

Analysis of party systems by measures of concentration of inequality and asymmetry of the Pareto curve

Gennady A. Grachev

Southern Federal University, Rostov-On-Don, Russia,

e-mail: grachev@sfedu.ru

Abstract

In this article, the Hoover index and asymmetry coefficient of the Pareto curve were tested to measure the state of the systems. When applying the new concept to the analysis of party systems in 18 European countries (158 election cases), we found that most of them showed a left-hand asymmetry of the Pareto curve and a concentration of inequality close to the Pareto principle.

Keywords

Party system; effective number of parties; number of successful parties; concentration of party systems; concentration typology

Introduction

Currently, the most widely used measure in political science for the state of party systems based on election results is the “effective number of parties, Eff NP” introduced by Laakso and Taagepera. The conceptual idea of the Eff NP and its varieties (Dunleavy & Boucek, 2016; Gaines & Taagepera, 2014; Golosov, 2010; Molinar, 1991; Rae, 1967; Taagepera, 1999) is to combine the number and size distribution of parties into a single fragmentation factor equal to the number of “important parties.” The advantage of the Eff NP is the simplicity of calculations, while the disadvantage is the ambiguity in measuring the state of the systems, since different scenarios for the distribution of votes in elections can correspond to the same value of the Eff NP. Also, being an artificial measure of the state of systems, the Eff NP has no intuitive meaning, which makes it more complicated to interpret the results obtained (Bogaards, 2004; Dunleavy & Boucek, 2016; Golosov, 2010; Magyar, 2022). In this regard, it is of interest to supplement the Eff NP with other parameters to reduce the ambiguity in the state of the systems.

Hereby we propose to use the Hoover index, HI, and the Pareto curve asymmetry coefficient, PAC, as additional parameters for the state of party systems. Conceptually, the HI is the simplest and most intuitive measure of the concentration of inequality, which determines the proportion of votes that must be redistributed to achieve equality of parties. The PAC determines which parties contribute the most to the overall inequality as measured with the HI.

Methods

Hoover index

Let the number of parties that received at least one vote in the elections be n . We denote the proportion of votes (party size) ranked in descending order by w_r . Then, we denote the cumulative sum of proportion values for votes l of parties as $S_l = \sum_{r=1}^l w_r$. By convention, $S_n = 1$.

The Eff NP is a real number calculated using the formula (Laakso & Taagepera, 1979):

$$\text{Eff NP} = \frac{1}{\sum_{r=1}^n w_r^2}. \quad (1)$$

Successful parties will mean parties that won a proportion of votes of not less than the average value of $\bar{w} = 1/n$ in the elections. Let the number of successful parties (Succ NP) be equal to the natural number m . Therefore, the proportion of successful parties $p_m = m/n$, and the number of votes gained by successful parties is equal to S_m .

The Hoover index can be mathematically defined as (Hoover, 1936):

$$\text{HI} = S_m - p_m, \quad (2)$$

It can be seen from (2) that HI is the portion of votes that would have to be redistributed, i.e., taken from successful parties to other parties to have equal distribution of votes. This is the reason the HI is often referred to as the Robin Hood index. The HI is also known as the Pietra index (Pietra, 1915) or the Schutz index (Schutz, 1951).

The Hoover index satisfies the inequality $0 \leq HI < 1$. The lower limit of the HI is obtained when there is a complete equality of votes, while the upper limit is reached when all voters voted for one party. On a Pareto chart, the HI is equivalent to the longest vertical distance between the Pareto curve and the 45-degree line representing perfect equality.

Pareto curve

The piecewise linear graph $S_i = S(p_i)$ is referred to as the Pareto curve, PC. PCs can be symmetrical or asymmetrical with respect to the alternate diagonal drawn from (0, 1) to (1, 0) of the unit square. PC symmetry means that the curve to the left of the alternative diagonal is a mirror image of the portion of the curve to the right of that diagonal. The PC symmetry condition is mathematically defined by the following equation (Kakwani, 1980):

$$p_m + S_m = 1. \quad (3)$$

When substituting (3) into (2), we find that the proportion of successful parties for symmetrical PCs is equal to

$$p_m = 0.5(1 - HI). \quad (4)$$

Asymmetric PCs are skewed up or down. For up-skewed PCs, the "longer part" is to the left of p_m (the left-hand asymmetry), while for down-skewed PCs, the "longer part" is to the right of p_m (the right-hand asymmetry). To measure the PC skewness, we will calculate the asymmetry coefficient (PAC) using the formula

$$PAC = 1 - p_m - S_m. \quad (5)$$

It can be seen from relation (5) that for PCs with right-hand asymmetry $PAC > 0$, and for PCs with left-hand asymmetry, $PAC < 0$.

