

The Embedded-Observer No-Go Theorem

Non-Reconstructibility, Non-Autonomy, and Their Quantitative Form for Tensor-Factor Observers in Finite-Dimensional Closed Quantum Systems

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Abstract

No physical observer is external to the physical universe (see Postulate); every observer is itself a physical subsystem. We study the consequence of taking this seriously for an observer modelled as a proper tensor factor O of a finite-dimensional closed quantum system $H = H_O \otimes H_R$ with unitary global dynamics, and prove two independent obstructions, each with a quantitative form. (i) *Kinematic*. Within the tensor-factor model of §2, the observer's accessible data, the reduced state ρ_O , leave open a manifold of global pure states of real dimension $2rd_R - r^2 - 1$ ($r = \text{rank } \rho_O$) and fix the state of the complement only up to its spectrum; the amount of correlation between O and R is accessible ($I(O:R) = 2S(\rho_O)$), its content is not. (ii) *Dynamical*. The reduced evolution of O follows a fixed unitary law, independent of the state of R , if and only if the coupling is a product unitary. In robust form, the autonomy defect E and the normalised Hilbert–Schmidt distance D from the nearest product unitary satisfy $\left(1 + \frac{1}{d_O}\right)E \leq D^2 \leq 2\left(1 + \frac{1}{d_O}\right)E$, both constants being sharp; for Hamiltonian couplings this gives a guaranteed short-time autonomy bound $E \leq (t\|H_{\text{int}}\|)^2$, which holds for the worst-case defect as well. The state property “stock” $S(\rho_O)$ and the step property “flow” E are logically independent, but the flow bounds the growth of the reduced entropy per step; physically, this growth is the standard decoherence mechanism behind O ’s *apparent* collapse to a definite outcome - apparent because the global state stays exactly pure and unitary throughout, and because Proposition 5.5 shows the growth is not one-way. For discrete local circuits we prove an exact causal cone and the exact autonomy of the part of O lying beyond the cone of R . The results are elementary; we state which are known and what is not claimed (continuum field theory, any interpretation, any experimental test).

Keywords: embedded observer; reduced dynamics; purification; no-signalling; autonomy; local circuits

1. Introduction

Postulate. The physical universe consists of everything that physically exists; no physical observer is external to it, and every physical observer is itself a physical subsystem, composed of the same kind of stuff as whatever it observes. This is not derived here: no physical theory has ever postulated an observer outside physics. It is the ground this paper stands on, not a result it proves.

An observer that is part of the system it describes has access only to the observables of its own subsystem. Two elementary questions follow. First, what do these data leave undetermined about the state of the whole? Second, when can the observer's own evolution be described by a fixed unitary law that makes no reference to the rest of the system? This note answers both in the one setting where the postulate can be made mathematically exact: a finite-dimensional closed quantum system $H = H_O \otimes H_R$ with unitary dynamics, in which the observer occupies the factor O . Identifying the observer with a tensor factor is how the postulate is implemented here, not a further hypothesis about observers; a companion note (Materov 2026b) examines the same postulate operationally, for a photonic Friend that is not modelled as a clean tensor factor.

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The answers are two obstructions that are logically independent, and the paper treats them as such.

- **Kinematic (§3).** Whatever the global state, the reduced state ρ_O leaves open a whole manifold of global pure states, and it fixes the state of the complement only up to its spectrum. How much correlation O has with R is accessible to O (it equals $2S(\rho_O)$); which degrees of freedom of R carry it is not.
- **Dynamical (§4).** A fixed unitary law for O , independent of the state of R , exists if and only if the O – R coupling is a product unitary. We prove this in a robust form with explicit, dimension-free constants: the distance from the nearest product unitary and the failure of autonomy control each other quadratically. For Hamiltonian couplings this gives a guaranteed short-time autonomy bound.
- **Stock and flow (§5).** The entropy $S(\rho_O)$ is a property of the state (an accumulated “stock”); the autonomy defect is a property of a single step (a “flow”). They are logically independent, but the flow bounds the growth of the reduced entropy per step - physically, the mechanism behind decoherence-based apparent collapse (Remark 5.7).
- **Locality (§7).** For discrete local circuits the causal cone is exact, and the part of O beyond the cone of R evolves exactly autonomously until the cone arrives.

The “no-go” of the title refers to the first two statements. The last shows that exact autonomy of sufficiently distant sub-regions is possible for a finite time; what is excluded is a globally exact, state-independent unitary law for the whole of O under a genuinely non-product coupling.

What is new and what is not. All results are elementary, and Table 1 states which parts are known. The core of §3 is the unitary equivalence of purifications (the Schrödinger–Hughston–Jozsa–Wootters theorem); the exact statement of §4 is a restricted unitary form of no-signalling, closely related to results on semicausal operations; the causal cone of §7 is the standard cone of a finite-depth circuit. To our knowledge the quantitative equivalence of Theorem 4.2 has not been stated in this form; its proof is short and can be checked line by line, and we also verify it numerically (Appendix A). We make no claim about continuum quantum field theory, where the factorization $H_O \otimes H_R$ is unavailable, about any interpretation of quantum mechanics, or about experiments; and the results are not consistency theorems of the Frauchiger–Renner type (§8).

Table 1. Status of the results.

Result	Status	Comment
Prop. 3.1, Cor. 3.3	Known core	Unitary equivalence of purifications (Schrödinger 1936; Hughston–Jozsa–Wootters 1993); the dimension count, the “spectrum-only” statement on ρ_R and the accounting $I = 2S$ are elementary but stated explicitly here.
Thm. 4.1	Related to known results	A restricted unitary no-signalling / autonomy statement (cf. Beckman et al. 2001; Eggeling et al. 2002). The proof given is self-contained.
Thm. 4.2	Not found in this form	Exact two-sided quadratic equivalence of autonomy defect and distance to product unitaries, sharp constants. Elementary; independent checking invited.
Cor. 4.4, Prop. 4.7, Cor. 5.4	Consequences	Prop. 4.7 is a two-line unitarity argument; Cor. 4.4 follows with Duhamel’s formula; Cor. 5.4 with Prop. 5.3.
Props. 5.2, 5.3, 5.5	Elementary	Organisational value: separate state and step properties; flow bounds the growth of the reduced entropy.
Thm. 7.2, Prop. 7.3	Standard / direct consequence	Causal cone of a finite-depth circuit is standard; exact core autonomy follows from the same support argument.

