament network under controlled spectral flow. The present section asks whether these filaments support stable mechanical observables.

The word *mechanics* is used here in a finite-graph and spectral sense. It does not mean that the channels possess physical string tension, a continuum worldsheet, a Nambu–Goto action, a Polyakov action, quantized oscillator modes, or a target-space embedding. Instead, the question is whether each filamentary path admits reproducible finite-graph observables analogous to length, action, tension, stiffness, and mode spectrum, and whether these observables remain stable under controlled spectral flow.

The frozen mechanical audit is GEO-06. It uses the same baseline population:

$$|\mathcal{C}_0| = 15056,$$

and the same dynamical active population:

$$13612$$

active carriers and

49217

active episodes. The final GEO-06 verdict is

EFFECTIVE_FINITE_GRAPH_FILAMENT_MECHANICS_PASS,

with high confidence, 8/8 final gates passed, and final score

0.867671.

Section-level mechanical claim

Pre-string filaments support stable finite-graph mechanical observables. Across 15056 baseline channels, the finite-tension fraction is 1.000000. The median hop length is 3, the median spectral length is 12.069761, the median spectral action is 44.124460, and the median tension proxy is 3.629348. Under controlled spectral flow, 13612 active carriers and 49217 active episodes retain finite dynamic mechanics with finite dynamic mechanics fraction 1.000000.

5.1 Spectral length and action

Let

$$P = (v_0, v_1, \dots, v_m)$$

be a pre-string filament path in the organizational graph. Each vertex v_ℓ corresponds to a spectral state with modular coordinate vector $k(v_\ell)$. The spectral edge length between consecutive vertices is defined by

$$\ell_\alpha(P) = d_{\text{spec}}(v_{\alpha-1}, v_\alpha) = \|k(v_\alpha) - k(v_{\alpha-1})\|_2, \quad \alpha = 1, \dots, m.$$

Definition 5.1 (Finite-graph spectral length). The spectral length of a filament path

$$P = (v_0, \dots, v_m)$$

is

$$L(P) = \sum_{\alpha=1}^m \ell_\alpha(P),$$

where

$$\ell_\alpha(P) = d_{\text{spec}}(v_{\alpha-1}, v_\alpha).$$

The corresponding finite-graph spectral action is defined as the quadratic edge-length sum:

$$A(P) = \sum_{\alpha=1}^m \ell_\alpha(P)^2.$$

Definition 5.2 (Finite-graph spectral action). The spectral action of a filament path P is

$$A(P) = \sum_{\alpha=1}^m d_{\text{spec}}(v_{\alpha-1}, v_{\alpha})^2.$$

This action is not a Nambu–Goto or Polyakov action. It is a finite-graph quadratic spectral cost assigned to a path. Its purpose is to measure how much modular spectral displacement is accumulated along the filament.

The frozen GEO-06 action/tension audit evaluates these observables on the full baseline population of

15056

channels. The finite-tension fraction is

1.000000.

The median hop length is

3,

the median spectral length is

12.069761,

and the median spectral action is

44.124460.

Table 18: Frozen GEO-06 spectral length and action audit.

Quantity	Frozen value
Baseline channel count	15056
Finite-tension fraction	1.000000
Median hop length	3
Median spectral length	12.069761
Median spectral action	44.124460
Action/tension stage gates	8/8
Action/tension stage score	0.945812
Verdict	action/tension proxy pass

Lemma 5.1 (Finite spectral length and action). *Every frozen baseline pre-string filament in the GEO-06 audit has finite spectral length and finite spectral action.*

Proof. Each filament is a finite path in a finite organizational graph. Each vertex corresponds to a finite modular coordinate vector $k_i = -\log \lambda_i$, where λ_i is positive and normalized. Hence every spectral edge length $\ell_{\alpha}(P)$ is finite. Since each path has finite hop length, both

$$L(P) = \sum_{\alpha} \ell_{\alpha}(P)$$

and

$$A(P) = \sum_{\alpha} \ell_{\alpha}(P)^2$$

are finite. The frozen audit confirms this numerically across the full baseline population of 15056 channels. $\square$

5.2 Tension proxy

A path with finite length and finite quadratic action admits a natural finite-graph tension proxy:

$$\tau(P) = \frac{A(P)}{L(P)}.$$

This quantity measures average quadratic spectral cost per unit spectral length. It is analogous to a tension-like stiffness-per-length measure, but it is not physical string tension.

Definition 5.3 (Tension proxy). For a filament path P with spectral length $L(P) > 0$ and spectral action $A(P)$, the finite-graph tension proxy is

$$\tau(P) = \frac{A(P)}{L(P)}.$$

The frozen median tension proxy is

$$3.629348.$$

The coefficient of variation of the tension proxy is

$$0.189067.$$

The action/tension stability score is

$$0.840995,$$

and the control neutrality check passes. The full action/tension stage passes

$$8/8$$

gates with score

$$0.945812.$$

Table 19: Frozen GEO-06 tension-proxy audit.

Quantity	Frozen value
Median tension proxy	3.629348
Tension coefficient of variation	0.189067
Action/tension stability	0.840995
Control neutrality	pass
Gates passed	8/8
Stage score	0.945812
Verdict	effective filament action/tension proxy pass

Proposition 5.1 (Non-degenerate finite-graph tension proxy). *The frozen pre-string filament population supports a non-degenerate finite-graph tension proxy.*

Proof. The tension proxy is finite for every baseline filament, since the finite-tension fraction is 1.000000. The median value is 3.629348, and the coefficient of variation is 0.189067, which shows that the observable is neither identically zero nor wildly uncontrolled across the population. The action/tension stage passes 8/8 gates with score 0.945812. Therefore the tension proxy is a stable finite-graph mechanical observable in the frozen ensemble. $\square$

Remark 5.1 (No physical string tension). The tension proxy $\tau(P)$ is a finite-graph spectral observable. It should not be identified with physical string tension. No continuum worldsheet area, target-space metric, or physical string action enters its definition.

5.3 Stiffness density and mode spectrum

The next mechanical layer assigns an effective stiffness operator to a filament path. Given edge lengths $\ell_\alpha(P)$ and tension proxy $\tau(P)$, one may define local edge stiffnesses of the form

$$\kappa_\alpha(P) = \frac{\tau(P)}{\ell_\alpha(P)}.$$

These edge stiffnesses define a weighted path Laplacian, or stiffness matrix, on the vertices of the path.

Definition 5.4 (Finite-graph stiffness matrix). Let $P = (v_0, \dots, v_m)$ be a filament path with edge stiffnesses

$$\kappa_\alpha(P) = \frac{\tau(P)}{\ell_\alpha(P)}.$$

The finite-graph stiffness matrix $M_P \in \mathbb{R}^{(m+1) \times (m+1)}$ is the weighted path Laplacian defined by

$$(M_P)_{\alpha-1, \alpha-1} \leftarrow (M_P)_{\alpha-1, \alpha-1} + \kappa_\alpha,$$

$$(M_P)_{\alpha, \alpha} \leftarrow (M_P)_{\alpha, \alpha} + \kappa_\alpha,$$

$$(M_P)_{\alpha-1, \alpha} = (M_P)_{\alpha, \alpha-1} = -\kappa_\alpha,$$

for $\alpha = 1, \dots, m$.

The eigenvalues of M_P define an effective finite-path mode spectrum:

$$0 = \mu_0(P) \leq \mu_1(P) \leq \dots \leq \mu_m(P).$$

The first nonzero eigenvalue

$$\mu_1(P)$$

is the mode gap. A scaled mode gap can be formed by compensating for path length and tension:

$$\hat{\mu}_1(P) = \frac{\mu_1(P)m^2}{\tau(P)}.$$

A stiffness density is defined by the positive spectral trace per hop:

$$s(P) = \frac{1}{m} \sum_{\mu_j(P) > 0} \mu_j(P).$$

Definition 5.5 (Finite-graph mode spectrum). The finite-graph mode spectrum of a filament P is the positive eigenvalue spectrum of its stiffness matrix M_P :

$$\text{Spec}_+(M_P) = \{\mu_j(P) > 0\}.$$

The first positive eigenvalue is the mode gap, and the positive trace per hop defines the stiffness density.

GEO-06 Stage 2 evaluates this spectrum on all 15056 baseline filaments. The finite spectrum fraction is

$$1.000000.$$

The median mode gap is

$$0.584116,$$

the median scaled mode gap is

$$1.647594,$$

and the median stiffness density is

$$2.125080.$$

The median mode entropy is

$$0.823461.$$

The mode-gap stability score is

$$0.845116,$$

and the stiffness stability score is

$$0.913176.$$

The stage passes

$$8/8$$

gates with score

0.954797.

Table 20: Frozen GEO-06 mode-spectrum and stiffness audit.

Quantity	Frozen value
Baseline channel count	15056
Finite spectrum fraction	1.000000
Median mode gap	0.584116
Median scaled mode gap	1.647594
Median stiffness density	2.125080
Median mode entropy	0.823461
Mode-gap stability	0.845116
Stiffness stability	0.913176
Control neutrality	pass
Gates passed	8/8
Stage score	0.954797
Verdict	filament mode spectrum/stiffness pass

Lemma 5.2 (Finite mode spectrum). *Every frozen baseline filament in the GEO-06 audit admits a finite positive mode spectrum.*

Proof. Each filament path is finite and has finite positive spectral edge lengths. The stiffness matrix M_P is therefore a finite symmetric weighted path Laplacian. Its eigenvalue spectrum is finite and real. The frozen audit confirms that the finite spectrum fraction is 1.000000 across all 15056 baseline channels. Hence every audited filament admits a finite positive mode spectrum. $\square$

Proposition 5.2 (Stable stiffness and mode observables). *The frozen pre-string filament population supports stable finite-graph stiffness and mode-spectrum observables.*

Proof. The GEO-06 mode-spectrum audit gives finite spectrum fraction 1.000000, median mode gap 0.584116, median scaled mode gap 1.647594, median stiffness density 2.125080, and median mode entropy 0.823461. It also reports mode-gap stability 0.845116 and stiffness stability 0.913176, with 8/8 gates passed and score 0.954797. Therefore stiffness and mode-spectrum observables are finite and stable under the frozen audit. $\square$

5.4 Mechanical stability under controlled spectral flow

A mechanical observable is meaningful only if it remains stable under the same controlled spectral flow used in Section 4. GEO-06 Stage 3 therefore evaluates dynamic mechanical ratios along active filament representatives.

Let P_0 be a baseline filament and P_t an active representative at flow step t . For an observable

X , define the multiplicative dynamic ratio

$$R_X(t) = \frac{X(P_t)}{X(P_0)}$$

or the corresponding stepwise representative ratio under the frozen tracking rule. A ratio near 1 indicates mechanical stability.

The frozen dynamic audit uses

$$13612$$

active carriers and

$$49217$$

active episodes. The finite dynamic mechanics fraction is

$$1.000000.$$

The median dynamic ratios are:

$$R_L = 1.008466,$$

$$R_A = 1.015823,$$

$$R_\tau = 1.006086,$$

$$R_{\hat{\mu}} = 0.996652,$$

$$R_s = 0.999797.$$

The combined mechanics stability score is

$$0.872177.$$

The stage passes

$$8/8$$

gates with score

$$0.865993.$$

Table 21: Frozen GEO-06 mechanical stability under controlled spectral flow.

Quantity	Frozen value
Active carriers	13612
Active episodes	49217
Finite dynamic mechanics fraction	1.000000
Median length ratio	1.008466
Median action ratio	1.015823
Median tension ratio	1.006086
Median scaled-gap ratio	0.996652
Median stiffness ratio	0.999797
Combined mechanics stability	0.872177
Gates passed	8/8
Stage score	0.865993
Verdict	mechanical stability under flow pass

Lemma 5.3 (Near-unit dynamic mechanical ratios). *Under controlled spectral flow, the frozen active filament representatives preserve finite-graph mechanical observables with median ratios close to unity.*

Proof. The dynamic mechanical audit reports median ratios 1.008466 for length, 1.015823 for action, 1.006086 for tension proxy, 0.996652 for scaled mode gap, and 0.999797 for stiffness density. Each is close to 1. The finite dynamic mechanics fraction is 1.000000, and the stage passes 8/8 gates. Therefore the active filament representatives preserve finite-graph mechanical observables under controlled flow. $\square$

This result is especially important because the previous section showed that the filament network is dynamically renewing rather than rigid. The mechanical stability result therefore does not mean that every filament preserves the same endpoint path. It means that the active filament representatives, including endpoint-persistent and reconnected representatives, retain stable finite-graph mechanical structure.

Proposition 5.3 (Mechanical stability without rigid identity). *The frozen filament network preserves mechanical observables under flow even though strict endpoint identity is not generally preserved.*

Proof. GEO-05 shows that strict endpoint persistence is only weak partial: the rigid fraction is 0.097635, while the active carrier fraction is 0.904091. GEO-06 shows that across the same active dynamical population, finite dynamic mechanics fraction is 1.000000, median mechanical ratios remain close to unity, and the mechanical stability under flow audit passes 8/8 gates. Therefore mechanical stability is a property of the renewing filament representative network, not merely of rigid endpoint-preserved paths. $\square$

5.5 Final GEO-06 mechanical verdict

The final GEO-06 synthesis integrates the action/tension audit, the mode-spectrum/stiffness audit, and the mechanical-stability-under-flow audit. It closes with high confidence:

EFFECTIVE_FINITE_GRAPH_FILAMENT_MECHANICS_PASS.

The final stage passes

8/8

gates with score

0.867671.

GEO-06 frozen result

The effective filament mechanics audit closes as

EFFECTIVE_FINITE_GRAPH_FILAMENT_MECHANICS_PASS.

Across 15056 baseline channels, the finite-tension fraction is 1.000000, median spectral length is 12.069761, median spectral action is 44.124460, median tension proxy is 3.629348, finite spectrum fraction is 1.000000, median mode gap is 0.584116, median scaled mode gap is 1.647594, and median stiffness density is 2.125080. Under controlled spectral flow, 13612 active carriers and 49217 active episodes retain finite dynamic mechanics with finite dynamic mechanics fraction 1.000000. The final GEO-06 audit passes 8/8 gates with score 0.867671.

5.6 Mechanical theorem

The mechanical layer can now be stated as a theorem.

Theorem 5.1 (Effective finite-graph filament mechanics). *In the frozen $N = 64$, $d = 16$, 24-seed ensemble over $k = 3, \dots, 6$, the recurrent dynamically renewing pre-string filament population supports stable finite-graph mechanical observables. These include spectral length, spectral action, a tension proxy, stiffness density, and finite mode-spectrum proxies. The result holds for 15056 baseline channels and for the active dynamical population of 13612 carriers and 49217 active episodes under controlled spectral flow.*

Proof. For each finite filament path P , spectral length $L(P)$, spectral action $A(P)$, and tension proxy $\tau(P) = A(P)/L(P)$ are well-defined from finite modular spectral edge lengths. GEO-06 Stage 1 evaluates these observables across 15056 baseline channels, finds finite-tension fraction 1.000000, median length 12.069761, median action 44.124460, median tension proxy 3.629348, tension coefficient of variation 0.189067, and passes 8/8 gates with score 0.945812.

The stiffness matrix M_P is a finite symmetric weighted path Laplacian and therefore has a finite real spectrum. GEO-06 Stage 2 finds finite spectrum fraction 1.000000, median mode gap 0.584116, median scaled mode gap 1.647594, median stiffness density 2.125080, median mode

entropy 0.823461, mode-gap stability 0.845116, stiffness stability 0.913176, and passes 8/8 gates with score 0.954797.

Under controlled spectral flow, GEO-06 Stage 3 evaluates 13612 active carriers and 49217 active episodes. It finds finite dynamic mechanics fraction 1.000000, median length ratio 1.008466, median action ratio 1.015823, median tension ratio 1.006086, median scaled-gap ratio 0.996652, median stiffness ratio 0.999797, combined mechanics stability 0.872177, and passes 8/8 gates with score 0.865993. The final GEO-06 synthesis passes 8/8 gates with score 0.867671. Therefore the frozen pre-string filament population supports stable effective finite-graph mechanics. $\square$

5.7 Interpretation: finite-graph mechanics rather than string dynamics

The mechanical result is strong but must be interpreted conservatively. It establishes a finite-graph mechanical layer for pre-string filaments. It does not establish physical string mechanics.

The following claims are supported:

1. each filament has finite spectral length;
2. each filament has finite spectral action;
3. each filament has a finite tension proxy;
4. each filament admits a finite stiffness matrix;
5. each filament has a finite mode-spectrum proxy;
6. these observables are stable across the baseline population;
7. active representatives preserve these observables under controlled spectral flow.

The following claims are not supported:

1. the tension proxy is physical string tension;
2. the spectral action is a Nambu–Goto action;
3. the stiffness matrix is a continuum string operator;
4. the mode spectrum is a quantized physical string spectrum;
5. the filament has a worldsheet;
6. the filament is embedded in a target spacetime;
7. a physical mass spectrum or scattering law is derived.

Limitation 5.1 (Finite-graph mechanics only). GEO-06 establishes stable effective finite-graph mechanical observables for pre-string filaments. It does not establish physical string tension, continuum worldsheet dynamics, quantized string modes, a Nambu–Goto action, or physical scattering behavior.

This distinction is central to the paper’s conservative claim. The result is not that fundamental strings have been derived. The result is that the pre-string filament population generated by spectral locality possesses a reproducible mechanical layer before any physical string interpretation is imposed.

5.8 Mechanical summary

The mechanical layer can be summarized as

pre-string channel $\longrightarrow$ spectral length $\longrightarrow$ spectral action $\longrightarrow$ tension proxy
 $\longrightarrow$ stiffness matrix $\longrightarrow$ mode spectrum $\longrightarrow$ mechanical stability under flow.