PAC describes an important aspect of the Pareto curve shape. It shows which parties contribute the most to the overall inequality of parties as measured with the HI. If $PAC < 0$, then the inequality is primarily due to the relatively large number of small parties. If $PAC > 0$,

then the inequality is primarily associated with the few largest parties. Note that the asymmetry coefficient (5) describes the asymmetry $S(p)$ only in the neighborhood of p_m .

As an example, dots in **Fig. 1** show the PCs of voting results of the parliamentary elections in Norway in 2001 (a) and Bulgaria in 1994. (b).

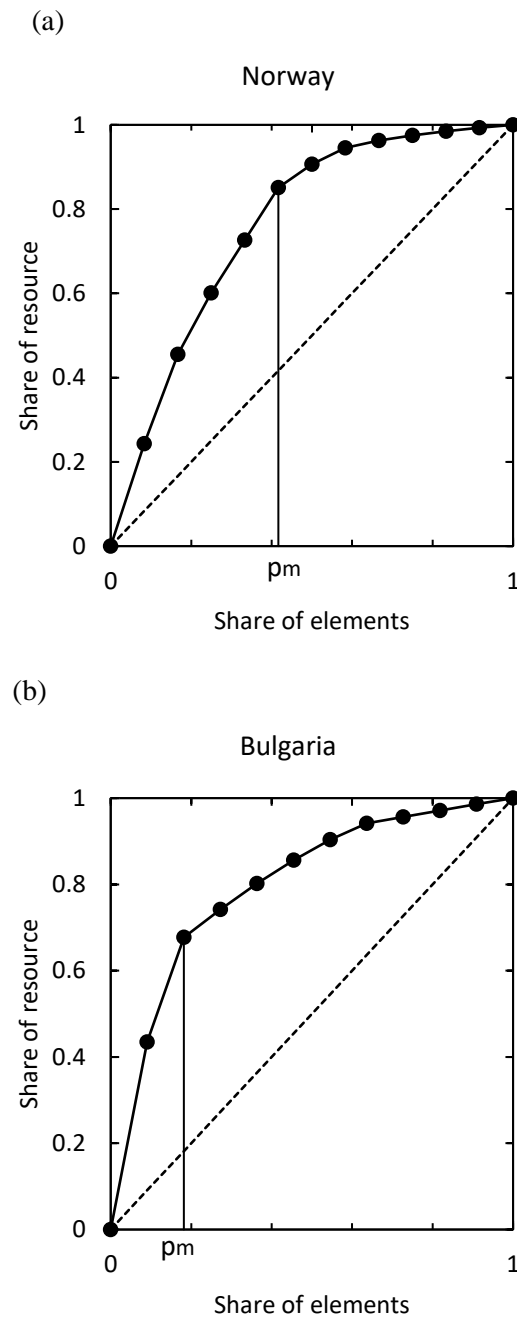


Fig. 1. Pareto curves of voting results in Norway in 2001 (a) and Bulgaria in 1994 (b)

We can see from the charts in **Fig. 1** that the PC of voting results of the parliamentary elections in Norway has a left-hand asymmetry, while in Bulgaria it has a right-hand asymmetry.

Typology of parties

In the late 19th century, Vilfredo Pareto published his research findings on wealth inequality in Italy in the form of a 20/80 ratio — 20% of families owned approximately 80% of all land (Pareto & Page, 1971). In the mid-20th century, Joseph M. Juran, after reading the work of Vilfredo Pareto, came to the conclusion that the 20/80 proportion adequately describes the principle of quality management that he had previously discovered, i.e. "the vital few and the trivial many", and later renamed his quality principle as Pareto principle (Juran, 1954, 1975). In 1996, Epstein and Axtell, using an agent-based model called SugarScape, showed that the 20/80 ratio is a natural phenomenon (Epstein & Axtell, 1996).

The principle of concentration of inequality discovered by Vilfredo Pareto gave rise to numerous studies of similar patterns in systems of different nature. The findings showed that in social systems, approximately 20-30% of a resource utilized provide 70-80% of results associated with this resource; accordingly, the remaining 80-70% provide only 30-20% of results (Zipf, 1949). The boundaries of the Pareto principle expanded by George Kingsley Zipf have embraced fundings on marketing systems (McCarthy & Winer, 2019; Sharp et al., 2019), economic systems (Grachev, 2009), and party systems (Grachev, 2011, 2012, 2013).

From Zipf's development of the Pareto principle, it follows that most common HI values should be in the range from 0.4 to 0.6. Based on this, we can classify the concentration of inequality into three types:

- Type I — high concentration, $0.6 < HI$,
- Type II — moderate concentration, $0.4 \leq HI \leq 0.6$,
- Type III — low concentration, $HI < 0.4$.

Applying to European countries

Empirical data

The public website https://o.nsd.no/european_election_database/ and the official government websites of electoral commissions were used as empirical data sources. 18 European countries were selected to measure the state of party systems. A total of 158 election cases was processed.