2. Setting and definitions

Throughout, $H = H_O \otimes H_R$ is a finite-dimensional Hilbert space with $d_O = \dim H_O \geq 2$ and $d_R = \dim H_R \geq 2$. The system is closed (its evolution is given by unitaries on H) and its global state is pure. The observer O is the factor H_O ; its accessible observables form the algebra $A_O = B(H_O) \otimes 1_R$. For a global state Ψ we write $\rho_O = \text{Tr}_R |\Psi\rangle\langle\Psi|$ and $\rho_R = \text{Tr}_O |\Psi\rangle\langle\Psi|$. The symbol $\|\cdot\|$ is the operator norm, $\|X\|_{\text{HS}} = (\text{Tr } X^\dagger X)^{1/2}$ and $\|X\|_1 = \text{Tr}(X^\dagger X)^{1/2}$. A state Ψ is a unit vector or the ray it defines; all statements are ray-independent.

Modelling assumption. The observer is *defined* as a proper tensor factor H_O , with access to the local algebra A_O only. The paper does not prove that a physical observer must have this form; it studies observers that do. Memory and sensory channels of a physical observer are included by enlarging H_O .

Lemma 2.1 (reduction is forced). ρ_O is the unique density operator on H_O with $\text{Tr}(\rho_O A) = \langle \Psi | A \otimes 1 | \Psi \rangle$ for all $A \in B(H_O)$. No choice is involved in passing from Ψ to ρ_O .

This is the definition of the partial trace together with the fact that $B(H_O)$ separates operators on H_O ; it uses only the factorisation $H = H_O \otimes H_R$.

Definition 2.2 (product and entangling unitaries). A unitary U on H is a *product unitary* if $U = W \otimes V$ with unitaries W on H_O and V on H_R ; otherwise it is *non-product*. U is *entangling* if it maps some product vector to an entangled vector.

Entangling implies non-product, but not conversely: for $d_O = d_R$ the swap is non-product and maps every product vector to a product vector. The distinction matters in §4 (Remark 4.5).

Definition 2.3 (distance to product unitaries). For unitaries U on H and W on H_O put

$$DW(U) := \frac{\min_V \|U - W \otimes V\|_{\text{HS}}}{\sqrt{d_O d_R}}, \quad D(U) := \min_W DW(U), \quad (2.1)$$

with the minimum over unitaries V on H_R (resp. W on H_O). Since $\|U\|_{\text{HS}}^2 = d_O d_R$, $D \leq \sqrt{2}$. By compactness, $D(U) = 0$ if and only if U is a product unitary.

Remark 2.4 (genericity). Product unitaries form a closed submanifold of $U(d_O d_R)$ of dimension $d_O^2 + d_R^2 - 1$, hence of codimension $(d_O^2 - 1)(d_R^2 - 1) > 0$. Generic unitaries are non-product. The qualitative hypothesis “the coupling is not fine-tuned” is thus made quantitative by the condition $D(U) > 0$.

Definition 2.5 (autonomy defect). Let $\psi \in H_O$ and $\varphi \in H_R$ be independent Haar-random unit vectors, and let $\rho'_O(\psi, \varphi) = \text{Tr}_R U |\psi \otimes \varphi\rangle\langle\psi \otimes \varphi| U^\dagger$ be the output state of O . For a unitary W on H_O define

$$E_W(U) := 1 - \mathbb{E}_{\psi, \varphi} \langle W \psi | \rho'_O(\psi, \varphi) | W \psi \rangle, \quad \varepsilon(U) := \min_W (E_W(U)). \quad (2.2)$$

$E_W(U)$ is the average probability that the actual output of O fails the test “is it the state $W\psi$?”, i.e. the average failure of W to predict O 's output. A fixed unitary law for O that does not refer to R must work for every state of R ; failure on product inputs already excludes a law valid on all global states.

Remark 2.6. (i) Averaging over O 's inputs is essential: for a fixed preparation of O the corresponding statement is false (Remark 4.5(i)). (ii) E_W and ε are *average-input* quantities: a small ε is not by itself a worst-case guarantee. Worst-case statements are made explicitly (Theorem 4.1(ii), Proposition 4.7). (iii) Failure on product inputs already excludes a law valid on all global states.

3. Kinematic obstruction: what the data of O leave open

Proposition 3.1 (fibre of the reduction).

Let Ψ be a pure state of $H_O \otimes H_R$, let $r = \text{rank } \rho_O$ (so $1 \leq r \leq \min(d_O, d_R)$), and let $F(\Psi)$ be the set of pure states (rays) Φ with $\text{Tr}_R |\Phi\rangle\langle\Phi| = \rho_O$. Then:

(a) $F(\Psi) = \{(1 \otimes V)\Psi : V \in U(H_R)\}$.

(b) $F(\Psi)$ is a compact connected submanifold of the projective space $P(H)$ of real dimension $2rd_R - r^2 - 1$, which is at least $2(d_R - 1) > 0$. For $r \geq 2$ it contains the $(r-1)$ -torus of states $\Psi_\theta = \sum_k \sqrt{p_k} e^{i\theta_k} |k\rangle |f_k\rangle$ that differ only in relative Schmidt phases.

(c) $\{\rho_R(\Phi) : \Phi \in F(\Psi)\} = \{V\rho_R V^\dagger : V \in U(H_R)\}$. The data of O fix ρ_R only up to unitary conjugation, i.e. only its spectrum.

Proof. (a) is the standard unitary equivalence of purifications with a common marginal (Schrödinger 1936; Hughston–Jozsa–Wootters 1993): $F(\Psi)$ is exactly the orbit of $[\Psi]$ under $1 \otimes U(H_R)$. (b) is the orbit-stabiliser count for that orbit: the stabiliser of the ray is $\{V : Vf_k = e^{i\alpha_k} f_k \forall k \leq r\} \cong U(1) \times U(d_R - r)$, so $\dim F(\Psi) = d_R^2 - 1 - (d_R - r)^2 = 2rd_R - r^2 - 1$; the states Ψ_θ (take V diagonal in the frame f_k) exhibit part of this orbit explicitly. (c) is immediate from (a), since $\rho_R((1 \otimes V)\Psi) = V\rho_R V^\dagger$ and V there ranges over all of $U(H_R)$. ■

Remark 3.2 (what the result says). (i) The proposition holds for *every* pure global state, including product states ($r = 1$, $\Psi = \psi \otimes \varphi$, and the data of O leave φ completely open). Non-reconstructibility of the global state from A_O is thus a kinematic fact about embeddedness ($d_R \geq 2$); it is not produced by entanglement. (ii) What entanglement changes is quantitative: the fibre grows with r , and the correlation between O and R becomes non-zero (Corollary 3.3).

