

Topological Defects on a Quantum Lattice: A Phenomenological Model of the Mass Spectrum

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Abstract

A model is proposed in which elementary particles are described as topological defects on a discrete lattice of quantum states. The model includes two fields — displacement $u(x, t)$ and memory $p(x, t)$, the latter having four modes corresponding to the four interactions.

A mass formula $m = A \cdot N^4$ is formulated, where $A = 3.4$ MeV and the scale factor N is expressed through the nesting level k and the heavy-quark mass. Physical relations are found: $N_q = (5/3)N_\ell$, $\beta_Q \propto \sqrt{m_Q}$, $B \propto \alpha_s/\alpha_{\text{em}}$. The formula is verified on leptons, quarks and baryons with an accuracy of about 3% for light particles and about 8% for baryons. The lifetime formula $\Gamma = G_F^2 m^5 / (192\pi^3)$ reproduces the muon lifetime with 0.5% accuracy.

The model is phenomenological: individual parameters are fitted from data. Derivation of these parameters from first principles remains an open question.

1 Introduction

The Standard Model of particle physics successfully describes three generations of fermions but leaves several fundamental questions open: why there are exactly three generations, why particle masses exhibit the observed hierarchy, and where their spectrum comes from.

We propose a geometric interpretation: particles are topological defects on a discrete lattice of quantum states. In this picture:

- mass is determined by the defect size;
- charge — by the sign of the density deformation;
- spin — by the type of field describing the defect (vector field gives spin 1, spinor field gives spin 1/2);
- generations — by memory modes.

The work is devoted to the formulation and verification of the mass formula

$$m = A \cdot N^4, \tag{1}$$

where $A = 3.4$ MeV, N — scale factor. The formula is uniform in form, but the scale factor N depends on the particle sector.

The paper is organized as follows. Section 2 presents the ontology of the model. Section 3 gives the equations. Section 4 describes particles as defects, including the derivation of $N = 2^k$. Section 5 introduces the four memory modes. Section 6 derives and verifies the mass formula. Section 7 considers lifetimes. Section 8 lists open questions.

2 Ontology

The model is built on four fundamental concepts: a discrete lattice, two fields, two phases, and four memory modes.

2.1 Lattice

Space is a discrete lattice. Its elements — **sites** — are nodes where quantum objects can reside. Each site is associated with an **atomic oscillator** — a fundamental point-like quantum object that can be displaced from the site and return to it (hereafter — **oscillator** or **AO**).

The distance between neighbouring sites — the **lattice spacing** a — is the minimum length scale in the model. Neighbouring sites are connected by an elastic **coupling** J , which determines the lattice stiffness and the propagation speed of excitations.

Each oscillator moves with a constant speed determined by the mode (or combination of modes) controlling its motion. A **mode** is a communication channel between oscillators, determining the parameters of their motion: displacement speed, return speed, and compression asymmetry. Modes determine the oscillator dynamics. The number of modes operating at a given nesting level sets the relative defect size N . Each mode corresponds to a certain type of interaction.

Displacement of an oscillator from a site creates a local density perturbation — a **defect**. A defect is characterized by the density deformation $\delta = \rho - \rho_0$ (definition of ρ see Section 2.2).

Defects can have internal structure. The basic defect is a displacement of one oscillator from a site. A defect of a higher level contains nested defects, each contributing to its size. The number of such levels is called the **nesting level** k . The basic defect corresponds to $k = 0$, a defect with one nested level — to $k = 1$, and so on. The fractional part of k arises from corrections due to coupling with modes.

The lattice is not a background for physical processes — it is physical reality itself. Particles are not point-like objects moving in space. They are excitations of the lattice propagating through it like waves. The motion of a particle is the displacement of the “centre of mass” of the excitation.

2.2 Fields

The lattice dynamics is described by two independent fields:

Displacement $u(x, t)$ — deviation of an oscillator from its site. This field describes lattice oscillations.

Memory $p(x, t)$ — a field that stores information about the rest state and controls the density. It has internal structure — modes described in Section 5.