The preparation of empirical data for further analysis of party systems included selecting parties that received at least one vote in parliamentary elections and normalizing the sum of proportion values of all parties to 1.

Results

Fig. 2 shows a scatterplot of Share of Succ NP and HI, and **Fig. 3** shows a scatterplot of the ratio of the number of successful parties to the number of effective parties and HI.

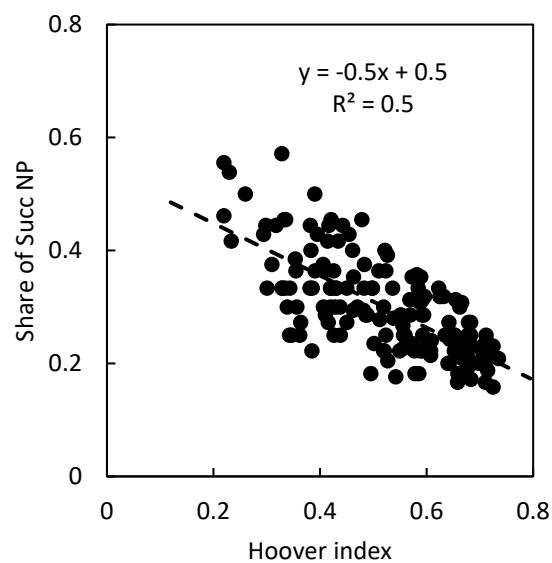


Fig. 2. Scatterplot of Succ NP and HI

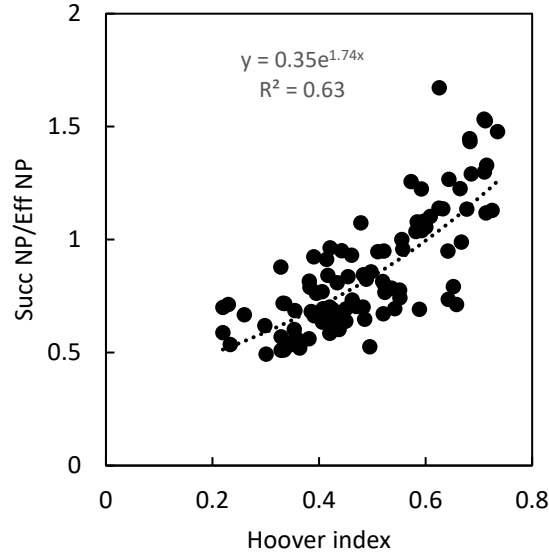


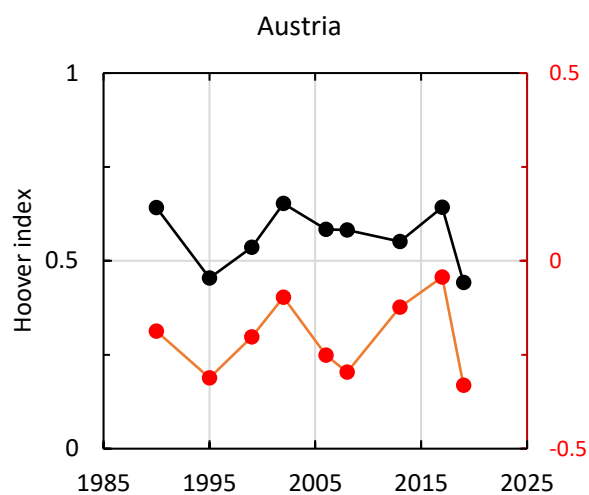
Fig. 3. Scatterplot of the ratio of the Succ NP to the Eff NP and HI

It can be seen from the scatterplot on **Fig. 2** that the regression line describing the relationship between the proportion of successful parties and the Hoover index matches equation (4) that describes the theoretical dependence of the proportion of successful parties on the Hoover index in systems with symmetrical PCs.

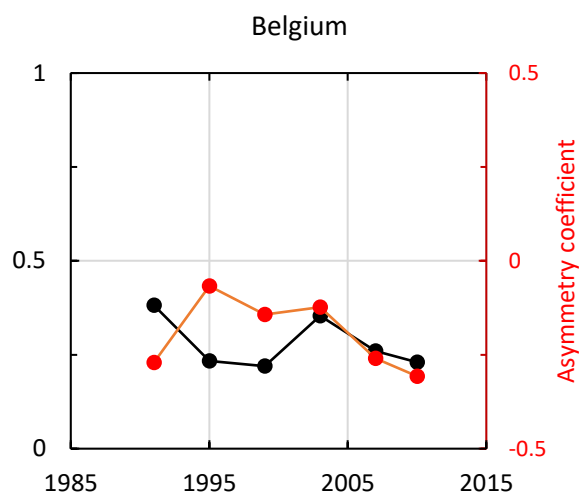
The scatterplot in **Fig. 3** shows that for low and moderate concentration party systems, the number of successful parties is less than the effective number of parties as introduced by Laakso and Taagepera. And conversely, in high concentration party systems, Succ NP values are higher than Eff NP values.