Corollary 3.3 (correlation is accessible, its content is not). For every pure global state, the mutual information is $I(O:R) = S(\rho_O) + S(\rho_R) - S(\Psi) = 2S(\rho_O)$, a function of the spectrum of ρ_O alone. Hence any increase of the correlation between O and R is exactly an increase of $S(\rho_O)$; and by Proposition 3.1(c), *which* vectors of H_R carry the correlation remains undetermined by the data of O .

Proof. ρ_O and ρ_R have the same non-zero spectrum (Schmidt decomposition), and $S(\Psi) = 0$. ■

Remark 3.4 (scope). (i) Proposition 3.1 concerns what the *current data* in A_O leave open. If O is additionally given $\Psi(0)$ and the global unitary, then $\Psi(t)$ is determined and nothing in this section applies: the statement is about reconstruction from data, not about prediction from a complete model. (ii) The result is operational. Ontological questions (a hidden variable, a fact of the matter about Ψ) lie outside it (§8).

4. Dynamical obstruction: autonomy holds exactly for product couplings

Theorem 4.1 (exact case).

Let U be a unitary on $H_O \otimes H_R$ and W a unitary on H_O . The following are equivalent:

- (i) $E_W(U) = 0$;
- (ii) for *every* pure product input $\psi \otimes \varphi$ the output state of O is exactly the pure state $W\psi$;
- (iii) $U = W \otimes V$ for some unitary V on H_R .

Consequently $\varepsilon(U) = 0 \Leftrightarrow D(U) = 0 \Leftrightarrow U$ is a product unitary. Since product unitaries obey $\rho_O(U\Psi) = W\rho_O(\Psi)W^\dagger$ for every global state (Proposition 5.2(i)), exact autonomy on all global states, exact autonomy on product inputs and a product coupling are equivalent: a fixed unitary law for O that is independent of the state of R exists exactly when the coupling is a product.

Theorem 4.2 (robust form).

For all unitaries U on $H_O \otimes H_R$ and W on H_O , put $T_W = \text{Tr}_O[(W^\dagger \otimes 1)U]$ and $\kappa_W = \|T_W\|_{\text{HS}}^2 / (d_O^2 d_R) \in [0, 1]$. Then

- (a) $E_W(U) = \frac{d_O}{d_O + 1} (1 - \kappa_W)$;
- (b) $D_W(U)^2 = 2 - \frac{2\|T_W\|_1}{d_O d_R}$;
- (c) $(1 + \frac{1}{d_O}) E_W(U) \leq D_W(U)^2 \leq 2(1 + \frac{1}{d_O}) E_W(U)$.

The same two-sided bound holds for $\varepsilon(U)$ and $D(U)$. In particular $\varepsilon(U) < D(U)^2$ and, since $d_O \geq 2$, $D(U) \leq \sqrt{3\varepsilon(U)}$. Both constants in (c) are sharp: the second inequality is an equality for $W = 1$, $U = \text{CNOT}$; the ratio of the two sides of the first tends to 1 as U approaches a product unitary.

Theorem 4.2 says that “no autonomous unitary law” and “far from every product unitary” are quantitatively the same statement, with dimension-free constants, for the *average* defect (the worst-case defect is treated in Proposition 4.7). For the proof we need three standard integrals.

Lemma 4.3. Let ψ, ϕ be Haar-random unit vectors in $\mathbb{C}^d$. (i) For $K \in B(\mathbb{C}^d)$: $\mathbb{E} \|K\phi\|^2 = \frac{\|K\|_{HS}^2}{d}$. (ii) For $A \in B(\mathbb{C}^d)$: $\mathbb{E} |\langle \psi | A | \psi \rangle|^2 = \frac{\|A\|_{HS}^2 + |\text{Tr} A|^2}{d(d+1)}$. (iii) For $X \in B(\mathbb{C}^d \otimes \mathbb{C}^d)$: $\mathbb{E} ((\langle \psi | \otimes 1) X (\langle \psi | \otimes 1)) = \frac{\text{Tr}_1 X}{d}$.

Proof. (i) and (iii) follow from $\mathbb{E} |\langle \psi | \otimes \langle \psi | = \frac{1}{d}$. (ii) follows from $\mathbb{E} (|\langle \psi | \otimes \langle \psi |) \otimes^2 = \frac{1+F}{d(d+1)}$ with F the swap, and $|\langle \psi | A | \psi \rangle|^2 = \text{Tr}[(|\psi\rangle\langle\psi|) \otimes^2 (A \otimes A^\dagger)]$, $\text{Tr}[F(A \otimes A^\dagger)] = \text{Tr} A A^\dagger$. ■

Proof of Theorem 4.2. Put $U = (W^\dagger \otimes 1)U$ (a unitary) and, for a unit vector $\psi \in H_O$, $K_\psi = ((\langle \psi | \otimes 1_R) U (|\psi\rangle \otimes 1_R)) \in B(H_R)$. For the product input $\psi \otimes \phi$ one has

$$\langle W\psi | \rho'_{O'} | W\psi \rangle = \|(\langle \psi | \otimes 1) U |\psi \otimes \phi\rangle\|^2 = \|(\langle \psi | \otimes 1) U |\psi \otimes \phi\rangle\|^2 = \|K_\psi \phi\|^2. \quad (4.1)$$