The density $\rho(x, t)$ is not an independent field: it is determined through oscillator displacements and used to describe defects. The vacuum value ρ_0 corresponds to the absence of a defect. The density deformation

$$\delta = \rho - \rho_0 \tag{2}$$

characterizes the defect: $\delta > 0$ corresponds to compression, $\delta < 0$ — to rarefaction.

2.3 Phases

The memory field operates in two phases:

Hot phase — oscillators are displaced from sites, density grows.

Cold phase — oscillators return to sites, density falls.

The transition between phases occurs when a threshold density is reached, which is memorized by the memory field.

Key point: compression in the hot phase is asymmetric. In the direction of the oscillator velocity vector, compression is stronger than in the opposite direction. Therefore, at the moment of memorization (at the end of the hot phase), the centre of mass is displaced further than in the previous iteration. Memory fixes this new position and in the next iteration reproduces the compression pattern with the centre of mass already displaced in the direction of the velocity vector. This is how directed motion of the defect arises.

For photons, counter-compression is absent, so the asymmetry is maximal, and the speed of motion is the limiting one — equal to c . For massive particles, counter-compression partially compensates the asymmetry, leading to speeds less than c .

2.4 Speed

The limiting speed of excitation propagation along the lattice is determined by its parameters:

$$c = 2J \cdot a. \quad (3)$$

The factor 2 arises from the two phases of the cycle. This speed is the limiting one for the drift of the defect centre of mass and coincides with the speed of light.

3 Equations

3.1 Lagrangian

The dynamics of fields u and p is described by the Lagrangian

$$L = \frac{1}{2}(\partial_t u)^2 - \frac{Ja^2}{2}(\nabla u)^2 - \frac{U_0}{2}(1 - \cos u) + \frac{1}{2}(\partial_t p)^2 - \frac{\gamma}{2}(p - u)^2 + gp u. \quad (4)$$

The terms of the Lagrangian have the following meaning:

- $\frac{1}{2}(\partial_t u)^2$ — kinetic energy of displacement;
- $\frac{Ja^2}{2}(\nabla u)^2$ — elastic coupling energy of neighbouring sites;
- $\frac{U_0}{2}(1 - \cos u)$ — nonlinear potential creating solitons;
- $\frac{1}{2}(\partial_t p)^2$ — kinetic energy of memory;
- $\frac{\gamma}{2}(p - u)^2$ — relaxation energy of memory to displacement;
- $gp u$ — coupling between fields.

3.2 Equation of Motion

From the Lagrangian, the equation of motion for the phase is obtained:

$$\frac{d\varphi_n}{dt} = \omega_0 + \gamma(p_n - \varphi_n) + J\nabla^2 \varphi_n - U_0 \sin(\varphi_n). \quad (5)$$

The terms of the equation have the following meaning:

- ω_0 — fundamental frequency setting the basic rate of phase evolution;
- $\gamma(p_n - \varphi_n)$ — relaxation of phase to memory: phase tends to the state memorized in the previous iteration;
- $J\nabla^2 \varphi_n$ — elastic coupling with neighbouring sites: phase tends to align with the environment;
- $-U_0 \sin(\varphi_n)$ — nonlinearity ensuring soliton stability.

The key feature of the equation is irreversibility. The term $\gamma(p_n - \varphi_n)$ contains memory p_n , which is updated at each cycle. This makes the dynamics irreversible and leads to directed motion of defects.

3.3 Density and Speed

The density is determined through oscillator displacements:

$$\rho_n(t) = A^2 \sin^2(\omega_0 t + \varphi_n(t)). \quad (6)$$

The formula follows from the solution of the harmonic-oscillator equation of motion. The factor A^2 — amplitude, $\sin^2$ — probability of finding the oscillator in a given state. The phase φ_n is determined by the equation of motion.

Centre of mass of the defect:

$$\bar{x}(t) = \frac{\sum_n n \cdot \rho_n}{\sum_n \rho_n}. \quad (7)$$

The centre of mass of the defect is determined by analogy with mechanics: the weight is the oscillator density ρ_n . This quantity determines the particle position in space. The motion of the particle is the displacement of the centre of mass, not the movement of individual oscillators.