The behavior of HI and PAC of party systems in 18 European countries is shown in **Fig. 4**.

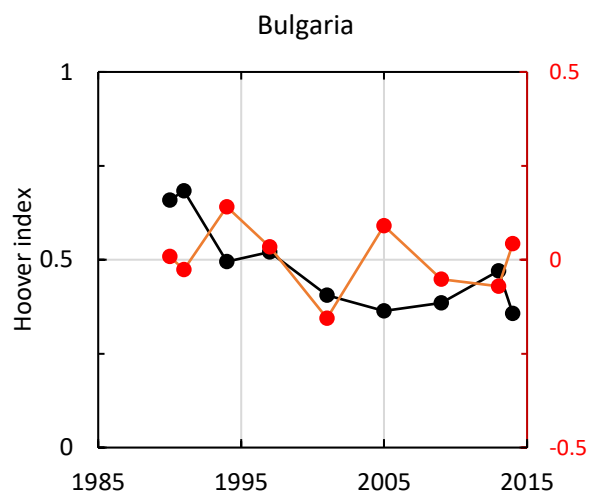
(a)



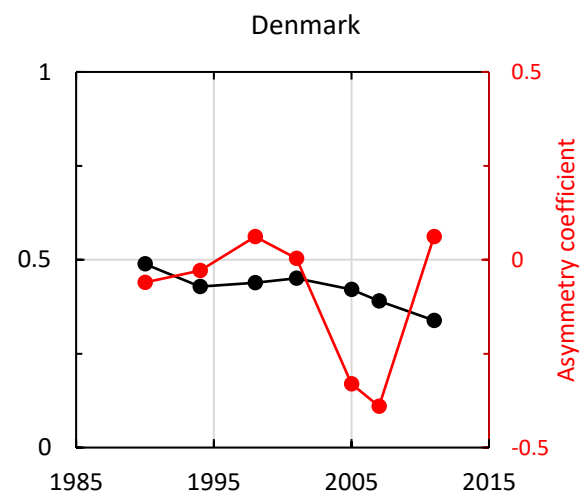
(b)



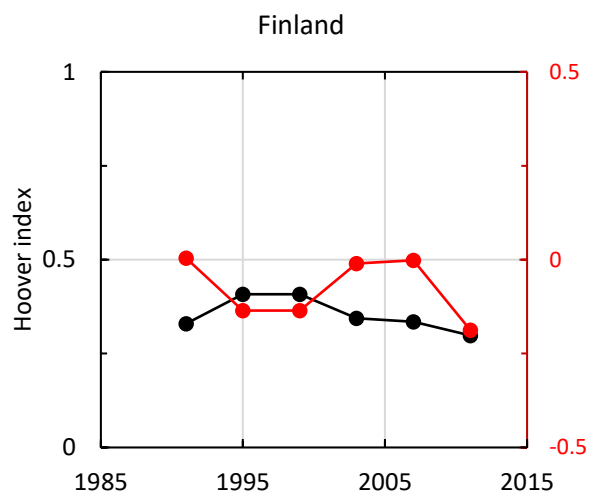
(c)



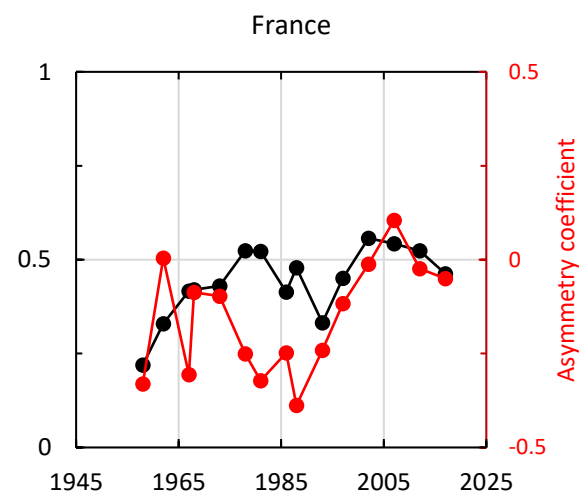
(d)



(e)