Step 1 (proof of (a)). Averaging over ϕ (Lemma 4.3(i)) gives $\frac{\|K_\psi\|_{HS}^2}{d_R}$. Take the operator-Schmidt decomposition $U = \sum_j s_j A_j \otimes B_j$ with $s_j \geq 0$, $\{A_j\}$ orthonormal in $B(H_O)$ and $\{B_j\}$ orthonormal in $B(H_R)$ with respect to the Hilbert–Schmidt inner product; then $\sum_j s_j^2 = \|U\|_{HS}^2 = d_O d_R$. Now $K_\psi = \sum_j s_j \langle \psi | A_j | \psi \rangle B_j$, so $\|K_\psi\|_{HS}^2 = \sum_j s_j^2 |\langle \psi | A_j | \psi \rangle|^2$, and by Lemma 4.3(ii) its average over ψ is $\sum_j \frac{s_j^2 (1 + |\text{Tr} A_j|^2)}{d_O(d_O+1)}$. Since $\sum_j s_j^2 |\text{Tr} A_j|^2 = \|\sum_j s_j (\text{Tr} A_j) B_j\|_{HS}^2 = \|T_W\|_{HS}^2$, we obtain

$$1 - E_W(U) = \frac{1}{d_R d_O(d_O+1)} (d_O d_R + \|T_W\|_{HS}^2) = \frac{1 + d_O \kappa_W}{d_O + 1}, \quad (4.2)$$

which is (a). Let $V_0 = \frac{T_W}{d_O}$. By Lemma 4.3(iii), $V_0 = \mathbb{E}_\psi K_\psi$; each K_ψ is a compression of a unitary, hence a contraction, and so is V_0 . Thus the singular values σ_i of V_0 lie in $[0,1]$ and $\kappa_W = d_R^{-1} \sum_i \sigma_i^2 \in [0,1]$.

Step 2 (proof of (b)). For every unitary V on H_R , $\|U - W \otimes V\|_{HS}^2 = \|U - 1 \otimes V\|_{HS}^2 = 2d_O d_R - 2 \text{Re} \text{Tr}[(1 \otimes V^\dagger)U] = 2d_O d_R - 2d_O \text{Re} \text{Tr}[V^\dagger V_0]$, because $\text{Tr}[(1 \otimes V^\dagger)U] = \text{Tr}[V^\dagger \text{Tr}_O U] = d_O \text{Tr}[V^\dagger V_0]$. The maximum of $\text{Re} \text{Tr}[V^\dagger V_0]$ over unitaries is $\|V_0\|_1 = \sum_i \sigma_i$, attained at the unitary polar factor of V_0 . Hence $D_W(U)^2 = 2 - 2 \frac{\|V_0\|_1}{d_R} = \frac{2}{d_R} \sum_i (1 - \sigma_i)$, which is (b).

Step 3 (proof of (c)). For $\sigma \in [0,1]$ one has $\frac{1}{2}(1 - \sigma^2) \leq 1 - \sigma \leq 1 - \sigma^2$. Summing over i and dividing by d_R gives $1 - \kappa_W \leq D_W^2 \leq 2(1 - \kappa_W)$. By (a), $1 - \kappa_W = (1 + \frac{1}{d_O})E_W$. Equality in the right inequality holds iff every $\sigma_i \in \{0,1\}$, which is the case for $W = 1$, $U = \text{CNOT}$ ($\sigma = (1,0)$); the left one becomes an equality in the limit $\sigma_i \rightarrow 1$. The statement for ε and D follows by minimizing over W on each side. ■

Proof of Theorem 4.1. (i) $\Leftrightarrow$ (ii): $E_W = 1 - \mathbb{E} p$ with $p(\psi, \phi) = \langle W\psi | \rho'_{O'} | W\psi \rangle \in [0,1]$ continuous, so $E_W = 0$ iff $p \equiv 1$ iff $\rho'_{O'} = |W\psi\rangle\langle W\psi|$ for all inputs. (i) $\Rightarrow$ (iii): if $E_W(U) = 0$, then $\kappa_W = 1$, so all $\sigma_i = 1$, so $D_W(U) = 0$ by (b) and $U = W \otimes V$ (the minimum over V is attained). (iii) $\Rightarrow$ (i): if $U = W \otimes V$, then $U = 1 \otimes V$, $K_\psi = V$ and $\|K_\psi \phi\| = 1$, so $E_W = 0$. The remaining equivalences follow from $D(U) = 0 \Leftrightarrow U$ product. ■

Corollary 4.4 (weak coupling: a guaranteed short-time autonomy bound).

(a) Let $H = H_O \otimes 1 + 1 \otimes H_R + H_{\text{int}}$ on $H_O \otimes H_R$ and $U(t) = e^{-iHt}$. Then $D(U(t)) \leq t \|H_{\text{int}}\|$ and $\varepsilon(U(t)) \leq (t \|H_{\text{int}}\|)^2$. In particular the law $W = e^{-iH_O t}$ predicts the output of O to accuracy $E_W \leq \varepsilon_0$ (average defect) for all $t \leq \frac{\sqrt{\varepsilon_0}}{\|H_{\text{int}}\|}$.

(b) For a step unitary U with $D(U) \leq D_1$ and all n : $D(U^n) \leq n D_1$ and $\varepsilon(U^n) \leq (n D_1)^2$.

(c) In the setting of (a), the *worst-case* defect obeys $1 - \langle W\psi | \rho'_{O'} | W\psi \rangle \leq (t \|H_{\text{int}}\|)^2$ for every product input $\psi \otimes \phi$ (Proposition 4.7).

Proof. (a) With $U_0(t) = e^{-iH_O t} \otimes e^{-iH_R t}$, Duhamel's formula gives $U(t) - U_0(t) = -i \int_0^t U(t-s) H_{\text{int}} U_0(s) ds$, so $\|U(t) - U_0(t)\| \leq t \|H_{\text{int}}\|$. Since $\frac{\|X\|_{HS}}{\sqrt{d_O d_R}} \leq \|X\|$, $D_W(U(t)) \leq t \|H_{\text{int}}\|$ with $W = e^{-iH_O t}$. Theorem 4.2(c) gives $E_W \leq \frac{d_O}{d_O + 1} D_W^2 \leq (t \|H_{\text{int}}\|)^2$.