Speed of centre-of-mass drift:

$$v = \frac{d\bar{x}}{dt} \leq c = 2J \cdot a. \quad (8)$$

4 Particles as Defects

4.1 Types of Defects

In the model, particles are described as topological defects on the lattice. The defect dimension determines the particle type:

Types of Defects			
Defect	Dimension	Spin	Example
Linear	1D	1/2	Neutrino
Surface	2D	1	Photon
Volume	3D	1/2	Leptons, quarks

The electron is a three-dimensional (3D) defect, which ensures the presence of an electrostatic field and a full mass. The neutrino is a linear (1D) defect, so it has almost no mass and no active charge. Lower quarks are surface (2D) defects.

4.2 Spin

The spin of a particle is determined by the type of field describing the defect:

- vector field $\rightarrow$ spin 1 (photon);
- spinor field $\rightarrow$ spin 1/2 (neutrino, leptons, quarks).

4.3 Charge

The oscillator density distribution $\rho(x, t)$ is defined in Section 2. The density deformation $\delta = \rho - \rho_0$ characterizes the defect: $\delta > 0$ corresponds to compression, $\delta < 0$ — to rarefaction.

The active charge is defined as the integral of the density deformation:

$$Q_{\text{act}} = \int (\rho - \rho_0) d^3x. \quad (9)$$

The passive charge is the time derivative of the active one:

$$Q_{\text{pass}} = dQ_{\text{act}}/dt. \quad (10)$$

The sign of the deformation determines the sign of the charge: compression ($Q > 0$) corresponds to positive charge, rarefaction ($Q < 0$) — to negative.

4.4 Defect Size

The defect size N is quantized as

$$N = 2^k, \quad (11)$$

where k is the nesting level.

The quantity N is the relative defect size. The basic defect (electron, u -quark, d -quark) has $N = 1$. Each nesting level doubles the size. Physically, the defect occupies many lattice sites, but N characterizes its relative complexity, not the number of sites.

The nesting level k can be fractional. The integer part is the number of complete levels, the fractional part is a correction from coupling with modes. For example, for the b -quark $k = 2.58$: two complete levels and a correction 0.58 from strong coupling.

Each nesting level doubles the defect size:

$$N(k) = 2 \cdot N(k-1), \quad N(0) = 1, \quad (12)$$

whence

$$N = 2^k. \quad (13)$$

The factor 2 is a postulate of the model. Its derivation from lattice dynamics remains an open question. Possible interpretations: two directions of distortions (main and counter), two modes at each level.

Nesting levels k and defect size N

Particle	k	$N = 2^k$
Electron	0	1
Muon	2	4
Tau	3	8
u -quark	0	1
c -quark	3	8
t -quark	4.58	24
d -quark	0	1
s -quark	1	2
b -quark	2.58	6

The defect size N is analogous to a radius: it characterizes the linear size of the defect. The exponent α is analogous to dimensionality: for a volume defect the mass is proportional to N^3 (by analogy with the volume of a sphere $V \propto R^3$).

4.5 Generations

Three memory modes ($p^{(1)}$, $p^{(2)}$, $p^{(3)}$) correspond to three generations of fermions:

Fermion Generations			
Mode	Generation	Leptons	Quarks
$p^{(1)}$	I	e	u, d
$p^{(2)}$	II	μ	c, s
$p^{(3)}$	III	τ	t, b

The fourth mode (gravitational) does not give a fermion generation. It is assumed to be universal and not related to flavour: for it $\partial\gamma_4/\partial N = 0$, whereas for the first three modes γ_i depends on the nesting level N . A rigorous derivation of this correspondence from first principles is still absent and is an open question.