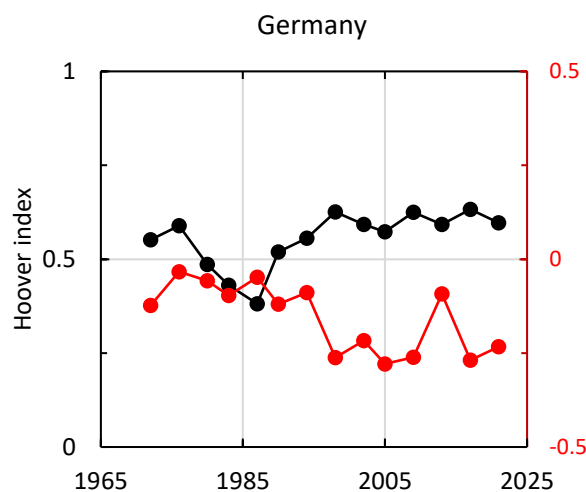


(f)

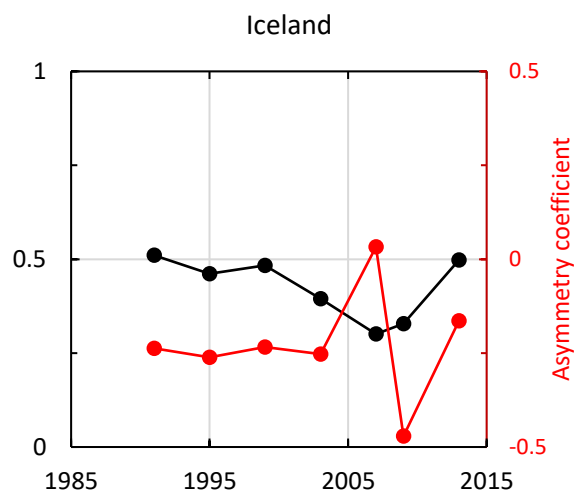


(g)

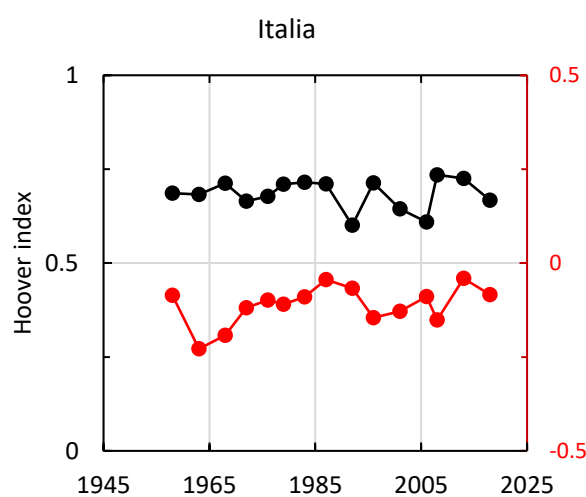
(h)



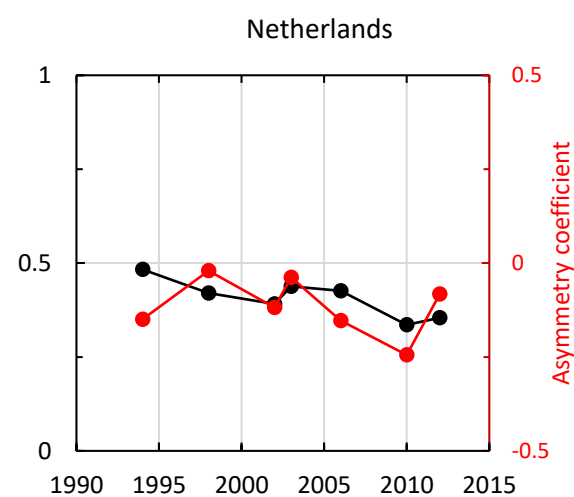
(i)



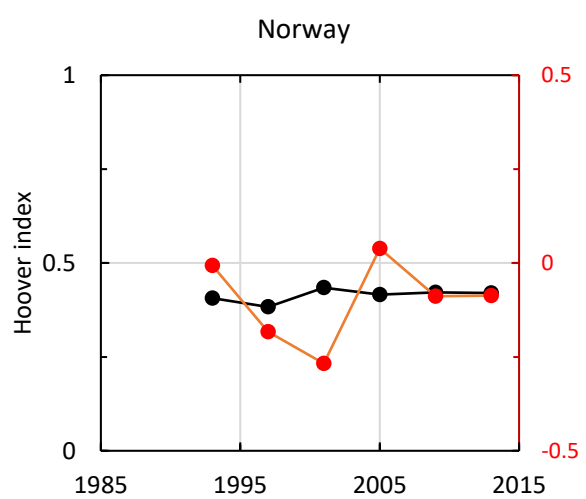
(j)