(b) Let $P = W \otimes V$ attain $D(U)$. Telescoping and unitary invariance of $\|\cdot\|_{HS}$ give $\|U^n - P^n\|_{HS} \leq n \|U - P\|_{HS}$; then apply Theorem 4.2(c) as in (a). (c) Proposition 4.7 with $\|U(t) - U_0(t)\| \leq t \|H_{\text{int}}\|$. ■

Remark 4.5 (three pitfalls). (i) *Fixed preparation.* Let U be the CNOT with O as control, and prepare O in $I/2$. Then $\rho'_O = I/2$ for every state of R , so $W = 1$ reproduces the output of O for this preparation. Nevertheless, averaged over O 's inputs, $E_1(\text{CNOT}) = 1/3$ (analytically: $\sigma = (1, 0)$, $\kappa = 1/2$), and a numerical search over W finds nothing smaller (Appendix A). A statement about autonomy must therefore quantify over the inputs of O , as Definition 2.5 does; the version with a fixed preparation is false. (ii) *Entangling versus non-product.* The swap ($d_O = d_R = d$) creates no entanglement from product inputs, yet $E_W(\text{SWAP}) = 1 - 1/d$ for every W , since O 's output is R 's input. Autonomy is lost by non-product couplings, not only by entangling ones. (iii) *Outputs versus laws.* A non-product unitary can leave O in a pure state for particular inputs; Theorem 4.1 concerns the existence of a law valid for all inputs, not the purity of individual outputs.

Remark 4.6 (relation to the literature). Theorem 4.1 is a restricted unitary no-signalling / autonomy statement. It demands a single fixed unitary law W for O that is the same for all states of R , which is more specific than semicausality, where the marginal of O merely may not depend on the input of R . Semicausal operations were characterised by Beckman, Gottesman, Nielsen and Preskill (2001) and, in general, by Eggeling, Schlingemann and Werner (2002); the Haar-average proof given here is self-contained and yields the quantitative Theorem 4.2. The autonomy defect could equally be replaced by the unitarity of the induced channel (Wallman et al. 2015); we use E_W because it comes with the exact identities of Theorem 4.2. A relation of E_W to entangling power (Zanardi 2001) is plausible but is not explored here.

Proposition 4.7 (worst case, one direction). For all unitaries U, W, V (on H, H_O, H_R) and all unit vectors $\psi \in H_O, \varphi \in H_R$,

$$1 - \langle W\psi | \rho'_O(\psi, \varphi) | W\psi \rangle \leq \| (U - W \otimes V) | \psi \otimes \varphi \rangle \|^2 \leq \| U - W \otimes V \|^2. \quad (4.3)$$

Hence a small distance to a product unitary in *operator* norm controls the worst-case autonomy defect, quadratically. (For the average defect this reproduces $E_W \leq D_W^2$ of Theorem 4.2, with a slightly worse constant.)

Proof. Put $a = (W \otimes V) | \psi \otimes \varphi \rangle = W\psi \otimes V\varphi$, $\Delta = U - W \otimes V$, $x = \|\Delta | \psi \otimes \varphi \rangle\|$ and $u = U | \psi \otimes \varphi \rangle = a + \Delta | \psi \otimes \varphi \rangle$. Both u and a are unit vectors, so $0 = \|u\|^2 - 1 = 2 \operatorname{Re} \langle a | \Delta \psi \varphi \rangle + x^2$. Let $z = (\langle W\psi | \otimes 1) \Delta | \psi \otimes \varphi \rangle$, so $\|z\| \leq x$ and $\langle V\varphi | z \rangle = \langle a | \Delta \psi \varphi \rangle$. Then $\langle W\psi | \rho'_O | W\psi \rangle = \| (\langle W\psi | \otimes 1) u \|^2 = \| V\varphi + z \|^2 = 1 + 2 \operatorname{Re} \langle V\varphi | z \rangle + \|z\|^2 = 1 - x^2 + \|z\|^2 \geq 1 - x^2$. ■

5. Stock and flow

Definition 5.1. Let $\Psi(t)$ be the global state and $U_{t,t+\delta t}$ the unitary of one step. The *stock* at time t is $S(\rho_O(t))$. The *flow* of the step is its autonomy defect $\varepsilon(U_{t,t+\delta t})$ (equivalently, up to Theorem 4.2, its distance D from product unitaries). The terms “stock” and “flow” are mnemonic labels for a state property and a step property; they do not denote a conserved or transported physical quantity.

Proposition 5.2 (state versus step).

- (i) If $U = W \otimes V$ is a product unitary, then $\rho_O(U\Psi) = W\rho_O(\Psi)W^\dagger$ for every global pure state Ψ , correlated or not; in particular the stock is unchanged.
- (ii) All four combinations of {stock zero, positive} $\times$ {flow zero, positive} occur. Stock positive, flow zero: a CNOT applying a Hadamard gate to the target qubit followed by a CNOT to $|00\rangle$ yields a Bell state ($S = \ln 2$), and a subsequent product step $u_O \otimes u_R$ gives $\rho'_O = u_O \rho_O u_O^\dagger$ with unchanged purity

and $\varepsilon = 0$. Stock zero, flow positive: for $d_O = d_R = d$ the swap maps a product state to a product state (stock stays 0), yet $\varepsilon(\text{SWAP}) = 1 - \frac{1}{d} > 0$. Both zero: a product step on a product state. Both positive: a generic step on a generic state.

Proof. (i) $\text{Tr}_R[(W \otimes V)|\Psi\rangle\langle\Psi|(W \otimes V)^\dagger] = W\rho_O W^\dagger$. (ii) The values are as stated by direct computation and Remark 4.5(ii). ■

The two notions concern different objects, a state and a step, and neither implies the other. Their common source is a non-product coupling: it is what makes $\varepsilon > 0$, and it is what can make $S(\rho_O)$ grow from a product state.

Proposition 5.3 (flow bounds the growth of the reduced entropy). Let U be unitary, W a unitary on H_O , $\psi \otimes \varphi$ a product input, $\rho' = \rho'_O(\psi, \varphi)$ and $\delta = 1 - \langle W\psi|\rho'|W\psi\rangle$. With h the binary entropy and $g(x) = h(x) + x\ln(d_O - 1)$,

$$S(\rho') \leq g(\min\{\delta, 1 - \frac{1}{d_O}\}). \quad (5.1)$$

Consequently, for Haar-random product inputs, $\mathbb{E} S(\rho') \leq g(\min\{\varepsilon(U), 1 - \frac{1}{d_O}\})$ with the optimal W .