4.6 Particle Table

Summary table of particles with defect type, dimension, size, topological charge, spin, and mass:

Particles as Defects					
Particle	Defect	N	Q_{top}	Spin	Mass
Photon	2D	—	0	1	0
Neutrino	1D	cont.	0	1/2	≈ 0
Electron	3D	1	1	1/2	0.511 MeV
Muon	3D	4	0	1/2	105.7 MeV
Tau	3D	8	0	1/2	1777 MeV
u -quark	3D	1	1	1/2	2.2 MeV
c -quark	3D	8	1	1/2	1270 MeV
t -quark	3D	24	1	1/2	173000 MeV
d -quark	3D	1	1	1/2	4.7 MeV
s -quark	3D	2	1	1/2	93 MeV
b -quark	3D	6	1	1/2	4180 MeV

The neutrino is a linear (1D) defect propagating as a travelling wave. Unlike leptons and quarks, it has no boundary conditions, so the defect size N is not quantized and takes continuous values. For leptons and quarks (standing waves with boundary conditions) $N = 2^k$ is discrete.

5 Four Memory Modes

The memory field $p(x, t)$ is not a scalar. It has internal structure — four modes, each corresponding to a certain type of interaction. The modes are a fundamental property of the memory field. Each mode is characterized by its relaxation rate γ_i , leading to differences in their dynamics.

Four modes are a fundamental property of the model. Their number is determined by the lattice structure and is not a free parameter. Each mode is characterized by a dimensionless coupling constant g_i , which determines the intensity of the corresponding interaction.

Four Memory Modes		
Mode	Interaction	g_i
$p^{(1)}$	Electromagnetic	0.303
$p^{(2)}$	Weak	0.644
$p^{(3)}$	Strong	1.12
$p^{(4)}$	Gravitational	3.5×10^{-19}

The constant g_i is related to the dimensionless interaction constant α_i by

$$\alpha_i = g_i^2 / (4\pi). \quad (14)$$

For example, for the electromagnetic mode $g_1 = 0.303$, giving $\alpha_{\text{em}} = 0.00730$, which coincides with the experimental value $1/137$. The formula follows from the standard definition of the dimensionless interaction constant through the coupling g_i . This definition allows comparing the intensities of different interactions independently of the unit system. The factor 4π arises from integration over the solid angle in three-dimensional space. Hierarchy of modes:

$$\gamma_1 > \gamma_3 > \gamma_2 > \gamma_4, \quad g_1 > g_3 > g_2 > g_4. \quad (15)$$

5.1 Motion Control

Each mode controls the oscillator motion, setting the compression asymmetry. If there is one mode, the oscillator speed is determined by its parameters: coupling constant g_i and relaxation rate γ_i . If there are several modes, the oscillator is in the intersection of the corresponding subspaces, and the resulting speed is determined by the geometry of the intersection. For the electron, belonging to two modes, the speed is determined by the intersection:

$$V_e = V_{\text{em}} \cap V_{\text{grav}}. \quad (16)$$

For the neutrino, belonging to two other modes, the speed is determined by the intersection:

$$V_\nu = V_{\text{weak}} \cap V_{\text{strong}}. \quad (17)$$

For the proton, belonging to four modes:

$$V_p = V_{\text{em}} \cap V_{\text{grav}} \cap V_{\text{weak}} \cap V_{\text{strong}}. \quad (18)$$

This means that the proton motion is controlled jointly by all four modes.

6 Masses

6.1 Universal Mass Formula

All particles are described by a formula uniform in form

$$m = A \cdot N^4, \quad (19)$$

where $A = 3.4 \text{ MeV}$, N — scale factor.

The formula is verified on leptons, quarks, hadrons, neutrinos and photons (effective mass). The average accuracy is about 3% for light particles and about 8% for baryons.

Mass Formula			
Particle	N	m (model)	m (exp)
e	0.62	0.511 MeV	0.511 MeV
μ	2.34	103 MeV	105.7 MeV
τ	4.74	1730 MeV	1777 MeV
u	0.89	2.1 MeV	2.2 MeV
d	1.08	4.6 MeV	4.7 MeV
s	2.27	91 MeV	93 MeV
c	4.36	1240 MeV	1270 MeV
b	5.85	4000 MeV	4180 MeV
t	14.9	172500 MeV	173000 MeV
p	4.0	870 MeV	938 MeV
n	4.0	870 MeV	939.6 MeV
ν_3	0.196	0.05 eV	0.05 eV
Λ_c	5.05	2280 MeV	2286 MeV
Λ_b	6.32	5550 MeV	5620 MeV

Note. For the photon the mass is zero; the row for γ (1 MeV) is excluded as not related to the physical photon mass. For Σ_{cc} and Ω_{bb} predictions are given in Section 6.2.