The frozen numerical core of this section is:

$|\mathcal{C}_0| = 15056,$
 finite tension fraction = 1.000000,
 median hop length = 3,
 median spectral length = 12.069761,
 median spectral action = 44.124460,
 median tension proxy = 3.629348,
 tension coefficient of variation = 0.189067,
 finite spectrum fraction = 1.000000,
 median mode gap = 0.584116,
 median scaled mode gap = 1.647594,
 median stiffness density = 2.125080,
 median mode entropy = 0.823461,
 active carriers = 13612,
 active episodes = 49217,
 finite dynamic mechanics fraction = 1.000000,
 combined mechanics stability = 0.872177,
 final GEO-06 score = 0.867671.

These results justify treating the dynamically renewing pre-string channel population as a mechanically structured finite-graph filament network. The next section asks whether these mechanically structured filaments are isolated or whether they couple to and interact through the surrounding organizational core.

6 Filament-Core Coupling and Interaction

The preceding section established that dynamically renewing pre-string filaments support stable finite-graph mechanical observables. The present section asks whether these filaments are isolated

objects or whether they are embedded in, coupled to, and interacting through the surrounding organizational graph.

The answer is decisive at the level of filament-core embedding and filament-filament core-mediated interaction, but qualified at the level of reconnection/action-exchange universality. The frozen GEO-07 audit shows that pre-string filaments are not isolated path-like objects. They possess strong interface with the organizational core, exhibit large leakage into the core, and interact with one another through shared core neighborhoods and bridge-weight couplings. Reconnection events, when present, are core-mediated and mechanically finite. However, the reconnection/action-exchange sector does not pass as a universal smooth interaction law. GEO-07 therefore closes as

FILAMENT_CORE_COUPLED_INTERACTION_PARTIAL_PASS_WITH_RECONNECTION_LIMITATION.

The frozen baseline remains

$$|\mathcal{C}_0| = 15056$$

channels across 24 seeds and $k = 3, \dots, 6$. The active dynamical population remains

$$13612$$

active carriers and

$$49217$$

active episodes, inherited from the GEO-05 dynamical audit.

Section-level coupling claim

Pre-string filaments are embedded in a higher-dimensional organizational core and interact through that core. GEO-07 Stage 1 establishes universal filament-core interface with interface fraction 1.000000, median leakage 0.897536, and core-bridge channel fraction 0.997210. GEO-07 Stage 2 establishes core-mediated filament-filament interaction with interaction pair fraction 0.998623, nonlocal interaction pair fraction 0.592651, and interaction-graph largest component fraction 1.000000. GEO-07 Stage 3 establishes that observed reconnections are core-mediated with fraction 1.000000, but also freezes a reconnection/action-exchange limitation.

6.1 Filament-core interface and leakage coupling

Let

$$P = (v_0, v_1, \dots, v_m)$$

be a pre-string filament path in the organizational graph

$$\mathcal{G}_k = (\mathcal{V}, \mathcal{E}_k).$$

Its path vertex set is

$$V(P) = \{v_0, \dots, v_m\}.$$

The surrounding organizational core of P is represented, at the finite-graph level, by vertices outside the path but adjacent to it:

$$\mathcal{B}(P) = \{u \in \mathcal{V} \setminus V(P) : \exists v \in V(P) \text{ such that } \{u, v\} \in \mathcal{E}_k\}.$$

This set is the filament-core boundary neighborhood. It measures how the path is embedded in the graph rather than isolated from it.

Definition 6.1 (Filament-core interface). For a filament path P , the filament-core interface is the set of graph edges

$$\partial_{\text{core}} P = \{\{u, v\} \in \mathcal{E}_k : u \in V(P), v \in \mathcal{V} \setminus V(P)\}.$$

The filament has nonzero core interface if

$$|\partial_{\text{core}} P| > 0.$$

If the organizational graph is equipped with positive spectral edge weights ω_{ij} , the total path-internal weight and boundary weight are

$$W_{\text{path}}(P) = \sum_{\{u,v\} \in \mathcal{E}_k, u,v \in V(P)} \omega_{uv},$$

and

$$W_{\text{bdry}}(P) = \sum_{\{u,v\} \in \partial_{\text{core}} P} \omega_{uv}.$$

The leakage fraction is then a normalized measure of how strongly the filament is coupled to its surrounding core.

Definition 6.2 (Boundary leakage fraction). The boundary leakage fraction of a filament P is

$$\Lambda_{\text{core}}(P) = \frac{W_{\text{bdry}}(P)}{W_{\text{bdry}}(P) + W_{\text{path}}(P)}.$$

A value near 0 indicates an almost isolated path, while a value near 1 indicates strong coupling to the surrounding organizational core.

GEO-07 Stage 1 audits this interface and leakage structure across the full frozen baseline channel population. The results are strong at every k -level.

Table 22: Frozen GEO-07 Stage 1 filament-core interface by scale.

k	Channels	Interface	Boundary edges	Leakage	Boundary coupling	Core bridge	Nonlocal bridge
3	4725	1.000	23.000	0.847	0.847	0.997	0.564
4	5375	1.000	29.000	0.897	0.897	1.000	0.651
5	3843	1.000	37.000	0.925	0.925	1.000	0.698
6	1113	1.000	44.750	0.937	0.937	1.000	0.844

The interface fraction is exactly

1.000000

across the full population. Thus every audited pre-string filament has nonzero core interface.

The median boundary edge count is

29,

the median core-neighbor count is

23,

the median shared-core-node count is

5,

and the median core-bridge-pair count is

6.

The median leakage fraction is

0.897536,

and the boundary/path weight ratio is

8.759508.

The core-bridge channel fraction is

0.997210,

and the nonlocal bridge fraction is

0.632505.

Endpoint core exposure is

1.000000.

Table 23: Frozen GEO-07 Stage 1 overall filament-core coupling statistics.

Quantity	Frozen value
Baseline channel count	15056
Seed-scale cells	96
Frozen input regression	pass
Interface fraction	1.000000
Core-bridge channel fraction	0.997210
Nonlocal bridge fraction	0.632505
Endpoint core exposure	1.000000
Median boundary edge count	29
Median core-neighbor count	23
Median shared-core-node count	5
Median core-bridge-pair count	6
Median leakage fraction	0.897536
Boundary coupling fraction	0.897536
Boundary/path weight ratio	8.759508
Exposure score	0.959014
Control leakage	0.887813
Channel/control leakage ratio	1.009382
Control neutrality	pass
Core coupling score	1.000000
$k = 5$ leakage	0.924830
$k = 5$ core-bridge fraction	1.000000
Gates passed	8/8
Gate score	0.990604
Verdict	filament-core interface/leakage coupling pass

Lemma 6.1 (Universal filament-core interface). *Every frozen pre-string filament in the GEO-07 Stage 1 audit has nonzero interface with the surrounding organizational core.*

Proof. GEO-07 Stage 1 reports interface fraction 1.000000 across the full baseline population of 15056 channels. Therefore every audited filament has nonzero $\partial_{\text{core}}P$, and hence every filament is embedded in the surrounding organizational core. $\square$

Proposition 6.1 (Strong filament-core leakage). *The frozen pre-string filaments are strongly coupled to the surrounding organizational core through boundary leakage.*

Proof. The median leakage fraction is 0.897536, the boundary/path weight ratio is 8.759508, the median boundary edge count is 29, and endpoint core exposure is 1.000000. Moreover, the core-bridge channel fraction is 0.997210, and the nonlocal bridge fraction is 0.632505. These values are incompatible with an isolated-path interpretation. They show that the filaments are strongly embedded in and coupled to the surrounding organizational core. $\square$

GEO-07 Stage 1 frozen result

The filament-core interface and leakage audit closes as

FILAMENT_CORE_INTERFACE_LEAKAGE_COUPLING_PASS.

Across 15056 baseline channels, the interface fraction is 1.000000, the median leakage fraction is 0.897536, the boundary/path weight ratio is 8.759508, the core-bridge channel fraction is 0.997210, the nonlocal bridge fraction is 0.632505, and endpoint core exposure is 1.000000. The audit passes 8/8 gates with score 0.990604.

6.2 Core-mediated filament-filament interaction

The preceding subsection established that individual filaments are strongly embedded in the organizational core. The next question is whether distinct filaments interact through that core.

Let P and Q be two filament paths. Their direct path overlap can be measured by the path-node Jaccard index

$$J_{\text{path}}(P, Q) = \frac{|V(P) \cap V(Q)|}{|V(P) \cup V(Q)|}.$$

If $J_{\text{path}}(P, Q) = 0$, the filaments share no path vertices. Such pairs can still interact through the core if their core neighborhoods overlap or are bridge-connected.

Definition 6.3 (Shared core neighborhood). Two filaments P and Q have shared core neighborhood if

$$\mathcal{B}(P) \cap \mathcal{B}(Q) \neq \emptyset,$$

where $\mathcal{B}(P)$ and $\mathcal{B}(Q)$ are their filament-core boundary neighborhoods.

Definition 6.4 (Core-mediated filament interaction). Two filaments P and Q are core-mediated interacting if they possess shared core neighborhood, bridge-weight coupling, or both, under the frozen GEO-07 interaction rule.

The Stage 2 audit evaluates

1903884

filament pairs. The shared-core pair fraction is

0.998623,

and the interaction pair fraction is also

0.998623.

The nonlocal pair fraction is

0.593945,

and the nonlocal interaction pair fraction is

$$0.592651.$$

Conditional on being nonlocal, the nonlocal interaction fraction is

$$0.997822.$$

The median shared-core count is

$$11,$$

the median shared-bridge-node count is

$$1,$$

and the bridge-weight fraction is

$$0.441122.$$

The core-mediated score is

$$0.702568.$$

The median path-node Jaccard is

$$0.000000.$$

Thus the interaction is not primarily path overlap. It is mediated through the core.

Table 24: Frozen GEO-07 Stage 2 core-mediated interaction by scale.

k	Channels	Pairs	Shared	Interaction	Nonlocal	Shared core	Bridge frac.	Score	LCC
3	4725	587288	0.996	0.996	0.547	6.000	0.325	0.617	1.000
4	5375	924907	1.000	1.000	0.631	9.000	0.402	0.678	1.000
5	3843	356176	1.000	1.000	0.584	13.000	0.476	0.722	1.000
6	1113	35513	1.000	1.000	0.434	21.000	0.668	0.813	1.000

The interaction graph built from filament-pair interactions has largest connected component fraction

$$1.000000$$

and mean degree

$$120.5.$$

Thus the filament population is not a collection of separate independent objects. It forms a connected interaction system through the organizational core.

Table 25: Frozen GEO-07 Stage 2 overall filament-filament interaction statistics.

Quantity	Frozen value
Total filament pair count	1903884
Shared-core pair fraction	0.998623
Interaction pair fraction	0.998623
Nonlocal pair fraction	0.593945
Nonlocal interaction pair fraction	0.592651
Nonlocal interaction given nonlocal	0.997822
Median shared-core count	11
Median shared-bridge-node count	1
Bridge-weight fraction	0.441122
Core-mediated score	0.702568
Median path-node Jaccard	0.000000
Exposure pair score	0.963817
Boundary pair leakage	0.909457
Interaction graph largest component fraction	1.000000
Interaction graph mean degree	120.5
Control neutrality	pass
$k = 5$ pair count	356176
$k = 5$ shared-core pair fraction	0.999989
$k = 5$ nonlocal interaction fraction	0.583720
$k = 5$ core-mediated score	0.722426
Gates passed	9/9
Gate score	1.000000
Verdict	core-mediated filament-filament interaction pass

Lemma 6.2 (Interaction without path overlap). *The dominant filament-filament interaction detected in GEO-07 Stage 2 is core-mediated rather than direct path-overlap interaction.*

Proof. The median path-node Jaccard index is 0.000000, showing that typical interacting filament pairs do not share path vertices. Nevertheless, the interaction pair fraction is 0.998623, the shared-core pair fraction is 0.998623, and the nonlocal interaction pair fraction is 0.592651. Conditional on nonlocality, the interaction fraction is 0.997822. Hence the interaction is mediated through shared core neighborhoods and bridge couplings rather than by direct path overlap. $\square$

Proposition 6.2 (Connected filament interaction system). *The frozen pre-string filament population forms a connected core-mediated interaction system.*

Proof. GEO-07 Stage 2 constructs an interaction graph on the filament population. The largest connected component fraction of this interaction graph is 1.000000, and the mean degree is 120.5. Since almost all filament pairs interact through shared core structure, with interaction pair fraction 0.998623, the filament population forms a connected interaction system rather than a set of isolated filaments. $\square$

GEO-07 Stage 2 frozen result

The core-mediated filament-filament interaction audit closes as

CORE_MEDIATED_FILAMENT_FILAMENT_INTERACTION_PASS.

Across 1903884 filament pairs, the interaction pair fraction is 0.998623, the shared-core pair fraction is 0.998623, the nonlocal interaction pair fraction is 0.592651, and nonlocal interaction given nonlocality is 0.997822. The median path-node Jaccard is 0.000000, confirming that the interaction is core-mediated rather than path-overlap dominated. The interaction graph has largest connected component fraction 1.000000. The audit passes 9/9 gates with score 1.000000.

6.3 Core-mediated reconnection and action exchange

GEO-07 Stage 3 asks a stronger question. It is not enough to show that filaments share core neighborhoods; one must test whether reconnection events are mediated through the core and whether the mechanical exchange across such events is controlled.

Let P_0 be a baseline filament and P_t a reconnected active representative under controlled spectral flow. Reconnection is core-mediated if the old and new representatives are linked through shared core neighborhoods, bridge-weight support, or core exposure under the frozen matching rule.

Definition 6.5 (Core-mediated reconnection). A reconnection event

$$P_0 \rightsquigarrow P_t$$

is core-mediated if the baseline and reconnected representatives share core-neighborhood support or bridge-weight coupling sufficient to satisfy the frozen GEO-07 reconnection mediation rule.

Mechanical exchange is measured by changes in the finite-graph observables defined in [Section 5](#). For an observable X , a natural exchange magnitude is

$$\Delta_X(P_0, P_t) = \left| \log \frac{X(P_t)}{X(P_0)} \right|,$$

or the corresponding normalized absolute change used by the frozen audit. The total exchange aggregates length, action, tension, mode, scaled-gap, and stiffness changes.

Definition 6.6 (Finite-graph mechanical exchange). The finite-graph mechanical exchange of a reconnection event is the vector of normalized changes

$$\Delta_{\text{mech}} = (\Delta_L, \Delta_A, \Delta_\tau, \Delta_\mu, \Delta_{\hat{\mu}}, \Delta_s),$$

where L is spectral length, A is spectral action, τ is tension proxy, μ is mode-gap proxy, $\hat{\mu}$ is scaled-gap proxy, and s is stiffness density.

The frozen Stage 3 audit finds that reconnection events, when present, are strongly core-mediated. The core-mediated reconnection carrier fraction is

$$1.000000,$$

and the core-mediated reconnection count is

$$17513.$$

The median reconnection mediation score is

$$0.911786,$$

with median shared-core count

$$20,$$

median shared-bridge count

$$4,$$

bridge weight

$$0.890310,$$

leakage pair score

$$0.898973,$$

and exposure pair score

$$0.959590.$$

Table 26: Frozen GEO-07 Stage 3 reconnection mediation by scale.

k	Channels	Reconn. frac.	Core-med.	Nonlocal reconn.	Mediation	Shared core	Bridge	Exchange
3	4725	0.191	1.000	0.038	0.886	16.000	0.872	0.0139
4	5375	0.288	1.000	0.056	0.912	20.000	0.890	0.0105
5	3843	0.217	1.000	0.046	0.925	24.250	0.897	0.0095
6	1113	0.194	1.000	0.024	0.942	29.500	0.922	0.0095

The event-level transition structure shows that reconnected events are mechanically more exchange-active than endpoint-persistent events. The median endpoint total exchange is

$$0.008306,$$

whereas the median reconnected total exchange is

$$0.053836.$$

Thus the reconnected/endpoint exchange ratio is

$$6.481297.$$

This is too large for a smooth endpoint-compatible exchange interpretation.

Table 27: Frozen GEO-07 Stage 3 event-level transition classes.

Event class	Count	Fraction	Core mediation	Median exchange
decayed to decayed	15201	0.202	–	–
endpoint to endpoint	27182	0.361	–	0.0070
endpoint to reconnected	6823	0.091	1.000	0.1342
reconnected to reconnected	8612	0.114	1.000	0.0083
decayed to reconnected	2078	0.028	1.000	0.1502

The step-level exchange profile remains bounded. Across flow steps, the active and reconnected populations retain core mediation with median exchange values at the 10^{-2} scale for active representatives.

Table 28: Frozen GEO-07 Stage 3 step-level reconnection and exchange profile.

Step	s	Active	Reconnected	Core-med.	Mediation	Exchange	Action exch.	Tension exch.
1	0.2	11731	3029	1.000	0.897	0.0117	0.0198	0.0096
2	0.4	9952	3663	1.000	0.913	0.0105	0.0175	0.0084
3	0.6	9416	3730	1.000	0.921	0.0087	0.0145	0.0071
4	0.8	9401	3527	1.000	0.927	0.0109	0.0180	0.0088
5	1.0	8717	3564	1.000	0.910	0.0150	0.0244	0.0119

The overall Stage 3 statistics are:

Table 29: Frozen GEO-07 Stage 3 overall reconnection/action-exchange statistics.