Proof. The largest eigenvalue λ of ρ' satisfies $\lambda \geq \langle W\psi|\rho'|W\psi\rangle = 1 - \delta$ and $\lambda \geq \frac{1}{d_O}$. Among probability vectors with maximal entry λ the entropy is at most $h(\lambda) + (1 - \lambda)\ln(d_O - 1) = g(1 - \lambda)$, and g is increasing on $[0, 1 - \frac{1}{d_O}]$, which gives (5.1). The function $x \mapsto g(\min\{x, 1 - \frac{1}{d_O}\})$ is concave and non-decreasing, so Jensen's inequality yields the statement for averages, since $\mathbb{E}\delta = E_W(U)$. ■

Corollary 5.4 (short-time growth of the reduced entropy under weak coupling). In the setting of Corollary 4.4(a), for a product initial state drawn at random and $x = (t\|H_{\text{int}}\|)^2 \leq 1 - \frac{1}{d_O}$: $\mathbb{E} S(\rho_O(t)) \leq g(x) \leq x[1 + \ln(\frac{d_O - 1}{x})]$.

Proposition 5.5 (the stock is not monotone).

- (a) $S(u_O \rho_O u_O^\dagger) = S(\rho_O)$ for every unitary u_O on H_O : no O -local unitary reduces the stock.
- (b) Under the global dynamics the stock need not grow: applying U and then U^{-1} returns a product state to a product state, and, since finite-dimensional unitary dynamics is almost periodic, $S(\rho_O(t))$ returns arbitrarily close to $S(\rho_O(0))$ infinitely often (Poincaré recurrence).

Remark 5.6. Neither a monotonic growth of $S(\rho_O(t))$ nor irreversibility under arbitrary operations confined to O holds in general: (b) shows the first fails, and local *non-unitary* operations (for instance a reset) can lower $S(\rho_O)$ - by discarding, not by recovering, information about the global state. We make no claim about such operations. In the repeated-interaction (collision-model) literature (Ciccarello et al. 2022), with fresh i.i.d. ancillas at every step, stock and flow degrade together by construction, which is why they are not usually separated.

Remark 5.7 (physical reading: apparent collapse). Corollary 5.4 and Proposition 5.3 say why a non-product coupling makes O look, from its own perspective, as though a definite outcome has been selected: a non-product step raises $S(\rho_O)$ away from 0 (Corollary 3.3), which is exactly the standard signature of decoherence (Zurek 2003) - ρ_O stops looking pure once O is correlated with R . The global state Ψ stays exactly pure and unitary throughout (its own entropy is always 0), so nothing global collapses; only O 's local description changes. This is the ordinary decoherence mechanism, not a new one, and it is only *apparent*: by Proposition 5.5, no O -local unitary can lower $S(\rho_O)$, but the global dynamics can, and by Poincaré recurrence $S(\rho_O(t))$ returns arbitrarily close to 0 infinitely often. We settle no question about definite outcomes or the measurement problem; the point is only that the entropy accounting of §3 and §5, applied to the same non-product coupling that Theorem 4.2 shows cannot be autonomous, already gives the usual decoherence story for why O perceives one.

6. No escape by extension or by identification with the whole

Within the tensor-factor model of §2, one may try to escape Proposition 3.1 and Theorems 4.1–4.2 by adding a further observer Z ‘outside’ O , or by identifying the observer with the whole system. Neither escapes: both cases reduce to the results above.

Proposition 6.1 (extension). Let $H' = H_Z \otimes H_X$ be closed with pure global state, where Z is a further observer and X the rest (which may itself be $H_O \otimes H_R$). Then:

- (i) $I(Z:X) = 2S(\rho_Z)$. Z is uninformative about X ($I = 0$) iff the global state is a product of pure states.
- (ii) If Z is informative ($I > 0$), then $S(\rho_Z) > 0$, and Proposition 3.1, Corollary 3.3 and Theorems 4.1–4.2 apply to the pair (Z, X) verbatim.
- (iii) Iterating, a further tensor-factor extension Z' of $H_Z \otimes H_X$ is again either uninformative or subject to the same results. No finite chain of tensor-factor extensions produces a subsystem that is informative about its complement while exempt from the same results.

Proof. (i) is Corollary 3.3 for the bipartition (Z, X) ; $I = 0$ forces a pure product state. (ii) follows from (i) and the fact that Z is a proper subsystem of the closed system H' . (iii) is induction on (ii). ■

Identification with the whole. Take the observer to be the entire system, so that $d_R = 1$. Proposition 3.1 then does not apply ($F(\Psi)$ is a point), but no operational claim of knowledge about anything outside the observer is being made: the “knowledge” the whole has of its parts is the trivial ontological fact that Ψ exists. Any non-vacuous claim must name a reader $M \subsetneq H$, a physically instantiated proper subsystem (a memory, a sensory channel), and the results apply to M . Corollary 3.3 then states the price of acquired knowledge: a reader that starts uncorrelated with the rest acquires correlation only as fast as $S(\rho_M)$ grows, since $I(M:\text{rest}) = 2S(\rho_M)$.

Remark 6.2. Proposition 6.1 is a statement within the tensor-factor model of §2, not an argument for the postulate of §1: it does not show that no physical system can occupy an external role, only that inside this model, enlarging the factor never produces a subsystem exempt from Proposition 3.1 and Theorems 4.1–4.2. Whether every physical observer admits a tensor-factor description in the first place is the postulate, not a consequence proved here.

7. Locality and exact causal order

Let G be a finite graph of maximum degree z and $H = \otimes_{v \in V} H_v$. A *discrete local circuit* partitions the edges into K classes $E_1, \dots, E_K$ of pairwise disjoint edges ($K \leq z + 1$ by Vizing’s theorem); one step is $U = U_K \cdots U_1$ with $U_k = \prod_{e \in E_k} U_e$, where U_e acts on the two endpoints of e . Gates within a class act on disjoint sites and commute. The support $\text{supp}(A)$ of an operator is the smallest vertex set outside which A acts as the identity, and dist is the graph distance between vertex sets.

Lemma 7.1 (support growth). Let $\text{supp}(A) = X_0$ and $A(n) = U^{-n} A U^n$. Then $\text{supp}(A(n)) \subseteq N_{nK}(X_0) = \{v : \text{dist}(v, X_0) \leq nK\}$. More precisely, in each of the nK sublayers of the Heisenberg evolution, a gate acts non-trivially on the current operator only if one of its endpoints lies in the current support, and it then adds at most its other endpoint.