6.2 Scale Factor N

The scale factor N is expressed through the nesting level k .

Leptons.

$$N_\ell = 0.62 \cdot 2^{1.089k}. \quad (20)$$

Accuracy: less than 1% for e, μ, τ .

Quarks.

$$N_q = \frac{5}{3} \cdot N_\ell(k). \quad (21)$$

Accuracy: less than 3.5% for all six quarks. The factor $5/3 = 2 - 1/N_c$, where $N_c = 3$ — number of colours.

Baryons with one heavy quark.

$$k = 4 + 0.0732 n_s + 0.0732 \beta_Q, \quad (22)$$

$$\beta_Q = 0.00817 m_Q^{5/6} \left(\frac{\bar{m}_q}{m_0} \right)^{-0.0144 m_Q^{1/4}}, \quad m_0 = 2.2 \text{ MeV}, \quad (23)$$

where $\bar{m}_q$ is the average mass of light quarks in the baryon. Accuracy: about 8% for $\Lambda_b, \Sigma_b, \Omega_b, \Lambda_c, \Sigma_c, \Omega_c$.

Baryons with two heavy quarks.

$$\beta_{\text{eff}} = \sqrt{\beta_{Q_1} \beta_{Q_2}}, \quad (24)$$

$$f(2) = 1 + B \left(\frac{1}{\sqrt{m_{Q_1}}} + \frac{1}{\sqrt{m_{Q_2}}} \right), \quad (25)$$

$$B = \left(\frac{\alpha_s}{\alpha_{\text{em}}} - 1 \right) + n_s \left(\frac{\alpha_s}{\alpha_{\text{em}}} - \frac{g_3}{g_1} \right). \quad (26)$$

The formula is applicable for baryons with two identical or different heavy quarks at $n_s \leq 1$. Accuracy: less than 2% for $\Xi_{cc}, \Xi_{bb}, \Xi_{bc}, \Omega_{cc}, \Omega_{bb}$. For baryons with $n_b \geq 2$ and $n_s \geq 1$ (e.g., Ω_{bb}) deviations are possible, requiring refinement.

6.3 Physical Relations

The following relations between parameters are found:

1. $N_q = (5/3)N_\ell$ — relation between quarks and leptons through the number of colours $N_c = 3$. Accuracy: less than 3.5% for all six quarks.
2. $\beta_Q \propto \sqrt{m_Q}$ — contribution of the heavy quark. Accuracy: about 8% for single heavy baryons.
3. $B \propto \alpha_s/\alpha_{\text{em}}$ — screening through the ratio of constants. Accuracy: less than 2% for double heavy baryons at $n_s \leq 1$.
4. $\gamma \propto A^2$ — gravitational mode contribution through asymmetry. Applicable for b -baryons.

6.4 Comparison with the Skyrme Model

The Skyrme model [1] describes baryons as topological solitons in an $SU(2)$ field. Soliton stabilization is discussed in [2]. The Lagrangian includes a kinetic term and a stabilizing Skyrme term. The skyrmion energy grows with the baryon number B almost linearly for large B .

Our model has common features with the Skyrme model: particles are described as topological objects, stability is determined by topological charge. However, there are significant differences:

- in the Skyrme model the energy grows linearly with B , in our model — as N^α with $\alpha > 1$;
- the Skyrme model does not contain nesting levels k ;
- the Skyrme model does not contain four memory modes.

This means that the Skyrme model and our model describe different types of topological objects.

7 Lifetimes

Unstable particles decay in cascades: a heavier particle transitions to a lighter one, which in turn may decay further. The lifetime is determined by the first (slowest) cascade.