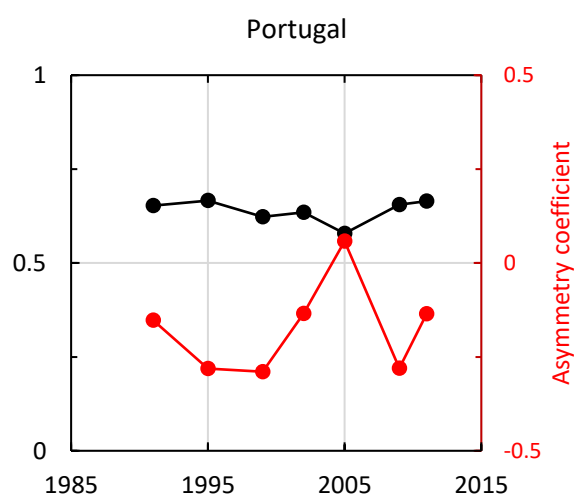
(k)



(l)



(m)



(n)

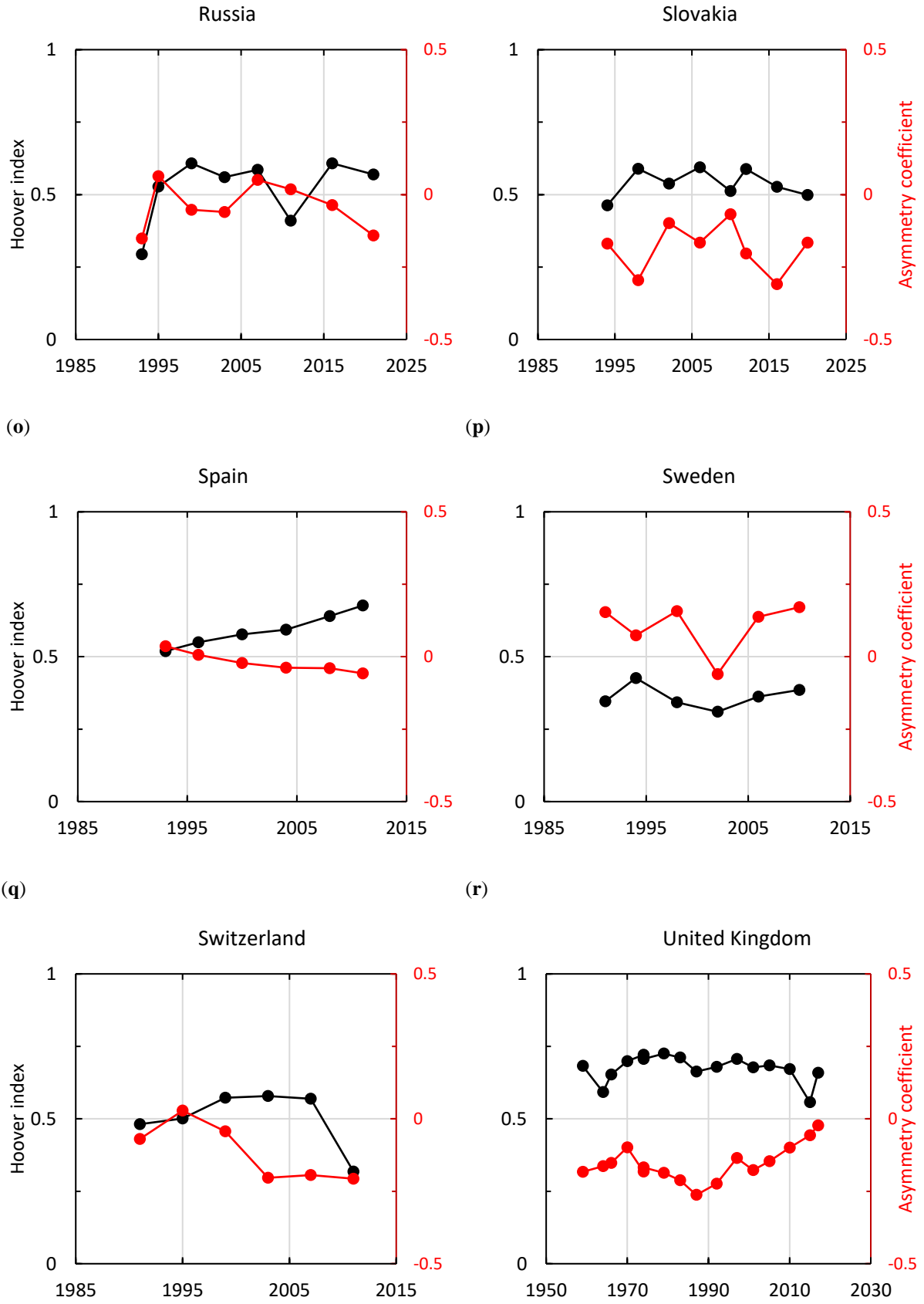


Fig. 4. The behavior of state of the party systems over time

The charts in **Fig. 4** show that each of the party systems under examination has come its path of development. Thus, the spike in the left-hand asymmetry of the PC of the party system in Germany occurred after the reunification of the FRG and the GDR took place. There are also general patterns. For example, right-hand PC asymmetry was seen in 10 countries — Bulgaria, Denmark, France, Iceland, Norway, Portugal, Russia, Spain, Sweden, and Switzerland. Only left-hand PC asymmetry occurred in Austria, Belgium, Finland, Germany, Italy, Netherlands, Slovakia, Switzerland, and United Kingdom. The rapid decline in left-hand PC asymmetry in the UK and France began about the same time.