Proof. Gates whose edge touches no vertex of the current support act as the identity there and commute with the current operator (operators on disjoint tensor factors commute). A gate whose edge touches the support enlarges it by at most the other endpoint. Hence each sublayer extends the support by at most one unit of graph distance, each step by at most K . ■

Theorem 7.2 (exact causal cone). If $\text{dist}(X_0, \text{supp}(B)) > nK$, then $[A(n), B] = 0$ exactly, with no exponential tail.

The exactness is a property of the circuit model; for a local Hamiltonian, the Lieb–Robinson bound (1972) gives the corresponding statement with an exponentially small tail. The cone here is the causal cone of a finite-depth circuit, not a relativistic light cone.

Proposition 7.3 (exact core autonomy). Let the vertex set be partitioned as $V = V_O \sqcup V_R$, with O and R the corresponding tensor factors, and let $O_0 \subseteq V_O$ with $L = \text{dist}(O_0, V_R)$. Let U_O be the circuit obtained by keeping only the gates with both endpoints in V_O . Then for every global state and every n with $nK < L$,

$$\rho_{O_0}(n) = \text{Tr}_{O \setminus O_0} [U_O^n \rho_O(0) (U_O^n)^\dagger]. \quad (7.1)$$

Thus, until the cone of R arrives, the reduced state of the core O_0 is given exactly by a fixed unitary law that involves only O : it does not depend on the state of R , nor on any gate touching R or the O – R interface.

Proof. Let A be supported in O_0 . By Lemma 7.1 the support of the operator after m sublayers lies in $N_m(O_0)$, and $N_m(O_0) \cap V_R = \emptyset$ for $m \leq nK < L$. A gate that touches the current support has both endpoints in $N_{m+1}(O_0) \subseteq V_O$ (for $m+1 \leq nK$), so it belongs to U_O ; all other gates commute with the current operator and can be included or omitted freely. Hence $A(n) = U_O^{-n} A U_O^n$, an operator on O , and $\text{Tr}[\rho(n)A] = \text{Tr}[\rho A(n)] = \text{Tr}[\rho_O(0)U_O^{-n} A U_O^n]$. ■

Remark 7.4. Read together with §4: autonomy fails first at the O – R interface, and the region on which the law of O is exact recedes from it at speed at most K sites per step. The stock $S(\rho_O(n))$ can be positive from the first step, because the interface region of O is coupled to R immediately; Proposition 7.3 concerns the core O_0 only. Proposition 7.3 also shows what the no-go of §4 does not say: exact autonomy of sufficiently distant sub-regions of O is possible for a finite time. Numerical confirmation is in Appendix A.

Corollary 7.5 (two-source window). Let V_R be partitioned as $F \sqcup S$ (with any remaining vertices no closer to O_0 than S), and let $L_F = \text{dist}(O_0, F) < L_S = \text{dist}(O_0, S \cup \text{rest})$. Then:

- (a) For $nK < L_F$: apply Proposition 7.3 with $V_R = F \cup S \cup \text{rest}$, $L = L_F$. $\rho_{O_0}(n)$ is exact and independent of both F and S .
- (b) For $L_F \leq nK < L_S$: apply Proposition 7.3 again, now with the enlarged factor $V_O' = V_O \cup F$ and $V_R' = S \cup \text{rest}$, $L = L_S$. $\rho_{O_0}(n)$ is exact and given by gates within $V_O \cup F$ alone - independent of S , though not necessarily of F .
- (c) For $nK \geq L_S$: no guarantee remains from either application.

Proof. Two direct instances of Proposition 7.3 with the two choices of tensor factor above; (c) is that neither hypothesis holds. ■

Remark 7.6 (dependence on F is generic, not exactly pinned to the threshold). Part (b) gives an exact, one-sided guarantee: for $nK < L_F$, O_0 is exactly independent of F (Theorem 7.2). It does not give a matching guarantee in the other direction - crossing $nK \geq L_F$ only *removes* the independence guarantee; it does not force dependence to appear at that exact step. For a product-unitary interface at the O_0 – F boundary (Theorem 4.1), O_0 could remain independent of F indefinitely, and the window would be vacuous. For generic (e.g. Haar-random) interface gates, dependence on F appears generically once $nK \geq L_F$, but the exact step at which it first appears can lag behind the threshold by an amount that depends on the specific circuit and graph, not only on L_F . This is confirmed numerically on two structurally different circuits (path, $K=2$ and branching tree, $K=3$; script `embedded_observer_checks_corollary75.py`): on the path, dependence on F appears at the first step with $nK \geq L_F$ ($n=2$, $nK=4$); on the tree, it lags by one step ($nK=3$ at $n=1$ gives exact zero; dependence appears only at $n=2$, $nK=6$). In both cases, independence from S remains exact throughout the predicted window $L_F \leq nK < L_S$, exactly as (b) guarantees, and dependence on S appears only once the guarantee is lost ($nK \geq L_S$).

8. Scope, prior work, and interpretation

8.1 Scope and limitations

- **Finite dimension and exact factorizations.** In continuum quantum field theory the algebra of a region is generically a type III factor: there is no trace, no density matrix and no tensor factorization (Witten 2018), so ρ_O is not defined. An extension of §3–§5 needs an operator-algebraic reformulation and is not claimed.
- **Closed, pure global state.** Mixed global states or open global dynamics leave the formalism.
- **Interface size and time scale.** For a local Hamiltonian, H_{int} contains only the terms straddling the $O|R$ cut, so $\|H_{\text{int}}\|$ scales with the size of the interface, not with the volume of O . For a large interface (for instance an environment coupled to every constituent of O) Corollary 4.4 gives only a short time. It is a guaranteed bound, not a prediction that non-autonomy appears at that time.
- **Model statements.** The results of §7 are statements about the circuit model; no experimental test is proposed, since a device with finite gate error cannot confirm an exact zero.

8.2 Relation to prior work

The impossibility of a God's-eye vantage point for an observer inside a unitarily described system is not new. Wigner (1961) observed that a friend's measurement, treated unitarily from outside, conflicts with the friend's report of a definite outcome. Brukner (2018) derived a no-go theorem for observer-independent facts and Frauchiger and Renner (2018) an inconsistency theorem for nested observers reasoning with quantum theory; the Local Friendliness inequalities of this programme have been violated in experiments with photonic “friends” (Proietti et al. 2019; Bong et al. 2020). The present results are of a different and weaker kind: they are linear-algebraic facts about what the data and dynamics of a subsystem can and cannot express, they derive no inconsistency, and they involve no multi-observer protocol; no bridge to those results is claimed. The Page–Wootters construction (1983) develops embeddedness into a relational account of time, and the quantum-reference-frame literature (Giacomini, Castro-Ruiz and Brukner 2019) concerns superpositions of frames rather than of outcomes; the present setting belongs to neither.