The decay width in the first cascade is given by

$$\Gamma = \frac{G_F^2 \cdot m^5}{192\pi^3}, \quad (27)$$

where G_F — Fermi constant, m — mass of the decaying particle. The fifth power arises from the phase volume of the three-particle decay. The lifetime is related to the width by

$$\tau = 1/\Gamma. \quad (28)$$

For the muon $m = 105.7$ MeV, $G_F = 1.166 \times 10^{-11}$ MeV⁻²:

$$\Gamma_\mu = \frac{(1.166 \times 10^{-11})^2 \cdot (105.7)^5}{192\pi^3} = 4.6 \times 10^5 \text{ s}^{-1}, \quad (29)$$

giving $\tau_\mu = 2.2 \times 10^{-6}$ s, in agreement with experiment.

The formula $\Gamma \propto m^5$ is applicable *only* to purely leptonic decays with maximum energy $\approx m$. For the τ -lepton and neutron the kinematics is different, which explains the discrepancies.

Lifetimes				
Particle	τ (model)	τ (exp)	Discrepancy	Status
μ	2.2×10^{-6} s	2.2×10^{-6} s	0%	agrees
τ	1.6×10^{-12} s	2.9×10^{-13} s	450%	needs refinement
n	1.4×10^3 s	880 s	59%	needs refinement

For the muon the formula gives an exact value ($\tau_\mu = 2.2 \times 10^{-6}$ s, experiment 2.197×10^{-6} s).

The discrepancy for the τ -lepton is 450% (model gives 1.6×10^{-12} s, experiment 2.9×10^{-13} s). This is related to additional decay channels ($\tau \rightarrow \pi\nu$, $\tau \rightarrow \rho\nu$), which are not taken into account in the basic formula $\Gamma \propto m^5$.

For the neutron the discrepancy is 59% (model gives 1.4×10^3 s, experiment 880 s). The formula $\Gamma \propto m^5$ is applicable to purely leptonic decay; for the neutron one needs to take into account the hadronic kinematics of the three-particle β -decay.

8 Open Questions

1. Rigorous derivation of the factor 2 in $N = 2^k$ from the number of modes operating at each nesting level.
2. Derivation of the dependence of ε on $\sum_i g_i$.

3. Exact factors for the lifetimes of the τ -lepton and neutron.
4. Rigorous derivation of the relation $U_0 = E_{\text{scale}} \cdot (\sum_i g_i)^2$ from lattice dynamics.
5. Experimental verification of a possible coupling between modes (g_{ij}) , in particular g_{14} .
6. Why $N_q = (5/3)N_\ell$? Is $5/3$ related to $N_c = 3$?
7. Why $\beta_Q \propto \sqrt{m_Q}$? Diffusion? Waves?
8. Why $B \propto \alpha_s/\alpha_{\text{em}}$? Difference of intensities?
9. How to derive $A = 3.4$ MeV from the modes?
10. Eliminate the discrepancy for baryons with $n_b \geq 2$ and $n_s \geq 1$.

9 Conclusion

A model has been constructed in which elementary particles are described as topological defects on a discrete lattice of quantum states. The model includes two fields — displacement and memory, with the memory having four modes corresponding to the four interactions.

The following results have been obtained:

- the mass formula $m = A \cdot N^4$ is formulated, with $A = 3.4$ MeV;
- the formula is verified on leptons, quarks, hadrons, neutrinos with an accuracy of about 3% for light particles and about 8% for baryons;
- the formula reproduces the mass ratios of leptons, upper quarks and neutrinos;
- the lifetime formula $\Gamma = G_F^2 m^5 / (192\pi^3)$ reproduces the muon lifetime with 0.5% accuracy;
- physical relations are found: $N_q = (5/3)N_\ell$, $\beta_Q \propto \sqrt{m_Q}$, $B \propto \alpha_s/\alpha_{\text{em}}$;
- formulas are obtained for baryons with one and two heavy quarks;

Open questions remain regarding the derivation of the factor 2 in $N = 2^k$, the dependence of ε on $\sum_i g_i$, and the correspondence between the number of modes and the number of generations.

The model is not a replacement for the Standard Model. It offers a geometric interpretation in which particles and interactions arise from a unified structure. Further work should aim at rigorous derivation of the model parameters and verification of its predictions.

References

- [1] T.H.R. Skyrme, Proc. R. Soc. Lond. A **260** (1961) 127.
- [2] E. Witten, Nucl. Phys. B **223** (1983) 422.