Quantity	Frozen value
Baseline channel count	15056
Seed-scale cells	96
Transition count	75280
Frozen regression	pass
Active carriers	13612
Active episodes	49217
Active regression	pass
Active episode fraction	0.653786
Endpoint episode fraction	0.421148
Reconnected episode fraction	0.232638
Decayed episode fraction	0.346214
Reconnected carrier fraction	0.508967
Core-mediated reconnection carrier fraction	1.000000
Core-mediated reconnection count	17513
Core-mediated reconnection fraction	1.000000
Nonlocal reconnection count	819
Nonlocal reconnection fraction	0.046765
Finite reconnected mechanics fraction	1.000000
Median reconnection mediation score	0.911786
Median shared-core count	20
Median shared-bridge count	4
Bridge weight	0.890310
Leakage pair score	0.898973
Exposure pair score	0.959590
Median endpoint total exchange	0.008306
Median reconnected total exchange	0.053836
Median active total exchange	0.011074
Reconnected/endpoint exchange ratio	6.481297
Exchange bound score	1.000000
$k = 5$ reconnected episode fraction	0.217174
$k = 5$ core-mediated reconnection fraction	1.000000
$k = 5$ nonlocal reconnection fraction	0.045531
$k = 5$ mediation score	0.925349
$k = 5$ active total exchange	0.009524
Gates passed	5/9
Gate score	0.885021
Verdict	weak partial reconnection/action-exchange

Lemma 6.3 (Core mediation of observed reconnection). *Every observed reconnection event in*

the frozen GEO-07 Stage 3 audit is core-mediated under the frozen reconnection rule.

Proof. The audit reports core-mediated reconnection carrier fraction 1.000000, core-mediated reconnection count 17513, and core-mediated reconnection fraction 1.000000. It also reports median reconnection mediation score 0.911786, median shared-core count 20, and bridge weight 0.890310. Therefore observed reconnections are mediated by the organizational core under the frozen rule. $\square$

Proposition 6.3 (Reconnection/action-exchange limitation). *The frozen reconnection sector is core-mediated and mechanically finite, but it does not establish a universal smooth reconnection/action-exchange regime.*

Proof. The positive side is strong: core-mediated reconnection fraction is 1.000000, finite reconnected mechanics fraction is 1.000000, and the exchange bound score is 1.000000. However, the reconnected episode fraction is 0.232638, below the 0.300000 threshold; the nonlocal reconnection fraction is 0.046765, slightly below the 0.050000 threshold; the reconnected/endpoint exchange ratio is 6.481297, above the allowed 2.500000 threshold; and the $k = 5$ reconnected episode fraction is 0.217174, below the 0.250000 threshold. Therefore the reconnection sector is real and core-mediated, but not universal as a smooth action-exchange law. $\square$

GEO-07 Stage 3 frozen result

The core-mediated reconnection/action-exchange audit closes as

CORE_MEDIATED_RECONNECTION_ACTION_EXCHANGE_WEAK_PARTIAL.

Observed reconnections are core-mediated with fraction 1.000000, and finite reconnected mechanics fraction is 1.000000. However, the reconnected episode fraction is $0.232638 < 0.300000$, the nonlocal reconnection fraction is $0.046765 < 0.050000$, the reconnected/endpoint exchange ratio is $6.481297 > 2.500000$, and the $k = 5$ reconnected episode fraction is $0.217174 < 0.250000$. The stage passes 5/9 gates with gate score 0.885021 and freezes as weak partial.

6.4 Final filament-core interaction verdict

GEO-07 Stage 4 integrates the three layers:

filament-core interface $\longrightarrow$ filament-filament core interaction $\longrightarrow$ core-mediated reconnection/action

The positive evidence is strong:

1. GEO-07 Stage 1 establishes universal filament-core interface:

interface fraction = 1.000000.

2. GEO-07 Stage 1 establishes strong leakage:

$$\text{median leakage} = 0.897536.$$

3. GEO-07 Stage 2 establishes filament-filament core-mediated interaction:

$$\text{interaction pair fraction} = 0.998623.$$

4. GEO-07 Stage 2 establishes nonlocal core-mediated interaction:

$$\text{nonlocal interaction pair fraction} = 0.592651.$$

5. GEO-07 Stage 3 establishes core mediation of observed reconnection:

$$\text{core-mediated reconnection fraction} = 1.000000.$$

The limitations are also frozen:

1. reconnected episode fraction:

$$0.232638 < 0.300000;$$

2. nonlocal reconnection fraction:

$$0.046765 < 0.050000;$$

3. reconnected/endpoint exchange ratio:

$$6.481297 > 2.500000;$$

4. $k = 5$ reconnected episode fraction:

$$0.217174 < 0.250000.$$

The final GEO-07 verdict is therefore

`FILAMENT_CORE_COUPLED_INTERACTION_PARTIAL_PASS_WITH_RECONNECTION_LIMITATION.`

The final validation passes. The audit passes

$$7/10$$

final gates with gate score

$$0.902049.$$

Table 30: Frozen GEO-07 final evidence and limitation summary.

Layer	Status	Frozen anchor
Filament-core interface	support	median leakage 0.897536
Filament-filament interaction	support	interaction pair fraction 0.998623
Reconnection core mediation	support	core-mediated reconnection fraction 1.000000
Reconnection population	limitation	reconnected episode fraction 0.232638
Nonlocal reconnection	limitation	nonlocal reconnection fraction 0.046765
Exchange compatibility	limitation	reconnected/endpoint ratio 6.481297
$k = 5$ reconnection support	limitation	$k = 5$ reconnected fraction 0.217174

GEO-07 final frozen result

The final GEO-07 verdict is

FILAMENT_CORE_COUPLED_INTERACTION_PARTIAL_PASS_WITH_RECONNECTION_LIMITATION.

The result is

PARTIAL / FROZEN.

The validation status is

PASS.

The compact claim is that pre-string filaments form an embedded core-mediated interaction system, with a qualified reconnection/action-exchange limitation. The final audit passes 7/10 gates with gate score 0.902049.

6.5 Embedded filament-core interaction theorem

The GEO-07 result can now be stated as a qualified theorem.

Theorem 6.1 (Embedded filament-core interaction). *In the frozen $N = 64$, $d = 16$, 24-seed ensemble over $k = 3, \dots, 6$, the dynamically renewing pre-string filaments form an embedded filament-core interaction system. Individual filaments have universal core interface and strong boundary leakage; filament pairs interact through shared core neighborhoods and bridge-weight coupling; observed reconnections are core-mediated and mechanically finite. However, the reconnection/action-exchange sector does not establish a universal smooth reconnection law.*

Proof. GEO-07 Stage 1 evaluates 15056 baseline filaments and finds interface fraction 1.000000, core-bridge channel fraction 0.997210, nonlocal bridge fraction 0.632505, endpoint core exposure 1.000000, median leakage fraction 0.897536, boundary/path weight ratio 8.759508, and 8/8 gates passed with score 0.990604. Therefore individual filaments are embedded in and strongly coupled to the organizational core.

GEO-07 Stage 2 evaluates 1903884 filament pairs and finds shared-core pair fraction 0.998623, interaction pair fraction 0.998623, nonlocal interaction pair fraction 0.592651, nonlocal interaction given nonlocality 0.997822, median path-node Jaccard 0.000000, interaction graph largest

component fraction 1.000000, and 9/9 gates passed with score 1.000000. Therefore the filament population forms a connected core-mediated interaction system rather than a set of isolated paths.

GEO-07 Stage 3 evaluates reconnection/action-exchange under controlled spectral flow. It finds core-mediated reconnection fraction 1.000000, finite reconnected mechanics fraction 1.000000, median reconnection mediation score 0.911786, median shared-core count 20, bridge weight 0.890310, and exchange bound score 1.000000. Therefore observed reconnections are core-mediated and mechanically finite.

However, the same audit finds reconnected episode fraction $0.232638 < 0.300000$, nonlocal reconnection fraction $0.046765 < 0.050000$, reconnected/endpoint exchange ratio $6.481297 > 2.500000$, and $k = 5$ reconnected episode fraction $0.217174 < 0.250000$. Thus the reconnection/action-exchange sector fails the universality criteria. The final GEO-07 synthesis passes 7/10 gates with gate score 0.902049 and freezes as partial. Therefore the theorem holds in the qualified form stated above. $\square$

6.6 Interpretation: embedded interaction, not physical force law

The GEO-07 result is one of the strongest pieces of evidence in the pre-string chain, but it must be interpreted carefully. It establishes embedded finite-graph interaction, not a physical force law.

The following claims are supported:

1. pre-string filaments are not isolated graph paths;
2. every audited filament has nonzero core interface;
3. filament-core leakage is strong;
4. filament pairs almost universally share core-mediated interaction structure;
5. many filament interactions are nonlocal in the sense of having no path-node overlap;
6. observed reconnection events are core-mediated;
7. reconnected representatives retain finite mechanical observables.

The following claims are not supported:

1. the core-mediated interaction is a physical force;
2. a gauge interaction has been derived;
3. a conserved charge has been identified;
4. a scattering amplitude has been computed;
5. reconnection obeys a universal smooth interaction law;
6. the exchange sector is equivalent to physical string interaction;

7. a continuum interaction field has been derived.

Limitation 6.1 (No physical force law). GEO-07 establishes finite-graph filament-core coupling and core-mediated filament-filament interaction. It does not establish a physical force law, gauge interaction, scattering amplitude, conserved charge, continuum interaction field, or physical string interaction.

The correct interpretation is therefore:

embedded organizational interaction $\neq$ physical force law.

The filaments interact because they are jointly embedded in the same organizational core. This interaction is measurable through shared core neighborhoods, bridge-weight coupling, leakage, nonlocal core mediation, and reconnection mediation. But the audit does not yet supply a closed analytic law governing all such interactions.

6.7 Summary of the filament-core coupling layer

The filament-core coupling layer can be summarized as

finite-graph filament $\longrightarrow$ core interface $\longrightarrow$ boundary leakage $\longrightarrow$ shared core neighborhoods
 $\longrightarrow$ core-mediated filament interaction $\longrightarrow$ core-mediated reconnection $\longrightarrow$ qualified reconnection/

The frozen numerical core of this section is:

$|\mathcal{C}_0| = 15056,$
 interface fraction = 1.000000,
 median leakage fraction = 0.897536,
 boundary/path weight ratio = 8.759508,
 core-bridge channel fraction = 0.997210,
 nonlocal bridge fraction = 0.632505,
 filament pair count = 1903884,
 interaction pair fraction = 0.998623,
 shared-core pair fraction = 0.998623,
 nonlocal interaction pair fraction = 0.592651,
 interaction graph LCC fraction = 1.000000,
 active carriers = 13612,
 active episodes = 49217,
 core-mediated reconnection fraction = 1.000000,
 finite reconnected mechanics fraction = 1.000000,
 reconnected episode fraction = 0.232638,
 nonlocal reconnection fraction = 0.046765,
 reconnected/endpoint exchange ratio = 6.481297,
 final GEO-07 gate score = 0.902049.

These results establish that the mechanically structured pre-string filament population is embedded in a higher-dimensional organizational core and interacts through that core. The next section asks whether this embedded interaction structure can be compressed into an effective interaction law, or whether it remains a descriptive graph-level coupling with only partial law-like regularity.

7 Effective Interaction-Law Discrimination

The preceding section established that pre-string filaments are embedded in a higher-dimensional organizational core and interact through shared core neighborhoods and bridge-weight couplings. It also established that observed reconnections are core-mediated and mechanically finite. The present section asks a stronger question: can the observed filament-core interaction structure be compressed into an effective interaction law?

This is a different question from whether interaction exists. GEO-07 already shows strong embedded interaction:

filament-core interface fraction = 1.000000,

filament-pair interaction fraction = 0.998623,

core-mediated reconnection fraction = 1.000000.

The task of GEO-08 is not to re-establish interaction, but to test whether pre-flow observables predict subsequent interaction outcomes in a reproducible law-like way.

The frozen outcome is partial. Refined pre-flow susceptibility, edge-margin, core-response, and mechanical observables produce a partial law-like signal for exchange and high-exchange behavior. However, they do not robustly discriminate reconnection events. Therefore the final GEO-08 verdict is

PARTIAL_EFFECTIVE_EXCHANGE_LAW_SIGNAL_WITH_RECONNECTION_NONDISCRIMINATION.

The result is

PARTIAL / FROZEN.

Section-level law-discrimination claim

The embedded filament-core system exhibits partial law-like regularity in exchange observables, but not a full reconnection law. GEO-08 Stage 1 finds only weak discrimination by simple candidate scores. GEO-08 Stage 2 improves high-exchange discrimination to best refined AUC 0.573874, obtains exchange-rank signal $|\rho| = 0.144145$, and shows seed-scale exchange-rank recurrence 0.926316. However, reconnection discrimination remains below threshold, with best refined reconnection AUC 0.526464 and gain over null 0.004032. The final GEO-08 audit passes 7/10 gates with score 0.693467.

7.1 Law-discrimination observables

Let P be a baseline pre-string filament and let P_t be its active representative under controlled spectral flow. The law-discrimination audit studies whether pre-flow scores

$$S(P)$$

can predict later dynamical outcomes. Three outcome classes are especially important:

1. reconnection outcome;
2. high-exchange outcome;
3. exchange intensity.

The reconnection outcome is binary:

$$R(P) = \begin{cases} 1, & \text{if } P \text{ becomes reconnected under the frozen matching rule,} \\ 0, & \text{otherwise.} \end{cases}$$

The high-exchange outcome is also binary:

$$H(P) = \begin{cases} 1, & \text{if the mechanical exchange of } P \text{ lies in the high-exchange class,} \\ 0, & \text{otherwise.} \end{cases}$$

The exchange intensity is a scalar:

$$E(P) \geq 0,$$

computed from normalized changes in the finite-graph mechanical observables

$$L, \quad A, \quad \tau, \quad \mu, \quad \hat{\mu}, \quad s.$$

Definition 7.1 (Candidate interaction-law score). A candidate interaction-law score is a pre-flow scalar observable

$$S(P)$$

assigned to a baseline filament P , constructed from core coupling, edge-margin, flow susceptibility, core-response, or finite-graph mechanical features, and evaluated by its ability to discriminate later outcomes $R(P)$, $H(P)$, or $E(P)$.

For binary outcomes R and H , discrimination is measured by the area under the receiver-operating curve:

$$\text{AUC}(S, R), \quad \text{AUC}(S, H).$$

For scalar exchange intensity E , rank-level association is measured by Spearman correlation:

$$\rho_S(S, E).$$

The comparison against a null baseline is measured by gain:

$$\Delta_{\text{null}}(S, Y) = \text{AUC}(S, Y) - \text{AUC}(S_{\text{null}}, Y).$$

Definition 7.2 (Law-like discrimination). A candidate score S exhibits law-like discrimination for an outcome Y if it produces reproducible discrimination above the frozen threshold and above the null score across seed-scale cells. In this paper, law-like discrimination is evaluated separately for reconnection, high exchange, and exchange intensity.

The distinction between these outcomes is essential. A score may partially predict exchange intensity without predicting reconnection. GEO-08 shows precisely this split.

7.2 Simple candidate interaction-law scores

GEO-08 Stage 1 tests simple candidate law scores. These include core-based scores, mechanical scores, hybrid scores, and null scores. The purpose is to determine whether the embedded coupling detected in GEO-07 immediately yields a scalar interaction law.

The answer is negative at the full-law level. Simple scores do not robustly discriminate

reconnection or high exchange. The frozen Stage 1 result is

CANDIDATE_INTERACTION_LAW_DISCRIMINATION_WEAK_PARTIAL.

At the k -level, the Stage 1 audit yields the following values.

Table 31: Frozen GEO-08 Stage 1 simple candidate interaction-law discrimination by scale.

k Score	Channels	AUCR core	AUCR mech.	AUCR hybrid	AUCR null	GainR	AUCH hybrid	GainH	SpearmanE hybrid
3 0.968	4725	0.502	0.480	0.498	0.513	-0.007	0.492	-0.031	-0.052
4 0.952	5375	0.520	0.462	0.503	0.488	0.020	0.486	0.004	-0.029
5 0.955	3843	0.541	0.502	0.543	0.504	0.026	0.506	-0.005	0.019
6 0.973	1113	0.607	0.533	0.588	0.458	0.104	0.499	0.005	-0.025

The overall Stage 1 population is the same frozen dynamical population used in GEO-05 and GEO-07:

$$|\mathcal{C}_0| = 15056,$$

$$\text{active carriers} = 13612,$$

$$\text{active episodes} = 49217.$$

The reconnected carrier fraction is

$$0.508967,$$

the median reconnected life is

$$0.2,$$

and the median total exchange is

$$0.014599.$$

The best simple reconnection AUC is only

$$0.526464$$

for the core-oriented score. The hybrid reconnection AUC is

$$0.514115,$$

and the null reconnection AUC is

$$0.502263.$$

The hybrid gain is therefore only

$$0.011852.$$

For high exchange, the core AUC is

0.500689,

the mechanical AUC is

0.463152,

the hybrid AUC is

0.485112,

and the null AUC is

0.505225.

Thus the hybrid high-exchange gain is negative:

−0.020113.

The strongest exchange-rank Spearman magnitude is only

0.002466.

Table 32: Frozen GEO-08 Stage 1 overall simple candidate-law statistics.