The results of identification of party systems by concentration of inequality are presented in **Table 1**.

Table 1. Results of identification of party systems in 18 European countries by concentration of inequality in fixed years of the 21st century

Country	Year	Concentration of inequality, HI			PAC
		High	Moderate	Low	
Austria	2019		0.44		-0.33
Belgium	2010			0.23	-0.31
Bulgaria	2014			0.36	0.04
Denmark	2011			0.34	0.06
Finland	2011			0.3	-0.19
France	2017		0.46		-0.05
Germany	2021	0.6			-0.23
Iceland	2013		0.50		-0.16
Italy	2018	0.67			-0.08
Netherlands	2012			0.35	-0.08
Norway	2013		0.42		-0.09
Portugal	2011	0.66			-0.13
Russia	2021		0.57		-0.14
Slovakia	2020		0.50		-0.17
Spain	2011		0.56		-0.06
Sweden	2010			0.38	0.17
Switzerland	2011			0.32	-0.21
United Kingdom	2017	0.66			-0.02

It can be seen from **Table 1** that in these years, high concentration of inequality was in 4 countries, while moderate and low concentration were spread evenly in the rest of the countries.

PC had left-hand asymmetry in 16 countries and right-hand asymmetry only in Bulgaria and Sweden.

Discussion and conclusions

In this work, the Hoover index and asymmetry coefficient of the Pareto curve were tested to address the state of the systems.

When applying the new concept to analyze party systems in 18 European countries (158 election cases), we found that most of them had a concentration of inequality close to the Pareto principle and a left-hand asymmetry PC, which distinguishes significantly the PC of party systems from the PC of income and urban settlement systems, which featured both right-hand and left-hand asymmetry (Clementi et al., 2019; Grachev, 2022).

The dominance of left-hand asymmetry in the PC of party systems can be explained by the struggle of political parties for votes of electors, while most of them are at the center of the ideological model (Downs, 1957). The cause for decline in the left-hand PC asymmetry which began in the UK in 1987 and France in 1988 may have been voters who sought alternatives that would better agree with their political preferences (Spoon & Klüver, 2019).

A slight asymmetry in 9 out of 18 countries $|PAC| \leq 0.1$ seems to suggest that at the beginning of the 21st century, those countries had similar conditions for the functioning of party systems. This conclusion follows from the universal principle of symmetry formulated by Pierre Curie (1894): the functioning of similar systems of any nature is only possible in symmetric media, while the functioning of systems with individual properties is only possible in asymmetric media.

It is known that a decrease in the concentration of inequality in party systems will result in increased cooperation between parties and a larger number of possible coalition alternatives (Vayrynen, 1972). As a similar effect occurs with an increase in left-hand PC asymmetry, one of the further research

objectives should be to look at the influence of PC asymmetry on the number of coalition alternatives, followed by the development of a typology of systems based on PC asymmetry.

To recap, we can state that the use of the Hoover index and the Pareto curve asymmetry coefficient provides new opportunities for understanding the development of party systems. The methodology for analyzing the systems proposed in this work can be considered as an independent tool, or as an addition to the methods used in political science for measuring the state of the party systems.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- Bogaards, M. (2004). Counting parties and identifying dominant party systems in Africa. *European Journal of Political Research*, 43(2), 173–197.
- Clementi, F., Gallegati, M., Gianmoena, L., Landini, S., & Stiglitz, J. E. (2019). Mis-measurement of inequality: a critical reflection and new insights. *Journal of Economic Interaction and Coordination*. <https://doi.org/10.1007/s11403-019-00257-2>
- Curie, P. (1894). Sur la symétrie dans les phénomènes physiques, symétrie d'un champ électrique et d'un champ magnétique. *J. Phys. Theor. Appl*, 1, 393–415. <https://doi.org/10.1051/jphysap:018940030039300i>
- Downs, A. (1957). An Economic Theory of Political Action in a Democracy. *Journal of Political Economy*, 65(2), 135–150.
- Dunleavy, P., & Boucek, F. (2016). Constructing the Number of Parties: *Party Politics*, 9(3), 291–315. <https://doi.org/10.1177/1354068803009003002>
- Epstein, J. M., & Axtell, R. L. (1996). Growing Artificial Societies: Social Science from the Bottom Up. *Growing Artificial Societies*. <https://doi.org/10.7551/MITPRESS/3374.001.0001>
- Gaines, B. J., & Taagepera, R. (2014). More on Measuring Two-Party Competition: A Response to Dunleavy. *Journal of Elections, Public Opinion and Parties*, 24(3), 386–392. <https://doi.org/10.1080/17457289.2014.913597>
- Golosov, G. V. (2010). The effective number of parties: A new approach. *Party Politics*, 16(2), 171–192. <https://doi.org/10.1177/1354068809339538>
- Grachev, G. A. (2009). Sistemnye zakonomernosti neravenstva dokhodov naseleniya. *Ekonomicheskie Nauki*, 8(57), 327–331. (In Russ.).
- Grachev, G. A. (2011). On the Assessment of Political Stability on the Basis of Results of