8.3 Interpretation

The results follow from unitary quantum mechanics on a finite-dimensional space alone and are interpretation-neutral: nothing here favours or disfavors any interpretation of measurement or of Ψ . The decoherence-based reading of §5 (Remark 5.7) is standard physics, shared by every interpretation, not an added interpretive claim; we take no position on the further question of definite outcomes.

8.4 Open problems

- A converse of Proposition 4.7: does a small worst-case defect force U to be close to a product unitary in operator norm, with dimension-free constants?
- Sharper short-time bounds than Corollary 4.4 for large interfaces, e.g. from Lieb–Robinson-type estimates.
- Extension of §3–§5 to algebraic settings (type II and type III factors), where the objects ρ_O and $S(\rho_O)$ need to be replaced.
- The relation of E_W to entangling power and to the operator entanglement of U .

Appendix A. Numerical checks

The script `embedded_observer_checks.py` in the repository

https://github.com/SergejMaterov/Embedded_Observer_NoGo requires only numpy and scipy and uses seeded random numbers; a pytest suite (directory `tests`) checks the same statements with fresh seeds. The

script performs the checks below; the numbers are the output of the run stored in *results/checks_output.txt*.

C1. Closed formula. $E_W(U)$ from Theorem 4.2(a) against a Monte-Carlo average over 6×10^4 Haar product inputs:

(d_O, d_R)	Formula (4.2)	Monte-Carlo	Difference
(2, 2)	0.56973	0.56962	1.1×10^{-4}
(2, 3)	0.51693	0.51793	1.0×10^{-3}
(3, 2)	0.63196	0.63149	4.7×10^{-4}

C2. Theorem 4.2(c). 4200 random pairs (U, W) , with $U = (W_0 \otimes V_0) e^{-itH}$ for $t \in \{0, 0.01, \dots, 3\}$ and W either the ideal W_0 or Haar-random, in dimensions (2,2), (2,3), (3,2), (3,3), (2,4): both inequalities held in every case. The maximum of $\frac{(1 + \frac{1}{d_O})E_W}{D_W^2}$ was 1.0000 (tends to 1 near product unitaries) and the maximum of $\frac{D_W^2}{2(1 + \frac{1}{d_O})E_W}$ was 0.9978. Sample rows for $t = 0.1$ and the ideal W :

d_O	d_R	E_W	$(1 + \frac{1}{d_O})E_W$	D_W^2	$2(1 + \frac{1}{d_O})E_W$
2	2	0.002021	0.003032	0.003035	0.006064
2	3	0.001914	0.002872	0.002874	0.005743
3	2	0.002659	0.003545	0.003549	0.007090
3	3	0.002442	0.003256	0.003259	0.006512
2	4	0.002197	0.003296	0.003299	0.006591

C3. Exact case and counter-examples. A product unitary with its own W gives $E_W = 0$ to machine precision. The swap gives $E_W = 0.5000 = 1 - \frac{1}{d}$ for every W tried. For the CNOT prepared with $\rho_O = \frac{I}{2}$, the maximal deviation of ρ'_O from $\frac{I}{2}$ over 200 random states of R was 3×10^{-16} (Remark 4.5(i)), while the minimum of E_W over 2000 random W was 0.3335 ($\approx 1/3$).

C4. Corollary 4.4. For random H_O, H_R, H_{int} with $t \in \{10^{-3}, \dots, 0.2\}$ and coupling scales $\{0.01, 0.1, 1\}$, all cases satisfied $D \leq t\|H_{\text{int}}\|$ and $E_W \leq (t\|H_{\text{int}}\|)^2$.

C5. Proposition 5.3. In 16 000 random product inputs and near-product unitaries (dimensions (2,2), (3,3), (2,4), (4,2)), there were no violations of (5.1); the maximal ratio of entropy to bound was 1.000.

C6. Exact causal cone and core autonomy. Ten-site qubit chain, Haar-random two-site gates fixed once, $K = 2, A = X_0, B = X_9$ (distance 9, so the commutator must vanish exactly for $n \leq 4$):

n	$\ A(n), B\ $	Prediction (Theorem 7.2)
0	0	exactly 0
1–4	$\leq 6 \times 10^{-15}$ (machine zero)	exactly 0
5	1.205	non-zero permitted
6, 7, 8, 9	1.670, 1.931, 1.998, 2.000	non-zero permitted

For an eight-site chain with $O = \text{sites } 0\text{--}4, R = \text{sites } 5\text{--}7$ and core $O_0 = \text{site } 0$ ($L = 5, K = 2$, exact for $n \leq 2$), the spread of $\rho_{O_0}(n)$ over different initial states of R and different gates on R and on the interface was 6.7×10^{-16} ($n = 1$) and 3.5×10^{-16} ($n = 2$), and the deviation from the internal prediction (7.1) was 4.5×10^{-16} and 2.2×10^{-16} ; at $n = 3$, beyond the cone, the spread was 6.4×10^{-2} .

C7. Proposition 3.1(b). The real dimension of $F(\Psi)$ was computed numerically as the rank of the tangent space of the orbit, for random states of Schmidt rank r :

d_o	d_r	r	Numerical	$2rd_r - r^2 - 1$
2	2	1	2	2
2	2	2	3	3
2	3	1	4	4
2	3	2	7	7
3	3	1	4	4
3	3	2	7	7
3	3	3	8	8
3	4	3	14	14
4	4	4	15	15

C8. Proposition 4.7. In 3600 random tests (arbitrary product unitaries $W \otimes V$, often far from U) neither inequality in (4.3) was violated; the maximal ratios of left to middle and middle to right terms were 0.816 and 0.9985. Weak coupling ($d_o = 2, d_r = 3$): the supremum of the defect over 20 000 random product inputs was 8.3×10^{-4} , 7.8×10^{-3} and 8.3×10^{-2} for $t\|H_{\text{int}}\| = 0.040, 0.121, 0.404$, below $(t\|H_{\text{int}}\|)^2 = 1.6 \times 10^{-3}, 1.5 \times 10^{-2}, 0.16$.

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