Quantity	Frozen value
Baseline channel count	15056
Active carriers	13612
Active episodes	49217
Reconnected carrier fraction	0.508967
Median reconnected life	0.2
Median total exchange	0.014599
Core AUC for reconnection	0.526464
Mechanical AUC for reconnection	0.481891
Hybrid AUC for reconnection	0.514115
Null AUC for reconnection	0.502263
Hybrid reconnection gain	0.011852
Core AUC for high exchange	0.500689
Mechanical AUC for high exchange	0.463152
Hybrid AUC for high exchange	0.485112
Null AUC for high exchange	0.505225
Hybrid high-exchange gain	−0.020113
Best exchange Spearman magnitude	0.002466
Law-discrimination score	0.630736
Seed-scale hybrid weak-AUC fraction	0.406250
Seed-scale hybrid positive-gain fraction	0.635417
Seed-scale exchange-Spearman fraction	0.568421
$k = 5$ hybrid AUC for reconnection	0.542634
$k = 5$ AUC for high exchange	0.506319
$k = 5$ reconnection gain	0.026422
$k = 5$ exchange Spearman	0.019181
$k = 5$ score	0.954928
Gates passed	5/9
Gate score	0.628373
Verdict	weak partial simple law discrimination

Lemma 7.1 (Simple scalar scores do not define a full interaction law). *The simple candidate scores tested in GEO-08 Stage 1 do not establish a full effective interaction law for the filament-core system.*

Proof. A full effective interaction law would require robust discrimination of relevant dynamical outcomes above null across the scale window. In Stage 1, the hybrid reconnection AUC is 0.514115, with gain only 0.011852. The best reconnection AUC is 0.526464, still weak. For high exchange, the hybrid AUC is 0.485112, below the null AUC 0.505225, giving negative gain −0.020113. The strongest exchange-rank Spearman magnitude is only 0.002466. Stage 1 passes only 5/9 gates and freezes as weak partial. Therefore the simple scores do not define a full

effective interaction law. □

GEO-08 Stage 1 frozen result

The simple candidate interaction-law audit closes as

`CANDIDATE_INTERACTION_LAW_DISCRIMINATION_WEAK_PARTIAL.`

The best reconnection AUC is 0.526464, the hybrid reconnection AUC is 0.514115, the hybrid gain is 0.011852, the hybrid high-exchange AUC is 0.485112, and the best exchange-rank Spearman magnitude is 0.002466. The stage passes 5/9 gates with gate score 0.628373.

7.3 Refined pre-flow susceptibility and core-response scores

GEO-08 Stage 2 refines the candidate scores. The goal is to test whether a more structured pre-flow feature set can recover a partial effective law signal. The refined scores include:

1. core-oriented pre-flow coupling;
2. mechanical pre-flow susceptibility;
3. flow-oriented response;
4. edge-margin score;
5. coupled score;
6. refined combined score;
7. response-oriented score;
8. null score.

Definition 7.3 (Refined interaction-law score). A refined interaction-law score is a pre-flow scalar observable constructed from a combination of core coupling, mechanical susceptibility, edge-margin, flow response, and filament-core response features. It is evaluated against reconnection, high exchange, and exchange intensity using the same frozen outcome population as Stage 1.

The refined Stage 2 result improves the exchange-related signal but still fails full reconnection-law discrimination.

At the k -level, Stage 2 gives:

Table 33: Frozen GEO-08 Stage 2 refined interaction-law discrimination by scale.

k Score	Channels	Core R	Flow R	Edge R	Coupled R	Refined R	Response R	Null R	Gain R	Refined E_H	Spearman E
3 1.000	4725	0.572	0.548	0.560	0.544	0.534	0.534	0.535	-0.003	0.567	0.093
4 1.000	5375	0.551	0.553	0.556	0.549	0.536	0.523	0.533	-0.013	0.548	0.104
5 1.000	3843	0.548	0.537	0.562	0.562	0.526	0.537	0.537	-0.011	0.566	0.155
6 1.000	1113	0.621	0.586	0.579	0.576	0.602	0.583	0.590	0.014	0.601	0.128

The overall refined audit preserves the frozen population regression:

$$|\mathcal{C}_0| = 15056, \quad \text{transition count} = 75280,$$

$$\text{active carriers} = 13612, \quad \text{active episodes} = 49217.$$

The reconnected carrier fraction is again

$$0.508967,$$

and median total exchange is

$$0.014599.$$

For reconnection, the refined score does not provide a strong improvement:

$$\text{best refined } \text{AUC}_R = 0.526464,$$

with gain

$$0.004032.$$

The response-oriented reconnection AUC is

$$0.503476,$$

and the null reconnection AUC is

$$0.502263.$$

Thus reconnection discrimination remains weak.

For high exchange, however, the refined features improve substantially:

$$\text{refined } \text{AUC}_H = 0.532982,$$

$$\text{response } \text{AUC}_H = 0.558363,$$

$$\text{null } \text{AUC}_H = 0.505225,$$

and the best refined high-exchange AUC reaches

$$0.573874.$$

The exchange-rank signal also improves:

$$|\rho_S|_{\text{best}} = 0.144145.$$

The seed-scale exchange-Spearman recurrence fraction is

$$0.926316.$$

Table 34: Frozen GEO-08 Stage 2 refined candidate-law statistics.

Quantity	Frozen value
Baseline channel count	15056
Seed-scale cells	96
Transition count	75280
Frozen regression	pass
Active carriers	13612
Active episodes	49217
Active regression	pass
Reconnected carrier fraction	0.508967
Median total exchange	0.014599
Core-oriented AUC for reconnection	0.526464
Mechanical-oriented AUC for reconnection	0.518109
Flow-oriented AUC for reconnection	0.500927
Edge-oriented AUC for reconnection	0.506174
Coupled-oriented AUC for reconnection	0.505540
Refined-oriented AUC for reconnection	0.506295
Response-oriented AUC for reconnection	0.503476
Null AUC for reconnection	0.502263
Best refined AUC for reconnection	0.526464
Best refined reconnection gain	0.004032
Refined-oriented AUC for high exchange	0.532982
Response-oriented AUC for high exchange	0.558363
Null AUC for high exchange	0.505225
Best refined AUC for high exchange	0.573874
Refined exchange Spearman	0.103513
Response exchange Spearman	0.144145
Flow exchange Spearman	0.132900
Best exchange Spearman magnitude	0.144145
Best reconnection Spearman magnitude	0.056693
Refined law-discrimination score	0.985029
Seed-scale refined weak-AUC fraction	0.395833
Seed-scale response weak-AUC fraction	0.312500
Seed-scale positive-gain fraction	0.625000
Seed-scale exchange-Spearman fraction	0.926316
Stable-window k -pass fraction	0.333333
$k = 5$ refined AUC for reconnection	0.526001
$k = 5$ response AUC for reconnection	0.536824
$k = 5$ null AUC for reconnection	0.536577
$k = 5$ refined gain	-0.010546
$k = 5$ response gain	0.001349
$k = 5$ high-exchange refined AUC	0.566458
$k = 5$ high-exchange response AUC	0.582627
$k = 5$ exchange Spearman	0.155385
$k = 5$ response Spearman	0.178481
$k = 5$ score	1.000000

Lemma 7.2 (Refined scores recover exchange-related signal). *The refined pre-flow scores recover a partial law-like signal for exchange and high-exchange behavior.*

Proof. Compared with Stage 1, Stage 2 raises the best high-exchange AUC to 0.573874, the response-oriented high-exchange AUC to 0.558363, and the exchange-rank Spearman magnitude to 0.144145. The seed-scale exchange-Spearman recurrence fraction is 0.926316, showing broad recurrence across seed-scale cells. Therefore refined pre-flow observables provide a partial law-like signal for exchange behavior. $\square$

Proposition 7.1 (Refined scores do not recover a reconnection law). *The refined score family does not establish robust reconnection-law discrimination.*

Proof. The best refined reconnection AUC is 0.526464, and the best gain over null is only 0.004032. The refined-oriented reconnection AUC is 0.506295, the response-oriented reconnection AUC is 0.503476, and the null AUC is 0.502263. The stable-window k -pass fraction is 0.333333, and the $k = 5$ response gain is only 0.001349. Therefore the refined scores do not establish robust reconnection-law discrimination. $\square$

GEO-08 Stage 2 frozen result

The refined candidate interaction-law audit closes as

REFINED_INTERACTION_LAW_CROSS_SCALE_PARTIAL_PASS.

The refined scores improve exchange-related behavior: best high-exchange AUC is 0.573874, best exchange-rank Spearman magnitude is 0.144145, and seed-scale exchange-Spearman recurrence is 0.926316. However, reconnection discrimination remains weak: best refined reconnection AUC is 0.526464, gain over null is 0.004032, stable-window k -pass fraction is 0.333333, and $k = 5$ reconnection-law support fails. The stage passes 7/10 gates with gate score 0.755117.

7.4 Exchange-law regularity

The positive content of GEO-08 is concentrated in exchange observables. The refined law candidates do not fully predict whether a filament reconnects, but they partially rank how exchange-active a filament becomes.

This distinction can be formalized as follows.

Definition 7.4 (Partial exchange-law signal). A partial exchange-law signal is present when pre-flow observables produce reproducible above-null discrimination of high-exchange behavior or recurrent rank-level association with exchange intensity, without satisfying the stronger criteria for a full interaction law.

The frozen exchange-law evidence consists of four anchors:

best refined high-exchange AUC = 0.573874,

response high-exchange AUC = 0.558363,

$$|\rho_S(S, E)|_{\text{best}} = 0.144145,$$

seed-scale exchange-Spearman recurrence = 0.926316.

At $k = 5$, the exchange-related behavior is especially visible:

high-exchange refined AUC = 0.566458,

high-exchange response AUC = 0.582627,

exchange Spearman = 0.155385,

response Spearman = 0.178481.

Table 35: Frozen GEO-08 positive exchange-law anchors.

Exchange-law anchor	Frozen value
Best refined high-exchange AUC	0.573874
Response-oriented high-exchange AUC	0.558363
Best exchange-rank Spearman magnitude	0.144145
Seed-scale exchange-Spearman recurrence	0.926316
$k = 5$ refined high-exchange AUC	0.566458
$k = 5$ response high-exchange AUC	0.582627
$k = 5$ exchange Spearman	0.155385
$k = 5$ response Spearman	0.178481

Proposition 7.2 (Partial law-like regularity of exchange). *The embedded filament-core system exhibits partial law-like regularity in exchange observables.*

Proof. The refined audit obtains best high-exchange AUC 0.573874, response high-exchange AUC 0.558363, best exchange-rank Spearman magnitude 0.144145, and seed-scale exchange-Spearman recurrence 0.926316. These values improve substantially over the Stage 1 simple-score results, where high-exchange hybrid AUC was 0.485112 and exchange-rank Spearman magnitude was only 0.002466. Therefore refined pre-flow observables provide a reproducible partial signal for exchange and high-exchange behavior. $\square$

The signal is partial for two reasons. First, the AUC values are above weak-discrimination level but far from strong prediction. Second, the signal is exchange-specific: it does not transfer to full reconnection-law discrimination.

7.5 Reconnection non-discrimination

The negative result of GEO-08 is equally important. The audit does not establish a reconnection law.

This must be distinguished from the GEO-07 result that reconnections are core-mediated. GEO-07 answers:

When reconnection occurs, is it mediated by the core?

The answer is yes:

core-mediated reconnection fraction = 1.000000.

GEO-08 asks a stronger predictive question:

Can pre-flow observables discriminate which filaments will reconnect?

The answer is no at the required threshold.

The frozen reconnection limitations are:

best refined reconnection AUC = 0.526464 < 0.560000,

best refined gain = 0.004032 < 0.020000,

stable-window k -pass fraction = 0.333333 < 0.666667,

$k = 5$ response gain = 0.001349 < 0.015000.

Table 36: Frozen GEO-08 reconnection-law limitations.

Limitation	Frozen value	Threshold
Best refined reconnection AUC	0.526464	0.560000
Best refined gain over null	0.004032	0.020000
Stable-window k -pass fraction	0.333333	0.666667
$k = 5$ response gain	0.001349	0.015000

Lemma 7.3 (Core-mediated reconnection is not the same as a reconnection law). *The fact that observed reconnections are core-mediated does not imply that the tested pre-flow observables define a reconnection law.*

Proof. GEO-07 establishes core-mediated reconnection fraction 1.000000, meaning that observed reconnections are mediated by the organizational core. GEO-08 tests predictive discrimination of reconnection using pre-flow scores. The best refined reconnection AUC is 0.526464, the gain over null is 0.004032, stable-window support is 0.333333, and $k = 5$ response gain is 0.001349. These values fail the frozen reconnection-law thresholds. Therefore core mediation of observed reconnection does not imply a predictive reconnection law. $\square$

Proposition 7.3 (Reconnection remains a core-mediated reconfiguration). *Within the present frozen construction, reconnection should be interpreted as a core-mediated reconfiguration phenomenon rather than as a closed effective interaction law.*

Proof. Observed reconnections are core-mediated by GEO-07, but the GEO-08 discrimination audit fails to produce robust predictive reconnection scores. The best reconnection AUC remains near weak-discrimination level, gain over null is small, stable-window support is incomplete, and $k = 5$ reconnection-law support fails. Therefore reconnection is established as a core-mediated reconfiguration mode, not as a closed law. $\square$

7.6 Final effective interaction-law verdict

GEO-08 Stage 3 integrates the weak simple-score result of Stage 1 and the refined partial result of Stage 2. The frozen input regression passes:

GEO-02 pass, GEO-03 pass, GEO-04 pass, GEO-05 pass, GEO-06 pass,

GEO-07 partial, GEO-08 Stage 1 weak partial, GEO-08 Stage 2 partial.

The baseline channel count, seed-scale count, transition count, and active population all remain consistent:

$$|\mathcal{C}_0| = 15056,$$

$$\text{seed-scale cells} = 96,$$

$$\text{transition count} = 75280,$$

$$\text{active carriers} = 13612,$$

$$\text{active episodes} = 49217.$$

The final evidence layers are:

1. Stage 1 simple candidate laws: weak partial, hybrid reconnection AUC 0.514115;
2. Stage 2 refined exchange-law signal: partial support, best refined high-exchange AUC 0.573874;
3. Stage 2 reconnection-law failure: limitation, best refined reconnection AUC 0.526464;
4. cross seed-scale exchange recurrence: support, exchange-Spearman recurrence 0.926316;
5. stable-window and $k = 5$ reconnection limitation: limitation, stable-window pass fraction 0.333333.

Table 37: Frozen GEO-08 final evidence and limitation summary.

Layer	Status	Frozen anchor
Simple candidate laws	weak partial	hybrid reconnection AUC 0.514115
Refined exchange-law signal	partial support	high-exchange AUC 0.573874
Exchange-rank recurrence	support	seed-scale recurrence 0.926316
Reconnection-law discrimination	limitation	reconnection AUC 0.526464
Reconnection gain over null	limitation	gain 0.004032
Stable-window law support	limitation	0.333333
$k = 5$ reconnection support	limitation	response gain 0.001349

The final gates are:

Table 38: Frozen GEO-08 final gate summary.

Gate	Status	Meaning
ELV-G1	pass	frozen chain integrity
ELV-G2	pass	Stage 1 weak law-candidate result retained
ELV-G3	pass	Stage 2 refined improvement retained
ELV-G4	pass	partial exchange-law signal
ELV-G5	pass	cross seed-scale exchange recurrence
ELV-G6	fail	reconnection-law discrimination
ELV-G7	fail	stable-window law stability
ELV-G8	fail	$k = 5$ reconnection-law support
ELV-G9	pass	descriptive coupling versus law boundary
ELV-G10	pass	conservative law scope

The final GEO-08 verdict is:

PARTIAL_EFFECTIVE_EXCHANGE_LAW_SIGNAL_WITH_RECONNECTION_NONDISCRIMINATION.

The confidence class is:

moderate_confidence_partial_exchange_law_signal.

The result is:

PARTIAL / FROZEN.

The final audit passes

7/10

gates with score

0.693467,

and validation status

PASS.

GEO-08 final frozen result

The final GEO-08 verdict is

PARTIAL_EFFECTIVE_EXCHANGE_LAW_SIGNAL_WITH_RECONNECTION_NONDISCRIMINATION.

The result is

PARTIAL / FROZEN,

with moderate confidence, 7/10 gates passed, gate score 0.693467, and validation status

PASS.

The audit supports partial exchange-law regularity, with best refined high-exchange AUC 0.573874, exchange-rank Spearman magnitude 0.144145, and seed-scale exchange-Spearman recurrence 0.926316. It does not establish a full reconnection law: best refined reconnection AUC is 0.526464, gain over null is 0.004032, stable-window support is 0.333333, and $k = 5$ reconnection support fails.

7.7 Partial effective exchange-law theorem

The final law-discrimination result can now be stated as a theorem with explicit limitation.

Theorem 7.1 (Partial effective exchange-law regularity). *In the frozen $N = 64$, $d = 16$, 24-seed ensemble over $k = 3, \dots, 6$, the embedded filament-core interaction system exhibits partial law-like regularity in exchange observables. Refined pre-flow susceptibility, edge-margin, core-response, and mechanical observables improve discrimination of high-exchange behavior and show recurrent rank-level association with exchange intensity. However, the same score family does not establish a full reconnection law or a universal effective interaction law.*

Proof. GEO-08 Stage 1 tests simple candidate scores and finds only weak partial discrimination. The hybrid reconnection AUC is 0.514115, hybrid reconnection gain is 0.011852, hybrid high-exchange AUC is 0.485112, hybrid high-exchange gain is -0.020113 , and best exchange-rank Spearman magnitude is 0.002466. Stage 1 passes only 5/9 gates with gate score 0.628373. Therefore simple scores do not define a full interaction law.

GEO-08 Stage 2 introduces refined pre-flow susceptibility, edge-margin, core-response, and mechanical scores. These improve exchange-related behavior: best refined high-exchange AUC reaches 0.573874, response high-exchange AUC is 0.558363, best exchange-rank Spearman magnitude is 0.144145, and seed-scale exchange-Spearman recurrence is 0.926316. Stage 2 passes 7/10 gates with gate score 0.755117. Therefore refined scores provide a partial law-like signal for exchange observables.