- Voting at the Elections. *POLIS-Politicheskiye Issledovaniya*, 5, 123-127. (In Russ.).
- Grachev, G. A. (2012). Assessment of the political stability at the latest Russian parliamentary and presidential elections. *POLIS-Politicheskiye Issledovaniya (In Russ.)*, 3, 30-35. (In Russ.).
- Grachev, G. A. (2013). Upravlenie strukturoi denezhnykh dokhodov naseleniya na osnove printsipa Pareto. *Terra Economicus (in Russ.)*, 11(1), 57–63.
- Grachev, G. A. (2022). Size distribution of states, counties, and cities in the USA: New inequality form information. *Physica A: Statistical Mechanics and Its Applications*, 592, 126831. <https://doi.org/10.1016/J.PHYSA.2021.126831>
- Hoover, E. (1936). The Measurement of Industrial Localization. *The Review of Economics and Statistics*, 18, 162–171. <https://doi.org/https://doi.org/10.2307/1927875>
- Juran, J. M. (1954). Universals in management planning and controlling. *Management Review*, 43(11), 748–761.
- Juran, J. M. (1975). The Non-Pareto Principle; mea culpa. *Quality Progress*, 8(5), 8–9.
- Kakwani, N. (1980). *Income inequality and poverty : methods of estimation and policy applications*. Published for the World Bank [by] Oxford University Press.
- Laakso, M., & Taagepera, R. (1979). “Effective” number of parties: A Measure with Application to West Europe. *Comparative Political Studies*, 12(1), 3–27. <https://doi.org/10.1177/001041407901200101>
- Magyar, Z. B. (2022). *What Makes Party Systems Different? A Principal Component Analysis of 17 Advanced Democracies 1970–2013*. 30(2), 250. <https://doi.org/https://doi.org/10.1017/pan.2021.21>
- McCarthy, D. M., & Winer, R. S. (2019). The Pareto rule in marketing revisited: is it 80/20 or 70/20? *Marketing Letters*, 30(2), 139–150. <https://doi.org/10.1007/s11002-019-09490-y>
- Molinar, J. (1991). Counting the Number of Parties: An Alternative IndexNo Title. *American Political Science Review*, 8(4), 1383–1391.
- Pareto, V., & Page, A. N. (1971). *Translation of Manuale di economia politica (‘Manual of political economy’)*. A.M. Kelley.
- Pietra, G. (1915). *Delle relazioni tra gli indici di variabilita: Vol. Note I, II*. Atti del Reale Istituto Veneto di Scienze.
- Rae, D. (1967). *The Political Consequences of Electoral Laws*. CT: Yale University Press.
- Schutz, R. R. (1951). On the Measurement of Income Inequality. *American Economic Review*, 41, 107–122.
- Sharp, B., Romaniuk, J., & Graham, C. (2019). Marketing’s 60/20 Pareto Law. *SSRN Electronic Journal*. <https://doi.org/10.2139/SSRN.3498097>
- Spoon, J.-J., & Klüver, H. (2019). Party convergence and vote switching: Explaining mainstream party decline across Europe. *European Journal of Political Research*Volume 58, Issue 4 p. 1021-1042, 58(4), 1021–1042.
- Taagepera, R. (1999). The Number of Parties as a Function of Heterogeneity and Electoral

System. *Comparative Political Studies*, 32(5), 531–548.

<https://doi.org/https://doi.org/10.1177/0010414099032005001>

Vayrynen, R. (1972). Analysis of Party Systems by Concentration, Fractionalization, and Entropy Measures. *Scandinavian Political Studies*, 7, 138–155.

Zipf, G. K. (1949). *Human behavior and the principle of least effort* (Issue Cambridge, (Mass.): Addison-Wesley,). Cambridge, (Mass.): Addison-Wesley.

[https://doi.org/10.1002/1097-4679\(195007\)6:3<306::aid-jclp2270060331>3.0.co;2-7](https://doi.org/10.1002/1097-4679(195007)6:3<306::aid-jclp2270060331>3.0.co;2-7)