However, reconnection-law discrimination remains weak. Best refined reconnection AUC is 0.526464, best gain over null is 0.004032, stable-window k -pass fraction is 0.333333, and

$k = 5$ response gain is 0.001349. These fail the frozen reconnection-law thresholds. The final GEO-08 synthesis therefore passes 7/10 gates with score 0.693467 and freezes as partial. Thus the theorem holds only in the stated partial exchange-law sense, not as a full reconnection or universal interaction law. $\square$

7.8 Interpretation: law-like exchange, not universal interaction dynamics

The GEO-08 result clarifies the boundary between established interaction and effective law.

The following claims are supported:

1. simple candidate laws are insufficient;
2. refined pre-flow observables improve exchange-related discrimination;
3. high-exchange behavior has partial above-null predictability;
4. exchange intensity has recurrent rank-level association across seed-scale cells;
5. the strongest law-like signal concerns exchange, not reconnection.

The following claims are not supported:

1. a full reconnection law;
2. a universal effective interaction law;
3. a physical force law;
4. a scattering amplitude;
5. a gauge interaction;
6. a conserved charge;
7. a continuum interaction field equation;
8. physical string interaction dynamics.

Limitation 7.1 (No universal interaction law). GEO-08 establishes only partial exchange-law regularity. It does not establish a full reconnection law, a universal effective interaction law, a physical force law, a scattering law, a gauge interaction, a conserved charge, or a continuum interaction field equation.

The correct interpretation is therefore:

$$\text{core-mediated interaction} + \text{partial exchange-law regularity} \neq \text{full interaction law}.$$

The embedded filament-core network has real interaction structure. It also has a weak but reproducible exchange-related law-like signal. But reconnection remains a core-mediated reconfiguration phenomenon rather than an outcome governed by a closed scalar law in the present construction.

7.9 Summary of the interaction-law discrimination layer

The interaction-law discrimination layer can be summarized as

embedded filament-core interaction $\longrightarrow$ candidate law scores $\longrightarrow$ weak simple-law result
 $\longrightarrow$ refined exchange-law signal $\longrightarrow$ reconnection non-discrimination $\longrightarrow$ partial effective exchange

The frozen numerical core of this section is:

$|\mathcal{C}_0| = 15056,$
 transition count = 75280,
 active carriers = 13612,
 active episodes = 49217,
 Stage 1 hybrid reconnection AUC = 0.514115,
 Stage 1 hybrid high-exchange AUC = 0.485112,
 Stage 1 best exchange Spearman = 0.002466,
 Stage 2 best high-exchange AUC = 0.573874,
 Stage 2 response high-exchange AUC = 0.558363,
 Stage 2 best exchange Spearman = 0.144145,
 seed-scale exchange-Spearman recurrence = 0.926316,
 best refined reconnection AUC = 0.526464,
 best refined reconnection gain = 0.004032,
 stable-window k -pass fraction = 0.333333,
 $k = 5$ response gain = 0.001349,
 final GEO-08 gate score = 0.693467.

Together with the previous sections, this completes the main computational theorem chain:

spectral locality $\Rightarrow$ pre-string channels $\Rightarrow$ effective one-dimensionality $\Rightarrow$ dynamical filaments $\Rightarrow$ finite-graph m

The next section discusses why this chain should be interpreted as pre-string organizational emergence rather than as fundamental string theory or a completed physical interaction law.

8 Discussion

The preceding sections establish a frozen theorem chain from spectral locality to pre-string organizational channels, effective one-dimensionality, dynamical filament renewal, finite-graph mechanics, embedded filament-core interaction, and partial exchange-law regularity. The purpose of this discussion is to clarify the scientific meaning of that chain.

The central result is not that fundamental strings have been derived. It is also not that a physical interaction law, gauge force, scattering amplitude, or continuum string dynamics has

been obtained. The result is more primitive and more conservative: a finite spectral-locality construction generates recurrent, effectively one-dimensional, dynamically renewing, mechanically structured, core-coupled organizational filaments. These are *pre-string* sectors: string-like in their emergent organizational role, but not fundamental strings in the conventional sense.

The full frozen evidence chain is:

spectral locality $\Rightarrow$ pre-string channels $\Rightarrow$ effective one-dimensionality $\Rightarrow$ dynamical filaments $\Rightarrow$ finite-graph n

The chain is supported by the following stage verdicts:

GEO-02 : PASS / FROZEN,
 GEO-03 : PASS / FROZEN,
 GEO-04 : PASS / FROZEN,
 GEO-05 : PASS / FROZEN,
 GEO-06 : PASS / FROZEN,
 GEO-07 : PARTIAL / FROZEN,
 GEO-08 : PARTIAL / FROZEN.

The first five stages establish the strong core of the paper. The last two stages establish embedded interaction and partial exchange-law regularity, while also fixing the limitations of reconnection and universal interaction-law claims.

Discussion-level synthesis

The frozen audits support the emergence of a finite-graph pre-string filament sector from spectral locality. This sector is recurrent across seeds and scales, effectively one-dimensional, dynamically renewing, mechanically structured, embedded in a higher-dimensional organizational core, and partially law-like in exchange observables. The same audits do not support a claim of fundamental strings, physical string tension, a Nambu–Goto action, a worldsheet, a physical force law, or a universal reconnection law.

8.1 Why the result is pre-string rather than string theory

The terminology *pre-string* is meant to mark a precise conceptual boundary. The objects studied in this paper resemble strings only in the following restricted sense:

1. they are extended;
2. they are effectively one-dimensional;
3. they persist or renew under a dynamical process;
4. they support finite mechanical observables;
5. they interact through an ambient structure.

However, they are not strings in the standard physical sense. They are not introduced as fundamental degrees of freedom. They do not propagate on a background spacetime. They

do not sweep out a worldsheet. They are not governed by a Nambu–Goto or Polyakov action. They do not possess physical string tension, oscillator quantization, target-space embedding, conformal symmetry, or a perturbative scattering expansion.

The difference can be summarized as:

$$\text{fundamental string} \neq \text{pre-string organizational filament}.$$

A fundamental string is assumed as an extended object in a theoretical framework. A pre-string organizational filament is inferred from spectral locality as an emergent finite-graph sector.

Table 39: Conceptual distinction between fundamental strings and pre-string organizational filaments.

Feature	Fundamental string	Pre-string organizational filament
Ontological status	postulated object	emergent organizational sector
Background	target spacetime	spectral-locality graph
Geometry	worldsheet geometry	finite-graph path geometry
Action	Nambu–Goto/Polyakov-type	spectral finite-graph action proxy
Tension	physical parameter	tension proxy A/L
Modes	quantized string modes	finite stiffness-spectrum proxies
Interaction	string splitting/joining/scattering	core-mediated graph reconfiguration
Law status	theoretical dynamical law	partial exchange-law regularity only

The strongest reason to use the prefix “pre” is that the entire construction is prior to physical spacetime. The starting point is not a manifold but a set of normalized spectral states:

$$\lambda_i \mapsto k_i = -\log \lambda_i \mapsto d_{\text{spec}} \mapsto \mathcal{G}_k.$$

The resulting channels are therefore not curves in space. They are paths in an organizational graph induced by modular spectral proximity.

This is precisely the point. If string-like organization appears before spacetime, then string-like structure need not be fundamental. It may instead be a stable channel of spectral organization. The frozen audits show that such channels can exist in a reproducible finite setting.

Proposition 8.1 (Pre-string interpretation). *The objects established in this paper should be interpreted as pre-string organizational filaments rather than physical strings.*

Proof. The audited objects are finite graph paths constructed from spectral distances between modular coordinates. They have no postulated worldsheet, no physical target-space embedding, no Nambu–Goto or Polyakov action, no physical string tension, and no quantized string spectrum. Nevertheless, they are recurrent, effectively one-dimensional, dynamically renewing, mechanically structured, and core-coupled. Therefore the appropriate interpretation is pre-string organizational filament, not physical fundamental string. $\square$

8.2 Finite-graph mechanics versus Nambu–Goto dynamics

GEO-06 establishes effective finite-graph mechanics. This is one of the strongest positive results of the paper, but it is also one of the easiest to overinterpret. The mechanical observables are:

$$L(P) = \sum_{\alpha} \ell_{\alpha}(P),$$

$$A(P) = \sum_{\alpha} \ell_{\alpha}(P)^2,$$

$$\tau(P) = \frac{A(P)}{L(P)},$$

together with stiffness-density and mode-spectrum proxies obtained from a finite weighted path Laplacian.

The frozen numerical anchors are:

$$\begin{aligned} |\mathcal{C}_0| &= 15056, \\ \text{finite tension fraction} &= 1.000000, \\ \text{median spectral length} &= 12.069761, \\ \text{median spectral action} &= 44.124460, \\ \text{median tension proxy} &= 3.629348, \\ \text{finite spectrum fraction} &= 1.000000, \\ \text{median mode gap} &= 0.584116, \\ \text{median scaled mode gap} &= 1.647594, \\ \text{median stiffness density} &= 2.125080, \\ \text{finite dynamic mechanics fraction} &= 1.000000. \end{aligned}$$

Under controlled spectral flow, the median mechanical ratios remain near unity:

$$\begin{aligned} R_L &= 1.008466, \\ R_A &= 1.015823, \\ R_{\tau} &= 1.006086, \\ R_{\hat{\mu}} &= 0.996652, \\ R_s &= 0.999797. \end{aligned}$$

This establishes stable finite-graph mechanics.

However, the action $A(P)$ is not a continuum worldsheet area. The tension proxy $\tau(P)$ is not physical string tension. The stiffness matrix is not a continuum string operator. The mode spectrum is not a quantized physical string spectrum.

The precise interpretation is:

$$\text{finite-graph mechanics} \neq \text{physical string dynamics}.$$

The finite-graph mechanics is nevertheless scientifically meaningful because it shows that the pre-string filaments are not merely detected paths. They carry stable scalar and spectral observables that survive controlled flow.

Lemma 8.1 (Mechanical observables are effective, not fundamental). *The mechanical quantities established in GEO-06 are effective finite-graph observables and should not be identified with fundamental string-theoretic quantities.*

Proof. Each mechanical observable is defined from finite spectral edge lengths along a graph path. The action is a quadratic spectral path cost, the tension proxy is A/L , and the mode spectrum is obtained from a finite weighted path Laplacian. None of these definitions involves a continuum worldsheet, a target-space metric, a physical action principle, or canonical quantization. Therefore the observables are effective finite-graph quantities, not fundamental string-theoretic quantities. $\square$

This distinction strengthens rather than weakens the result. It prevents the claim from becoming artificially inflated. The article does not claim a derivation of string theory. It claims the emergence of a mechanically structured pre-string layer.

8.3 Embedded interaction versus physical force law

GEO-07 establishes that pre-string filaments are not isolated. This is a major structural result. The frozen anchors are:

interface fraction = 1.000000,
median leakage fraction = 0.897536,
boundary/path weight ratio = 8.759508,
core-bridge channel fraction = 0.997210,
filament pair count = 1903884,
interaction pair fraction = 0.998623,
shared-core pair fraction = 0.998623,
nonlocal interaction pair fraction = 0.592651,
interaction graph LCC fraction = 1.000000.

These numbers show that the filament population forms an embedded interaction system. Individual filaments have universal core interface. Filament pairs interact through shared core neighborhoods. Many interactions are nonlocal in the finite-graph sense: the median path-node Jaccard is

0.000000,

yet the interaction pair fraction is

0.998623.

Thus interaction is not caused by direct path overlap. It is mediated by the organizational core.

The important conceptual point is:

$$\text{core-mediated interaction} \neq \text{physical force law.}$$

A physical force law would require additional structure: physical observables, conserved charges, coupling constants, field equations, or scattering amplitudes. None of these are derived in GEO-07. The audit establishes finite-graph organizational interaction, not physical dynamics in spacetime.

Proposition 8.2 (Embedded interaction without physical force). *The filament-core coupling established in GEO-07 is an embedded organizational interaction, not a physical force law.*

Proof. GEO-07 establishes universal filament-core interface, strong boundary leakage, shared-core filament-pair interaction, nonlocal core-mediated interaction, and connected interaction graph structure. These are finite-graph relational properties. They do not define a physical field, conserved charge, coupling constant, scattering amplitude, or spacetime force. Therefore the result is embedded organizational interaction, not a physical force law. $\square$

This distinction matters for the overall interpretation of UMD. The emergence of interaction-like structure before physical fields suggests that physical interactions might require a deeper organizational substrate. But the present article does not yet derive that physical layer. It identifies a pre-dynamical, pre-field interaction structure.

8.4 Reconnection as core-mediated reconfiguration

Reconnection is the most delicate part of the analysis. GEO-07 shows that observed reconnections are core-mediated:

$$\text{core-mediated reconnection fraction} = 1.000000.$$

The finite reconnected mechanics fraction is also

$$1.000000.$$

The median reconnection mediation score is

$$0.911786,$$

with median shared-core count

$$20,$$

and bridge weight

$$0.890310.$$

These are strong positive results.

However, GEO-07 also freezes a reconnection/action-exchange limitation:

$$\text{reconnected episode fraction} = 0.232638 < 0.300000,$$

$$\begin{aligned}\text{nonlocal reconnection fraction} &= 0.046765 < 0.050000, \\ \text{reconnected/endpoint exchange ratio} &= 6.481297 > 2.500000, \\ k = 5 \text{ reconnected episode fraction} &= 0.217174 < 0.250000.\end{aligned}$$

Thus reconnection is present, core-mediated, and mechanically finite, but it is not a smooth universal continuation of endpoint persistence.

GEO-08 strengthens this interpretation. It tests whether pre-flow scores discriminate reconnection outcomes. They do not. The frozen reconnection-law limitations are:

$$\begin{aligned}\text{best refined reconnection AUC} &= 0.526464, \\ \text{best refined gain} &= 0.004032, \\ \text{stable-window } k\text{-pass fraction} &= 0.333333, \\ k = 5 \text{ response gain} &= 0.001349.\end{aligned}$$

These values are below the thresholds for reconnection-law discrimination.

The correct interpretation is therefore:

$$\begin{aligned}\text{reconnection} &= \text{core-mediated reconfiguration}, \\ \text{not} \\ \text{reconnection} &= \text{closed effective interaction law}.\end{aligned}$$

Lemma 8.2 (Reconnection non-discrimination). *Within the frozen audit, reconnection is established as core-mediated but not as a predictively discriminated law-like outcome.*

Proof. GEO-07 establishes that observed reconnections are core-mediated, with core-mediated reconnection fraction 1.000000. GEO-08 tests whether pre-flow scores discriminate reconnection outcomes and finds best refined reconnection AUC 0.526464, gain over null 0.004032, stable-window support 0.333333, and $k = 5$ response gain 0.001349. These values fail the reconnection-law thresholds. Therefore reconnection is core-mediated but not law-discriminated in the present construction. $\square$

This is a scientifically useful negative result. It prevents the interaction claim from exceeding the evidence. It also identifies the next target: a stronger analytic theory of reconnection would need observables beyond the tested scalar score families.

8.5 Partial exchange-law regularity

Although reconnection-law discrimination fails, GEO-08 does identify a partial exchange-law signal. This is the positive law-like result of the article.

The frozen anchors are:

$$\text{best refined high-exchange AUC} = 0.573874,$$

response high-exchange AUC = 0.558363,
best exchange-rank Spearman magnitude = 0.144145,
seed-scale exchange-Spearman recurrence = 0.926316.

At $k = 5$, the exchange signal is also visible:

refined high-exchange AUC = 0.566458,
response high-exchange AUC = 0.582627,
exchange Spearman = 0.155385,
response Spearman = 0.178481.

This means that the embedded filament-core system contains some pre-flow information about later mechanical exchange. The signal is not strong enough to qualify as a universal interaction law. But it is strong enough to show that the interaction structure is not purely descriptive.

The most accurate statement is:

partial exchange-law regularity exists,

but

full reconnection law does not follow.

Proposition 8.3 (Exchange-law regularity is weaker than an interaction law). *The exchange-law signal established in GEO-08 is a partial regularity of mechanical exchange observables and is weaker than a universal effective interaction law.*

Proof. The refined audit improves high-exchange discrimination to best AUC 0.573874 and exchange-rank Spearman magnitude 0.144145, with seed-scale recurrence 0.926316. These values establish partial regularity. However, the reconnection AUC remains 0.526464, gain over null is 0.004032, and stable-window support is 0.333333. Therefore the exchange-law signal is real but insufficient for a universal interaction law. $\square$

This distinction should be preserved throughout the article. The interaction is real at the graph-organizational level. The exchange signal is partially law-like. The reconnection law is not established.

8.6 Scale windows and overconnection

A recurring theme in the audits is finite scale dependence. The strongest pre-string signature appears in the window

$$k = 3, \dots, 6.$$

In GEO-03, the scale audit gives:

k	status	mean score
3	pass	0.917
4	pass	0.892
5	pass	0.846
6	pass	0.791
7	fail/mixed	0.567
8	fail/mostly none	0.473
9	fail	0.466

This behavior has a natural interpretation. At small enough k , the graph preserves local spectral structure and allows extended low-clustering channels to remain visible. At larger k , additional edges increasingly connect the graph into a denser core. This overconnection reduces the contrast between filament and core, absorbing the pre-string channels into a higher-connectivity structure.

The finite scale window is therefore not merely a computational artifact. It is part of the phenomenon. The pre-string sector is a scale-resolved organizational structure, not an all-scale invariant object.

Lemma 8.3 (Pre-string channels are scale-resolved structures). *The frozen audits support pre-string channels as scale-resolved organizational structures in the finite window $k = 3, \dots, 6$, not as unrestricted all-scale graph invariants.*

Proof. GEO-03 passes for $k = 3, 4, 5, 6$ and fails for $k = 7, 8, 9$, with mean scores decreasing from 0.917 at $k = 3$ to 0.466 at $k = 9$. This shows that the pre-string signature is resolved in a finite scale window and degrades under overconnection. Therefore the channels are scale-resolved structures. $\square$

This observation also explains why some later law-discrimination gates fail. Interaction-law stability is harder than channel detection because it must survive not only the existence of channels, but also the consistency of exchange/reconnection behavior across the finite scale window. GEO-08 confirms partial exchange recurrence but not stable-window reconnection-law support.

8.7 Role of numerical evidence

The article is not based on a single visual graph or isolated numerical observation. Its claim is cumulative. The central frozen numerical anchors are:

baseline channel count = 15056,
 seed-scale cells = 96,
 ensemble detection fraction = 0.958333,
 detected-or-partial fraction = 1.000000,
 motif seed-scale coverage = 0.916667,
 one-dimensional channel fraction = 1.000000,
 active carrier count = 13612,
 active episode count = 49217,
 finite dynamic mechanics fraction = 1.000000,
 filament-core interface fraction = 1.000000,
 filament interaction pair fraction = 0.998623,
 core-mediated reconnection fraction = 1.000000,
 seed-scale exchange-Spearman recurrence = 0.926316.

These numbers play different roles. Some establish existence:

15056 baseline channels.

Some establish recurrence:

0.958333 ensemble detection fraction.

Some establish dimensional character:

1.000000 one-dimensional channel fraction.

Some establish dynamics:

13612 active carriers, 49217 active episodes.

Some establish mechanics:

1.000000 finite dynamic mechanics fraction.

Some establish interaction:

0.998623 interaction pair fraction.

Some establish law-like partiality:

0.144145 exchange-rank Spearman magnitude, 0.926316 seed-scale exchange recurrence.

The combined role of the numerical evidence is to support a staged theorem chain. No single

number carries the entire conclusion. The conclusion is carried by the frozen sequence of gates, regressions, and partial verdicts.

Proposition 8.4 (Cumulative numerical support). *The pre-string conclusion is supported cumulatively by existence, recurrence, dimensionality, dynamical persistence, mechanical stability, embedded interaction, and partial law-discrimination evidence.*

Proof. Existence is supported by 15056 baseline channels. Recurrence is supported by detection fraction 0.958333, detected-or-partial fraction 1.000000, and motif seed–scale coverage 0.916667. Dimensionality is supported by one-dimensional channel fraction 1.000000. Dynamics is supported by 13612 active carriers and 49217 active episodes. Mechanics is supported by finite dynamic mechanics fraction 1.000000. Interaction is supported by interaction pair fraction 0.998623 and core-mediated reconnection fraction 1.000000. Law-like partiality is supported by exchange-rank recurrence 0.926316. Therefore the conclusion is cumulative across independent audit layers. $\square$

8.8 Implications for Universal Modular Dynamics

Within Universal Modular Dynamics, the result has a clear conceptual implication. It shows that extended filamentary organization can arise from the spectral layer before spacetime geometry is assumed. This supports the broader UMD idea that geometry and matter-like organization may emerge from modular spectral distinguishability rather than from primitive spatial objects.

The result also refines the role of the organizational core. The core is not merely a passive background. It supports the channels, couples to them, mediates filament-filament interaction, and mediates reconnection events. Thus the emergent pre-string sector is not separate from the surrounding modular organization. It is embedded in it.

The UMD-level chain suggested by the audits is:

distinguishability $\Rightarrow$ modular spectrum $\Rightarrow$ spectral locality $\Rightarrow$ organizational geometry $\Rightarrow$ pre-string filaments

This chain is compatible with the broader view that spatial and material structures are not fundamental inputs but stabilized organizational outputs.

The result also indicates where the present construction remains incomplete. To move from pre-string organizational filaments toward physical theory, one would need at least:

1. analytic control of the finite scale window;
2. stronger interaction-law candidates;
3. a continuum-limit or thermodynamic-limit analysis;
4. a relation between finite-graph action proxies and physical action principles;
5. identification of physical observables, if any, associated with the filament sector;
6. a bridge from core-mediated exchange to field-theoretic or particle-like descriptions.

These are future steps. They are not claimed in the present article.

UMD-level implication

The frozen audits support a UMD interpretation in which string-like sectors are not fundamental degrees of freedom but emergent organizational channels of modular spectral locality. The resulting pre-string filaments are finite-graph structures embedded in and mediated by a higher-dimensional organizational core.

8.9 Discussion summary

The discussion can be summarized by four statements.

First, the positive result is strong:

spectral locality generates pre-string filament sectors.

These sectors are recurrent, effectively one-dimensional, dynamically renewing, mechanically structured, and core-coupled.

Second, the result is pre-string, not string-theoretic:

pre-string filament $\neq$ fundamental physical string.

No worldsheet, Nambu–Goto action, physical string tension, target-space embedding, or quantized string spectrum is derived.

Third, the interaction result is embedded and organizational:

core-mediated interaction $\neq$ physical force law.

The filaments interact through the organizational core, but no gauge interaction, conserved charge, scattering amplitude, or continuum field equation is obtained.

Fourth, the law-like result is partial:

partial exchange-law regularity $\neq$ universal interaction law.

Exchange observables show reproducible partial regularity, but reconnection remains a core-mediated reconfiguration rather than a discriminated closed law.

These distinctions define the proper scope of the conclusion. The article establishes an emergent finite-graph pre-string sector from spectral locality. It does not claim a completed theory of physical strings or physical interactions.

9 Conclusion

This paper investigated whether string-like organizational sectors can emerge from spectral locality without postulating fundamental strings, a background spacetime, a worldsheet, or a prior geometric substrate. Within the finite Universal Modular Dynamics construction studied here, normalized spectral states were transformed into modular coordinates, spectral distances, symmetrized nearest-neighbor organizational graphs, recurrent pre-string channels, dynamically

renewing filaments, finite-graph mechanical observables, embedded filament-core interactions, and finally partial exchange-law regularity.

The main conclusion is that the tested spectral-locality construction does generate a robust finite-graph pre-string sector. The result is not a derivation of physical string theory. It is a more primitive statement: spectral locality can produce recurrent, effectively one-dimensional, dynamically renewing, mechanically structured, core-coupled organizational filaments. These filaments behave as pre-string sectors in the sense of emergent organizational geometry, not as fundamental strings in the sense of conventional string theory.

The final frozen theorem chain is:

spectral locality $\Rightarrow$ pre-string channels $\Rightarrow$ effective one-dimensionality $\Rightarrow$ dynamical filament network $\Rightarrow$ finite-

9.1 Frozen theorem chain

The computational and conceptual result of the paper is organized into seven frozen stages:

```
GEO-02 : BASELINE_PRE_STRING_DETECTION_PASS,
GEO-03 : ENSEMBLE_PRE_STRING_ORGANIZATIONAL_CHANNELS_PASS,
GEO-04 : ONE_DIMENSIONAL_PRE_STRING_CHANNELS_PASS,
GEO-05 : DYNAMICAL_PRE_STRING_FILAMENT_NETWORK_PASS,
GEO-06 : EFFECTIVE_FINITE_GRAPH_FILAMENT_MECHANICS_PASS,
GEO-07 : FILAMENT_CORE_COUPLED_INTERACTION_PARTIAL_PASS,
GEO-08 : PARTIAL_EFFECTIVE_EXCHANGE_LAW_SIGNAL.
```

The first five stages close as

PASS / FROZEN.

The last two close as

PARTIAL / FROZEN,

not because the underlying interaction disappears, but because reconnection/action-exchange universality and full interaction-law discrimination are not established.

The final stage synthesis closes as

FINAL SYNTHESIS: PASS / FROZEN,

with manuscript readiness

95.00%.

The principal theorem-level conclusions are as follows.

Theorem 9.1 (Final pre-string emergence theorem). *In the frozen $N = 64$, $d = 16$, 24-seed spectral ensemble over the finite scale window $k = 3, \dots, 6$, spectral locality generates a recurrent finite population of pre-string organizational channels. These channels form effective one-dimensional, dynamically renewing, mechanically structured filament sectors embedded in a higher-dimensional organizational core.*

Proof. The construction begins with normalized spectral states λ_i , modular coordinates $k_i = -\log \lambda_i$, spectral distances d_{spec} , and symmetrized organizational graphs $\mathcal{G}_k$. GEO-02 establishes baseline pre-string detection with 100 candidates, score 0.825694, and 7/7 gates passed. GEO-03 establishes recurrence across 24 seeds, with detection fraction 0.958333, detected-or-partial fraction 1.000000, finite signature window $k = 3, \dots, 6$, and final score 0.863458. GEO-04 establishes effective one-dimensionality across the full frozen channel population of 15056 channels, with one-dimensional channel fraction 1.000000, stable-window contrast $71/72 = 0.986111$, and final score 0.994293. GEO-05 establishes dynamical renewal under controlled spectral flow with 13612 active carriers and 49217 active episodes. GEO-06 establishes finite-graph mechanics with finite tension fraction 1.000000, finite spectrum fraction 1.000000, and finite dynamic mechanics fraction 1.000000. GEO-07 establishes embedded filament-core interaction with interface fraction 1.000000 and interaction pair fraction 0.998623. Therefore the frozen ensemble supports the stated pre-string emergence theorem. $\square$

Theorem 9.2 (Final interaction limitation theorem). *The frozen pre-string filament population forms an embedded core-mediated interaction system with partial exchange-law regularity, but the present construction does not establish a full reconnection law or a universal physical interaction law.*

Proof. GEO-07 establishes strong embedded interaction: filament-core interface fraction 1.000000, median leakage fraction 0.897536, filament pair interaction fraction 0.998623, nonlocal interaction pair fraction 0.592651, and core-mediated reconnection fraction 1.000000. However, GEO-07 also freezes reconnection/action-exchange limitations: reconnected episode fraction $0.232638 < 0.300000$, nonlocal reconnection fraction $0.046765 < 0.050000$, reconnected/endpoint exchange ratio $6.481297 > 2.500000$, and $k = 5$ reconnected episode fraction $0.217174 < 0.250000$. GEO-08 then shows partial exchange-law regularity, with best refined high-exchange AUC 0.573874, best exchange-rank Spearman magnitude 0.144145, and seed-scale exchange-Spearman recurrence 0.926316. But reconnection-law discrimination remains weak: best refined reconnection AUC 0.526464, gain over null 0.004032, stable-window support 0.333333, and $k = 5$ response gain 0.001349. Therefore embedded interaction and partial exchange-law regularity are established, while full reconnection and universal interaction-law claims are not. $\square$

9.2 Numerical summary of the evidence

The frozen numerical base of the article is summarized by the following anchors:

$$\begin{aligned}
N &= 64, \\
d &= 16, \\
\text{seeds} &= 24, \\
k\text{-window} &= \{3, 4, 5, 6\}, \\
\text{seed-scale cells} &= 96, \\
\text{baseline channel count} &= 15056.
\end{aligned}$$

The emergence and recurrence layer is supported by:

GEO-02 baseline candidates = 100,
GEO-02 detection score = 0.825694,
GEO-02 gates passed = 7/7,
GEO-03 detected seeds = 23/24,
GEO-03 partial seeds = 1/24,
GEO-03 non-detections = 0/24,
ensemble detection fraction = 0.958333,
detected-or-partial fraction = 1.000000,
mean detection score = 0.791310,
median detection score = 0.804230,
motif seed coverage = 24/24,
motif seed-scale coverage = 88/96 = 0.916667.

The effective one-dimensionality layer is supported by:

channel count = 15056,
seed-scale coverage = 96/96,
median intrinsic dimension = 1.0,
one-dimensional channel fraction = 1.000000,
median ambient relative dimension = 3.0,
stable-window contrast coverage = 71/72 = 0.986111,
GEO-04 final score = 0.994293.

The dynamical filament layer is supported by:

baseline channels = 15056,
flow steps = 6,
transition count = 75280,
active carriers = 13612,
active carrier fraction = 0.904091,
active episodes = 49217,
active episode fraction = 0.653786,
endpoint episode fraction = 0.421148,
reconnected episode fraction = 0.232638,
decayed episode fraction = 0.346214,
rigid fraction = 0.097635,
dynamical mode entropy = 0.974802,
GEO-05 final score = 0.896787.

The finite-graph mechanics layer is supported by:

finite tension fraction = 1.000000,
median hop length = 3,
median spectral length = 12.069761,
median spectral action = 44.124460,
median tension proxy = 3.629348,
tension coefficient of variation = 0.189067,
finite spectrum fraction = 1.000000,
median mode gap = 0.584116,
median scaled mode gap = 1.647594,
median stiffness density = 2.125080,
finite dynamic mechanics fraction = 1.000000,
combined mechanics stability = 0.872177,
GEO-06 final score = 0.867671.

The filament-core interaction layer is supported by:

interface fraction = 1.000000,
 median leakage fraction = 0.897536,
 boundary/path weight ratio = 8.759508,
 core-bridge channel fraction = 0.997210,
 nonlocal bridge fraction = 0.632505,
 filament pair count = 1903884,
 interaction pair fraction = 0.998623,
 shared-core pair fraction = 0.998623,
 nonlocal interaction pair fraction = 0.592651,
 interaction graph LCC fraction = 1.000000,
 core-mediated reconnection fraction = 1.000000,
 finite reconnected mechanics fraction = 1.000000.

The limitation and partial law layer is supported by:

reconnected episode fraction = 0.232638,
 nonlocal reconnection fraction = 0.046765,
 reconnected/endpoint exchange ratio = 6.481297,
 best refined high-exchange AUC = 0.573874,
 response high-exchange AUC = 0.558363,
 best exchange-rank Spearman magnitude = 0.144145,
 seed-scale exchange-Spearman recurrence = 0.926316,
 best refined reconnection AUC = 0.526464,
 best refined reconnection gain = 0.004032,
 stable-window law support = 0.333333,
 GEO-08 final score = 0.693467.

These values define the quantitative backbone of the paper. The claim is not based on a single observation, but on a staged frozen chain of detection, recurrence, dimensionality, dynamics, mechanics, coupling, and law-discrimination tests.

9.3 Scientific interpretation

The scientific interpretation of the result can be separated into three levels.

First, the strong established level:

spectral locality $\Rightarrow$ recurrent pre-string channels.

This is supported by GEO-02 and GEO-03. The channels are not arbitrary graph paths. They

recur across seeds, survive a finite scale window, and exhibit motif-level structure.

Second, the structural-dynamical level:

pre-string channels $\Rightarrow$ dynamical finite-graph filaments.

This is supported by GEO-04, GEO-05, and GEO-06. The channels are effectively one-dimensional, dynamically renewing, and mechanically structured. They are not rigid permanent strings; their identity is carried by endpoint persistence, reconnection, decay, recovery, and active representatives under controlled spectral flow.

Third, the interaction level:

filaments $\Rightarrow$ embedded core-mediated interaction.

This is supported by GEO-07 and qualified by GEO-08. The filaments are embedded in a higher-dimensional organizational core and interact through shared core neighborhoods. Reconnection is core-mediated when it occurs. Exchange observables exhibit partial law-like regularity. But a full reconnection law and a universal effective interaction law are not established.

The most compact final statement is therefore:

Spectral locality generates emergent pre-string filament sectors with finite-graph mechanics and embedded co

The required qualification is:

The construction supports partial exchange-law regularity, but not a full reconnection law or physical string

9.4 What has not been claimed

The conservative scope of the paper is essential. The following claims are not made:

1. that fundamental strings have been derived;
2. that a physical string tension has been obtained;
3. that a Nambu–Goto or Polyakov action has been derived;
4. that a continuum worldsheet has been constructed;
5. that quantized physical string modes have been obtained;
6. that a target spacetime has been assumed or derived;
7. that a physical force law has been obtained;
8. that a gauge interaction has been derived;
9. that a conserved charge has been identified;
10. that a scattering amplitude has been computed;
11. that a universal reconnection law has been established;

12. that a continuum interaction field equation has been derived.

The article instead establishes a finite-graph pre-string layer:

emergent spectral organizational mechanical core-coupled.

This is a weaker claim than physical string theory, but it is also more primitive. It shows that string-like sectors can appear as organizational consequences of spectral locality before physical string assumptions are introduced.

9.5 Outlook

The present work naturally leads to several next steps.

First, the finite scale window

$$k = 3, \dots, 6$$

should be studied analytically. The failure at larger k -values suggests that pre-string channels are scale-resolved structures that disappear under overconnection. A future analytic theory should explain when and why such windows appear.

Second, the partial exchange-law signal should be strengthened. GEO-08 shows that refined pre-flow observables partially predict high-exchange behavior, but reconnection-law discrimination remains weak. Stronger candidates may require non-scalar observables, local core topology, temporal memory, higher-order motifs, or operator-level flow quantities.

Third, the continuum or large- N behavior remains open. The present article is finite-graph based. A stronger physical interpretation would require studying whether the pre-string channel sector survives systematic limits in N , spectral dimension, ensemble family, and flow structure.

Fourth, the relation between finite-graph mechanical observables and physical action principles remains to be developed. GEO-06 establishes stable spectral length, action, tension proxy, stiffness density, and mode-spectrum proxies. A future theory would need to determine whether any of these quantities converge toward continuum mechanical or field-theoretic objects.

Fifth, the core-mediated interaction layer may provide a route toward emergent interactions in UMD. GEO-07 shows that filaments interact through the organizational core before any physical force law is postulated. This suggests that physical interactions, if derived in a later stage, may arise from deeper modular coupling structures.

9.6 Final statement

The final result of the paper is a conservative but nontrivial emergence theorem. Spectral locality alone, applied to finite modular spectral states, can generate recurrent pre-string organizational channels. These channels are effectively one-dimensional, dynamically renewing, mechanically structured, and embedded in a higher-dimensional organizational core. They interact through that core and exhibit partial exchange-law regularity.

The frozen final synthesis closes as:

FINAL SYNTHESIS: PASS / FROZEN.

The manuscript-ready claim is:

Spectral locality can generate recurrent, effectively one-dimensional, dynamically renewing

The frozen limitation is:

The present construction supports partial exchange-law regularity, but not a full reconnect.

This is the precise result established by the article.

10 Computational Protocol

This appendix specifies the frozen computational protocol used in the GEO-02 through GEO-08 audits. The purpose of the protocol is to make explicit how spectral states are generated, how organizational graphs are constructed, how pre-string channels are extracted, how controlled spectral flow is applied, how finite-graph mechanical and coupling observables are computed, and how frozen validation gates determine the final stage verdicts.

The protocol is finite, reproducible, and deliberately conservative. It does not assume a background spacetime, a continuum metric, a worldsheet, a physical string action, or a physical force law. All structures are computed from normalized spectral states and their modular-coordinate relations.

The frozen global configuration is:

$$N = 64, \quad d = 16, \quad k \in \{3, 4, 5, 6\}, \quad 24 \text{ seeds.}$$

Thus the full audit contains

$$24 \times 4 = 96$$

seed-scale cells. The frozen baseline channel population extracted from this protocol is

$$|\mathcal{C}_0| = 15056.$$

Appendix-level protocol anchor

The computational protocol defines a deterministic pipeline:

seeds $\rightarrow$ spectral states $\rightarrow$ modular coordinates $\rightarrow$ spectral distances $\rightarrow$ organizational graphs $\rightarrow$ pre-string

The frozen final synthesis validates this pipeline with

FINAL SYNTHESIS: PASS / FROZEN.

10.1 Frozen ensemble and seeds

The frozen ensemble consists of 24 independent pseudo-random seeds. Each seed generates one spectral ensemble of $N = 64$ normalized spectral states in dimension $d = 16$. Each seed is

evaluated over the finite organizational scale window

$$k = 3, 4, 5, 6.$$

The frozen seed list is:

31700033, 31700034, 31700035, 31700036, 31700037,
 41170021, 41170022, 41170023, 41170024,
 73001991, 73001992, 73001993, 73001994,
 99173003, 99173004, 99173005, 99173006,
 12031007, 15042011, 18052017, 21062023,
 24072029, 27083031, 30093037.

Table 40: Frozen ensemble configuration.

Quantity	Frozen value
Number of seeds	24
States per seed	$N = 64$
Spectral dimension	$d = 16$
Scale window	$k = 3, 4, 5, 6$
Seed-scale cells	96
Baseline channel count	15056
Primary flow steps	6
Noninitial transitions	75280
Active carriers under flow	13612
Active episodes under flow	49217

Protocol 10.1 (Frozen ensemble protocol). For each seed s in the frozen seed list:

1. initialize the pseudo-random generator with seed s ;
2. generate $N = 64$ spectral states in dimension $d = 16$;
3. construct modular coordinates;
4. build organizational graphs for $k = 3, 4, 5, 6$;
5. extract pre-string channels;
6. evaluate geometric, dynamical, mechanical, coupling, and law-discrimination metrics.

The use of a fixed seed list is essential. It prevents post-hoc selection of favorable realizations and allows every later stage to be checked against the same baseline population.

10.2 Spectral state generation

For each seed, a finite spectral ensemble is generated as a collection of normalized positive vectors:

$$\lambda_i = (\lambda_{i1}, \dots, \lambda_{id}), \quad i = 1, \dots, N, \quad d = 16.$$

The raw entries are drawn as positive random values:

$$x_{ia} > 0, \quad a = 1, \dots, d.$$

The normalized spectrum is then

$$\lambda_{ia} = \frac{x_{ia}}{\sum_{b=1}^d x_{ib}}.$$

Therefore

$$\lambda_{ia} > 0, \quad \sum_{a=1}^d \lambda_{ia} = 1.$$

The modular coordinate vector is computed by

$$k_{ia} = -\log \lambda_{ia}.$$

In numerical implementation, a small cutoff ϵ is used only as a protection against numerical underflow:

$$k_{ia} = -\log(\max(\lambda_{ia}, \epsilon)).$$

Since the generated values are positive before normalization, this cutoff does not change the conceptual construction.

Definition 10.1 (Generated spectral state). A generated spectral state is a normalized positive vector

$$\lambda_i \in \mathbb{R}_{>0}^{16}, \quad \sum_{a=1}^{16} \lambda_{ia} = 1.$$

Its modular-coordinate representation is

$$k_i = -\log \lambda_i.$$

Protocol 10.2 (Spectral state generation). For each seed s :

1. generate $N = 64$ raw positive vectors $x_i \in \mathbb{R}_{>0}^{16}$;
2. normalize each vector:

$$\lambda_i = x_i / \sum_a x_{ia};$$

3. compute modular coordinates:

$$k_i = -\log \lambda_i;$$

4. store the spectral ensemble

$$\mathcal{S}_s = \{(\lambda_i, k_i)\}_{i=1}^{64}.$$

The construction is intentionally minimal. No geometric coordinates, spatial embedding, metric tensor, or physical field variables are assigned to the states.

10.3 Graph construction

For each spectral ensemble, pairwise spectral distances are computed in modular-coordinate space:

$$d_{\text{spec}}(\sigma_i, \sigma_j) = \|k_i - k_j\|_2 = \left(\sum_{a=1}^{16} (k_{ia} - k_{ja})^2 \right)^{1/2}.$$

For each $k \in \{3, 4, 5, 6\}$, the directed k -nearest-neighbor relation is first computed:

$$\sigma_j \in \mathcal{N}_k(\sigma_i) \iff \sigma_j \text{ is among the } k \text{ nearest states to } \sigma_i \text{ under } d_{\text{spec}}.$$

This relation is then symmetrized. The organizational graph is

$$\mathcal{G}_{s,k} = (\mathcal{V}_s, \mathcal{E}_{s,k}),$$

where

$$\mathcal{V}_s = \{\sigma_1, \dots, \sigma_{64}\},$$

and

$$\{\sigma_i, \sigma_j\} \in \mathcal{E}_{s,k} \iff \sigma_j \in \mathcal{N}_k(\sigma_i) \text{ or } \sigma_i \in \mathcal{N}_k(\sigma_j).$$

Protocol 10.3 (Organizational graph construction). For each seed s and each $k \in \{3, 4, 5, 6\}$:

1. compute the full pairwise spectral distance matrix

$$D_{ij} = d_{\text{spec}}(\sigma_i, \sigma_j);$$

2. for each vertex σ_i , select the k nearest distinct vertices;
3. form the directed neighborhood relation;
4. symmetrize the relation to obtain the undirected edge set;
5. store $\mathcal{G}_{s,k}$.

The representative frozen GEO-02 graph at $k = 6$ has:

$$|\mathcal{V}| = 64, \quad |\mathcal{E}| = 308, \quad \langle d \rangle = 9.625,$$

and is connected:

$$c(\mathcal{G}) = 1.$$

It has no articulation points and no bridges:

$$n_{\text{articulation}} = 0, \quad n_{\text{bridge}} = 0.$$

This confirms that the baseline organizational graph is not a fragile tree-like object.

10.4 Channel extraction protocol

Pre-string channels are extracted as extended low-clustering path-like structures in the organizational graph. Let

$$P = (v_0, \dots, v_m)$$

be a simple path in $\mathcal{G}_{s,k}$. Its hop length is

$$h(P) = m.$$

The frozen minimum hop criterion is

$$h(P) \geq 3.$$

For an internal vertex v , let $C(v)$ be its local clustering coefficient. A path is classified as low-clustering if its internal clustering score satisfies the frozen threshold condition:

$$C_{\text{path}}(P) < 0.25.$$

The exact path score used in the implementation is evaluated consistently across all seeds and k -values; the threshold value

$$0.25$$

is frozen.

Definition 10.2 (Frozen channel candidate). A frozen pre-string channel candidate is a simple path P in $\mathcal{G}_{s,k}$ satisfying:

1. $h(P) \geq 3$;
2. P is low-clustering under the frozen threshold 0.25;
3. P contributes to a recurrent channel family under the ensemble and motif audits.

The channel extraction protocol also evaluates collective-mode alignment. In the representative GEO-02 graph, the Fiedler partition is

$$35/29,$$

the conductance is

$$0.266187,$$

and the low-cluster cross fraction is

$$0.757576.$$

Thus low-clustering channels align with a collective graph separation mode.

Protocol 10.4 (Channel extraction). For each graph $\mathcal{G}_{s,k}$:

1. enumerate candidate simple paths satisfying the minimum hop condition;
2. compute internal clustering scores;

3. retain low-clustering extended paths;
4. compute collective-mode alignment and Fiedler-crossing support;
5. deduplicate and classify channel candidates;
6. store the resulting channel set $\mathcal{C}_{s,k}$.

The frozen channel population is:

$$\mathcal{C}_0 = \bigcup_{s,k} \mathcal{C}_{s,k}, \quad |\mathcal{C}_0| = 15056.$$

The scale-level channel counts are:

k	channels
3	4725
4	5375
5	3843
6	1113

and therefore

$$4725 + 5375 + 3843 + 1113 = 15056.$$

Table 41: Frozen channel counts by scale.

k	Frozen channel count
3	4725
4	5375
5	3843
6	1113
Total	15056

The channel extraction protocol is validated by the GEO-02 and GEO-03 audits:

$$\text{GEO-02 baseline score} = 0.825694,$$

$$\text{GEO-03 detection fraction} = 0.958333,$$

$$\text{motif seed-scale coverage} = 88/96 = 0.916667.$$

10.5 Controlled spectral flow

Controlled spectral flow is used to test whether baseline pre-string channels remain dynamically meaningful under spectral deformation. For each seed and each flow step, the spectral ensemble is deformed, renormalized, and converted again into modular coordinates.

Let

$$s_t \in [0, 1], \quad t = 0, \dots, 5,$$

denote the six sampled flow steps. The baseline step is $s_0 = 0$. At each step,

$$\lambda_i(0) \mapsto \lambda_i(s_t),$$

with

$$\sum_a \lambda_{ia}(s_t) = 1, \quad \lambda_{ia}(s_t) > 0.$$

The modular coordinates and organizational graph are recomputed:

$$k_i(s_t) = -\log \lambda_i(s_t),$$

$$\mathcal{G}_{s,k}(s_t) = \mathcal{G}_k(\{\lambda_i(s_t)\}_{i=1}^{64}).$$

The channel set at flow step s_t is

$$\mathcal{C}_{s,k}(s_t).$$

The frozen flow amplitude is

$$0.12,$$

and the normalization error is at machine precision:

$$\approx 4.44 \times 10^{-16}.$$

Protocol 10.5 (Controlled spectral flow). For each seed s , scale k , and flow step s_t :

1. deform the baseline spectra by the frozen controlled-flow rule;
2. renormalize each spectrum;
3. recompute modular coordinates;
4. reconstruct the organizational graph;
5. re-extract channel representatives;
6. match baseline channels to active representatives;
7. classify each baseline channel state as endpoint-persistent, reconnected, or decayed.

A baseline channel has five noninitial flow opportunities, because the flow has six steps. Therefore the total frozen transition count is

$$15056 \times 5 = 75280.$$

The frozen dynamical population is:

$$\text{active carriers} = 13612,$$

$$\text{active episodes} = 49217.$$

The active carrier fraction is

$$0.904091,$$

and the active episode fraction is

$$0.653786.$$

The dynamical state fractions are:

$$f_{\text{endpoint}} = 0.421148,$$

$$f_{\text{reconnected}} = 0.232638,$$

$$f_{\text{decayed}} = 0.346214.$$

These quantities are used in GEO-05, GEO-06, GEO-07, and GEO-08.

Table 42: Frozen controlled-flow dynamical population.

Quantity	Frozen value
Baseline channels	15056
Flow steps	6
Noninitial transitions	75280
Active carriers	13612
Active-carrier fraction	0.904091
Active episodes	49217
Active-episode fraction	0.653786
Endpoint episode fraction	0.421148
Reconnected episode fraction	0.232638
Decayed episode fraction	0.346214
Rigid fraction	0.097635
Dynamical mode entropy	0.974802

10.6 Mechanical observables

For each filament path

$$P = (v_0, \dots, v_m),$$

the spectral edge length is

$$\ell_\alpha(P) = d_{\text{spec}}(v_{\alpha-1}, v_\alpha), \quad \alpha = 1, \dots, m.$$

The spectral length is

$$L(P) = \sum_{\alpha=1}^m \ell_\alpha(P).$$

The spectral action is

$$A(P) = \sum_{\alpha=1}^m \ell_\alpha(P)^2.$$

The tension proxy is

$$\tau(P) = \frac{A(P)}{L(P)}.$$

The finite-graph stiffness matrix is a weighted path Laplacian. With edge stiffness

$$\kappa_\alpha(P) = \frac{\tau(P)}{\ell_\alpha(P)},$$

the stiffness matrix M_P is defined by:

$$(M_P)_{\alpha-1,\alpha-1} \leftarrow (M_P)_{\alpha-1,\alpha-1} + \kappa_\alpha,$$

$$(M_P)_{\alpha,\alpha} \leftarrow (M_P)_{\alpha,\alpha} + \kappa_\alpha,$$

$$(M_P)_{\alpha-1,\alpha} = (M_P)_{\alpha,\alpha-1} = -\kappa_\alpha.$$

The positive spectrum of M_P defines the mode-spectrum proxy:

$$0 = \mu_0(P) \leq \mu_1(P) \leq \dots \leq \mu_m(P).$$

The mode gap is

$$\mu_1(P),$$

the scaled gap is

$$\hat{\mu}_1(P) = \frac{\mu_1(P)m^2}{\tau(P)},$$

and the stiffness density is

$$s(P) = \frac{1}{m} \sum_{\mu_j(P) > 0} \mu_j(P).$$

Protocol 10.6 (Mechanical observable computation). For each filament representative P :

1. compute spectral edge lengths $\ell_\alpha(P)$;
2. compute spectral length $L(P)$;
3. compute spectral action $A(P)$;
4. compute tension proxy $\tau(P) = A(P)/L(P)$;
5. construct the finite weighted path Laplacian M_P ;
6. compute the positive mode spectrum;
7. store mode gap, scaled gap, stiffness density, and mode entropy.

The frozen mechanical anchors are:

$$\text{finite tension fraction} = 1.000000,$$

$$\text{median spectral length} = 12.069761,$$

median spectral action = 44.124460,
 median tension proxy = 3.629348,
 finite spectrum fraction = 1.000000,
 median mode gap = 0.584116,
 median scaled mode gap = 1.647594,
 median stiffness density = 2.125080.

Under controlled spectral flow, dynamic mechanical ratios are computed between baseline and active representatives. The frozen median ratios are:

$$\begin{aligned}
 R_L &= 1.008466, \\
 R_A &= 1.015823, \\
 R_\tau &= 1.006086, \\
 R_{\hat{\mu}} &= 0.996652, \\
 R_s &= 0.999797.
 \end{aligned}$$

The finite dynamic mechanics fraction is

$$1.000000.$$

10.7 Filament-core coupling metrics

For a filament path P , let $V(P)$ be its vertex set. The filament-core boundary is the set of edges connecting the path to vertices outside the path:

$$\partial_{\text{core}}P = \{\{u, v\} \in \mathcal{E}_{s,k} : u \in V(P), v \notin V(P)\}.$$

The filament has core interface if

$$|\partial_{\text{core}}P| > 0.$$

Let $W_{\text{path}}(P)$ be the total weight of path-internal edges and $W_{\text{bdry}}(P)$ the total weight of boundary edges. The boundary leakage fraction is

$$\Lambda_{\text{core}}(P) = \frac{W_{\text{bdry}}(P)}{W_{\text{bdry}}(P) + W_{\text{path}}(P)}.$$

For two filaments P and Q , their path overlap is measured by

$$J_{\text{path}}(P, Q) = \frac{|V(P) \cap V(Q)|}{|V(P) \cup V(Q)|}.$$

Their shared-core neighborhood is

$$\mathcal{B}(P) \cap \mathcal{B}(Q),$$

where

$$\mathcal{B}(P) = \{u \notin V(P) : \exists v \in V(P), \{u, v\} \in \mathcal{E}_{s,k}\}.$$

Protocol 10.7 (Filament-core coupling audit). For each baseline filament P :

1. compute the core boundary $\partial_{\text{core}}P$;
2. compute interface indicator;
3. compute boundary edge count and boundary weight;
4. compute leakage fraction;
5. compute endpoint core exposure;
6. compute core-bridge support.

For each filament pair (P, Q) :

1. compute path-node Jaccard overlap;
2. compute shared-core neighborhood;
3. compute bridge-weight coupling;
4. classify core-mediated interaction;
5. construct the filament interaction graph.

The frozen GEO-07 Stage 1 coupling anchors are:

$$\text{interface fraction} = 1.000000,$$

$$\text{median leakage} = 0.897536,$$

$$\text{boundary/path weight ratio} = 8.759508,$$

$$\text{core-bridge channel fraction} = 0.997210,$$

$$\text{nonlocal bridge fraction} = 0.632505.$$

The frozen GEO-07 Stage 2 interaction anchors are:

$$\text{filament pair count} = 1903884,$$

$$\text{interaction pair fraction} = 0.998623,$$

$$\text{shared-core pair fraction} = 0.998623,$$

$$\text{nonlocal interaction pair fraction} = 0.592651,$$

$$\text{nonlocal interaction given nonlocality} = 0.997822,$$

$$\text{median path-node Jaccard} = 0.000000,$$

interaction graph LCC fraction = 1.000000.

The frozen reconnection coupling anchors are:

core-mediated reconnection fraction = 1.000000,

finite reconnected mechanics fraction = 1.000000,

median reconnection mediation score = 0.911786.

10.8 Interaction-law discrimination metrics

The interaction-law discrimination audit tests whether pre-flow observables predict later interaction outcomes. The baseline filament population remains

$$|\mathcal{C}_0| = 15056,$$

with active population

$$13612$$

carriers and

$$49217$$

episodes.

For each baseline filament P , the audit defines:

$$R(P) \in \{0, 1\},$$

where $R(P) = 1$ indicates reconnection under the frozen matching rule;

$$H(P) \in \{0, 1\},$$

where $H(P) = 1$ indicates high-exchange behavior; and

$$E(P) \geq 0,$$

the scalar exchange intensity.

A candidate law score is a pre-flow scalar:

$$S(P).$$

It may be core-oriented, mechanical, flow-oriented, edge-oriented, coupled, refined, response-oriented, or null.

For binary outcomes, discrimination is measured by AUC:

$$\text{AUC}(S, R), \quad \text{AUC}(S, H).$$

For scalar exchange intensity, rank association is measured by Spearman correlation:

$$\rho_S(S, E).$$

Gain over null is

$$\Delta_{\text{null}}(S, Y) = \text{AUC}(S, Y) - \text{AUC}(S_{\text{null}}, Y).$$

Protocol 10.8 (Interaction-law discrimination). For each baseline filament P :

1. compute pre-flow candidate scores $S(P)$;
2. determine reconnection outcome $R(P)$;
3. determine high-exchange outcome $H(P)$;
4. compute exchange intensity $E(P)$;
5. compute AUC for reconnection;
6. compute AUC for high exchange;
7. compute Spearman correlation with exchange intensity;
8. compute gain over null;
9. aggregate results over seed-scale cells.

GEO-08 Stage 1 tests simple candidate laws and freezes as weak partial:

$$\text{hybrid reconnection AUC} = 0.514115,$$

$$\text{hybrid high-exchange AUC} = 0.485112,$$

$$\text{best exchange Spearman} = 0.002466.$$

GEO-08 Stage 2 tests refined law candidates and obtains partial exchange-law support:

$$\text{best refined high-exchange AUC} = 0.573874,$$

$$\text{response high-exchange AUC} = 0.558363,$$

$$\text{best exchange Spearman} = 0.144145,$$

$$\text{seed-scale exchange-Spearman recurrence} = 0.926316.$$

The reconnection-law limitations are:

$$\text{best refined reconnection AUC} = 0.526464,$$

$$\text{best refined gain} = 0.004032,$$

$$\text{stable-window support} = 0.333333,$$

$$k = 5 \text{ response gain} = 0.001349.$$

Thus GEO-08 establishes:

partial exchange-law regularity,

but not:

full reconnection law.

10.9 Validation and frozen-gate logic

Each stage is evaluated by frozen gates. A gate is a predeclared numerical or logical condition. A stage verdict is assigned only after all required values are computed and compared with their thresholds.

The gate outcomes are classified as:

PASS, PARTIAL, WEAK PARTIAL, FAIL.

Definition 10.3 (Frozen gate). A frozen gate is a predetermined validation condition of the form

$$M \geq \theta, \quad M \leq \theta, \quad M = \theta,$$

or a Boolean regression condition. The metric M , threshold θ , and interpretation are fixed before the final stage verdict is assigned.

Definition 10.4 (Frozen regression). A frozen regression is a hard consistency check requiring that a later stage reproduce the baseline population, seed-scale count, transition count, or prior-stage result exactly. Failure of a hard frozen regression invalidates the stage as evidence.

The most important hard regressions are:

$$|\mathcal{C}_0| = 15056,$$

$$\text{seed-scale cells} = 96,$$

$$\text{transition count} = 75280,$$

$$\text{active carriers} = 13612,$$

$$\text{active episodes} = 49217.$$

The final stage verdicts are:

GEO-02 : PASS / FROZEN,
 GEO-03 : PASS / FROZEN,
 GEO-04 : PASS / FROZEN,
 GEO-05 : PASS / FROZEN,
 GEO-06 : PASS / FROZEN,
 GEO-07 : PARTIAL / FROZEN,
 GEO-08 : PARTIAL / FROZEN.

Table 43: Frozen validation summary by audit stage.

Stage	Final verdict	Result	Gate score
GEO-02	baseline pre-string detection	PASS / FROZEN	0.825694 detection score
GEO-03	ensemble recurrence	PASS / FROZEN	0.863458
GEO-04	one-dimensionality	PASS / FROZEN	0.994293
GEO-05	dynamical filament network	PASS / FROZEN	0.896787
GEO-06	finite-graph mechanics	PASS / FROZEN	0.867671
GEO-07	core-mediated interaction	PARTIAL / FROZEN	0.902049
GEO-08	partial exchange-law signal	PARTIAL / FROZEN	0.693467

The final synthesis validation checks the consistency of the entire theorem chain:

GEO-02, ..., GEO-08 $\longrightarrow$ FINAL SYNTHESIS.

The final synthesis closes as:

Final synthesis: PASS,

Manuscript readiness: 95.00%,

Validation: PASS.

Theorem 10.1 (Protocol reproducibility theorem). *Under the frozen seed list, spectral dimension $d = 16$, state count $N = 64$, scale window $k = 3, \dots, 6$, channel extraction rules, controlled-flow protocol, mechanical observables, coupling metrics, law-discrimination metrics, and frozen-gate logic described in this appendix, the audit chain GEO-02 through GEO-08 yields the validated final theorem chain reported in the main text.*

Proof. The appendix specifies all structural inputs of the finite protocol: the seed list, spectral state generation, modular-coordinate construction, spectral distance, symmetrized k -nearest-neighbor graph construction, channel extraction rules, controlled spectral flow, mechanical observables, coupling metrics, interaction-law metrics, and frozen validation gates. The frozen regression anchors are reproduced across stages: 15056 baseline channels, 96 seed-scale cells, 75280 transitions, 13612 active carriers, and 49217 active episodes. Each stage closes with a

frozen verdict. The final synthesis validates the stage chain and closes as PASS. Therefore the reported theorem chain is reproducible under the frozen protocol. $\square$

11 Frozen Numerical Anchors and Stage Registry

This appendix collects the frozen numerical anchors, stage verdicts, manuscript-ready claims, limitations, and output registry used throughout the paper. Its purpose is to make the evidential structure of the article explicit and auditable.

The paper’s main claim is not supported by a single numerical observation. It is supported by a frozen sequence of computational stages:

GEO-02 $\rightarrow$ GEO-03 $\rightarrow$ GEO-04 $\rightarrow$ GEO-05 $\rightarrow$ GEO-06 $\rightarrow$ GEO-07 $\rightarrow$ GEO-08 $\rightarrow$ FINAL SYNTHESIS.

The first five stages close as **PASS / FROZEN**. The final two scientific audits close as **PARTIAL / FROZEN**, reflecting the frozen limitations on reconnection/action-exchange universality and full interaction-law discrimination.

Appendix B anchor
The final theorem chain closes as: FINAL SYNTHESIS: PASS / FROZEN. The manuscript-ready result is: spectral locality generates recurrent, effectively one-dimensional, dynamically renewing pre-string filament s The frozen limitation is: partial exchange-law regularity is supported, but no full reconnection law or universal physical interaction l

11.1 Frozen stage chain

The frozen stage chain is summarized in [Table 44](#). Each stage is treated as evidence only after its internal validation passes and its hard regression anchors are preserved.

Table 44: Frozen stage chain and final verdicts.

Stage	Audit name	Frozen verdict	Scientific role
GEO-02	Baseline Pre-String De-tection	BASELINE_PRE_STRING_DETECTION_PASS	At spectral locality generates detectable pre-string organizational candidates in the representative frozen graph.

Stage	Audit name		Frozen verdict	Scientific role
GEO-03	Ensemble Universality Audit		ENSEMBLE_PRE_STRING_ORGANIZATION_CHANNELS_PASS	across 24 independent seeds, finite scale-window support, and motif recurrence.
GEO-04	Effective Dimensionality Audit	One-	ONE_DIMENSIONAL_PRE_STRING_CHANNELS_PASS	channels behave as effective one-dimensional finite-graph sectors relative to the core.
GEO-05	Dynamical Persistence and Transport Audit		DYNAMICAL_PRE_STRING_FILAMENT_NETWORK_PASS	form dynamically renewing filament networks under controlled spectral flow.
GEO-06	Effective Filament Mechanics Audit		EFFECTIVE_FINITE_GRAPH_FILAMENT_MECHANICS_PASS	graph mechanical observables: length, action, tension proxy, stiffness density, and mode-spectrum proxies.
GEO-07	Filament-Core Coupling and Interaction Audit		FILAMENT_CORE_COUPLED_INTERACTION_PARTIAL_PASS	filament-core interaction and core-mediated reconnection, with a frozen reconnection/action-exchange limitation.
GEO-08	Effective Interaction-Law Discrimination Audit		PARTIAL_EFFECTIVE_EXCHANGE_LAW_SIGNAL_PASS	exchange-law regularity, but not a full reconnection law or universal interaction law.
FINAL	Frozen Final Synthesis		FINAL_SYNTHESIS_PASS	Integrates the frozen theorem chain, limitations, and manuscript-ready claims.

The result classes are:

GEO-02 : PASS / FROZEN,
 GEO-03 : PASS / FROZEN,
 GEO-04 : PASS / FROZEN,
 GEO-05 : PASS / FROZEN,
 GEO-06 : PASS / FROZEN,
 GEO-07 : PARTIAL / FROZEN,
 GEO-08 : PARTIAL / FROZEN.

The frozen final synthesis reports:

Final synthesis: PASS,
 Manuscript readiness: 95.00%,
 Validation: PASS.

11.2 Numerical anchors

The primary frozen numerical anchors are listed in [Table 45](#). These values are reused across the main text, the computational protocol, and the final synthesis.

Table 45: Primary frozen numerical anchors.

Anchor	Frozen value	Meaning
Number of seeds	24	Independent frozen spectral realizations.
Spectral states per seed	64	Number of states in each finite spectral ensemble.
Spectral dimension	16	Dimension of each normalized spectral vector.
Scale window	$k = 3, 4, 5, 6$	Finite organizational scale window in which pre-string signatures recur.
Seed-scale cells	96	24 seeds times 4 k -values.
Baseline channel count	15056	Full frozen pre-string channel population.
GEO-02 baseline candidates	100	Representative frozen pre-string candidate count.
GEO-02 detection score	0.825694	Representative baseline detection score.
GEO-03 detection fraction	0.958333	23/24 detected seeds.
Detected-or-partial fraction	1.000000	23/24 detected plus 1/24 partial.
Motif seed-scale coverage	0.916667	88/96 seed-scale motif coverage.

Anchor	Frozen value	Meaning
One-dimensional channel fraction	1.000000	GEO-04 full channel-level one-dimensionality fraction.
Stable-window dimensional contrast	0.986111	71/72 stable-window contrast coverage.
Active carriers	13612	Baseline channels active at least once under controlled flow.
Active episodes	49217	Noninitial active filament episodes.
Transition count	75280	15056×5 possible noninitial transitions.
Finite dynamic mechanics fraction	1.000000	Active representatives with finite mechanical observables.
Filament-core interface fraction	1.000000	All audited filaments possess core interface.
Median filament-core leakage	0.897536	Strong boundary leakage into the organizational core.
Filament pair interaction fraction	0.998623	Fraction of filament pairs with core-mediated interaction.
Nonlocal interaction pair fraction	0.592651	Core-mediated interactions without path-node overlap.
Core-mediated reconnection fraction	1.000000	Observed reconnections are core-mediated.
Best refined high-exchange AUC	0.573874	Partial high-exchange law-like discrimination.
Exchange-rank Spearman magnitude	0.144145	Partial rank-level exchange association.
Seed-scale exchange recurrence	0.926316	Recurrence of exchange-rank signal across seed-scale cells.
Best refined reconnection AUC	0.526464	Reconnection-law discrimination remains weak.

The channel count decomposition by k -scale is:

k	frozen channel count
3	4725
4	5375
5	3843
6	1113

and therefore:

$$4725 + 5375 + 3843 + 1113 = 15056.$$

The dynamical state decomposition is:

active episode fraction = 0.653786,
endpoint episode fraction = 0.421148,
reconnected episode fraction = 0.232638,
decayed episode fraction = 0.346214,
rigid fraction = 0.097635,
dynamical mode entropy = 0.974802.

The mechanical anchor block is:

median hop length = 3,
median spectral length = 12.069761,
median spectral action = 44.124460,
median tension proxy = 3.629348,
tension coefficient of variation = 0.189067,
median mode gap = 0.584116,
median scaled mode gap = 1.647594,
median stiffness density = 2.125080,
combined mechanics stability = 0.872177.

The interaction limitation block is:

reconnected episode fraction = 0.232638 < 0.300000,
nonlocal reconnection fraction = 0.046765 < 0.050000,
reconnected/endpoint exchange ratio = 6.481297 > 2.500000,
 $k = 5$ reconnected episode fraction = 0.217174 < 0.250000.

The law-discrimination limitation block is:

best refined reconnection AUC = 0.526464 < 0.560000,
best refined reconnection gain = 0.004032 < 0.020000,
stable-window law support = 0.333333 < 0.666667,
 $k = 5$ response gain = 0.001349 < 0.015000.

11.3 Claims and limitations

The manuscript-ready claims are listed in [Table 46](#). The strength labels are part of the frozen final synthesis.

Table 46: Frozen manuscript-ready claims.

ID	Strength	Claim	Basis
C1	strong	Spectral locality alone can generate recurrent pre-string organizational channels in a finite ensemble of modular spectral states.	GEO-02/GEO-03
C2	strong	These channels behave as effective one-dimensional finite-graph sectors rather than arbitrary high-dimensional graph paths.	GEO-04
C3	strong	The channels form dynamically renewing leaky filament networks under controlled spectral flow.	GEO-05
C4	strong	The filaments support finite-graph mechanical observables, including action, tension proxy, stiffness density, and mode-spectrum proxies.	GEO-06
C5	qualified strong	The filaments are embedded in a higher-dimensional organizational core and interact through shared core neighborhoods.	GEO-07
C6	partial	The embedded filament-core system exhibits partial law-like regularity in exchange observables.	GEO-08
C7	negative/limiting	The present construction does not establish a full reconnection law or universal effective interaction law.	GEO-07/GEO-08

The frozen limitations are listed in [Table 47](#). These limitations are not incidental; they define the conservative scientific scope of the paper.

Table 47: Frozen limitations.

ID	Class	Statement	Implication
L1	scope	All results are finite-graph and ensemble-specific.	No continuum worldsheet, no physical string tension, and no physical quantization are claimed.
L2	interaction	GEO-07 closes only as partial because reconnection/action-exchange is limited.	Filament-core interaction is strong, but reconnection is not a full universal smooth exchange sector.

ID	Class	Statement	Implication
L3	law	GEO-08 establishes only a partial effective exchange-law signal.	No full physical interaction law or universal analytic coupling law is established.
L4	reconnection	Reconnection-law discrimination remains weak.	Reconnection should be described as core-mediated reconfiguration, not as a closed law.
L5	scale	Stable-window law support is incomplete.	The exchange-law signal is not yet uniformly stable across all tested k -scales.
L6	physics	No physical force, scattering amplitude, gauge interaction, conserved charge, or continuum field equation is derived.	The manuscript remains in the language of effective organizational dynamics.

The final claim/limitation pair can be written compactly as:

spectral locality $\Rightarrow$ pre-string filament sectors with finite-graph mechanics and embedded core-mediated inter

and

partial exchange-law regularity $\nRightarrow$ full reconnection law or universal physical interaction law.

11.4 Output-file registry

The frozen final synthesis generated the output registry listed in [Table 48](#). These files contain the structured final stage chain, numerical anchors, theorem chain, limitations, manuscript claims, article plan, and summary data.

Table 48: Final synthesis output-file registry.

Output file	Content
<code>final_synthesis_frozen_stage_chain.csv</code>	Frozen GEO-02 through GEO-08 stage chain, verdicts, confidence levels, and core claims.
<code>final_synthesis_numerical_anchors.csv</code>	Primary numerical anchors used in the manuscript: channel counts, active episodes, interaction fractions, and law-discrimination metrics.
<code>final_synthesis_theorem_chain.csv</code>	Frozen theorem chain from spectral-locality channel emergence to partial exchange-law regularity.
<code>final_synthesis_limitations.csv</code>	Frozen limitations L1–L6 defining conservative scope.

Output file	Content
<code>final_synthesis_manuscript_claims</code>	Manuscript-ready claims C1–C7 and their evidence basis.
<code>final_synthesis_article_sections</code>	Article blueprint: Abstract, Introduction, main sections, discussion, conclusion, appendices.
<code>final_synthesis_figures_tables</code>	Planned figures and tables for the article.
<code>final_synthesis_summary.json</code>	Machine-readable final synthesis summary containing all frozen structured outputs.
<code>final_synthesis_summary.txt</code>	Human-readable final synthesis summary containing stage chain, numerical anchors, claims, limitations, and article blueprint.

The final output block is:

Final synthesis: PASS,
Manuscript readiness: 95.00%,
Validation: PASS.

Theorem 11.1 (Frozen stage-registry closure). *The frozen numerical anchors, stage verdicts, claims, limitations, and output registry are mutually consistent with the final theorem chain of the paper.*

Proof. The stage registry preserves the frozen sequence GEO-02 through GEO-08. The numerical anchors reproduce the baseline channel count 15056, seed–scale count 96, transition count 75280, active carrier count 13612, and active episode count 49217. The claims C1–C7 are directly mapped to the supporting stages. The limitations L1–L6 are directly mapped to the partial results and failed law/reconnection gates in GEO-07 and GEO-08. The final synthesis output registry contains the structured files generated from this same frozen information. Therefore the appendix registry is consistent with the final theorem chain. $\square$

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