

Bismillahirrəhmanirrəhim (In the name of Allah, the Merciful and Compassionate).

Steps to proving the Fermat's last theorem

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There are many different approaches [3] to Fermat's last theorem (FLT). In this study, we will consider an interesting approach.

Theorem (FLT): For an arbitrary integer $n > 2$, there are no integers a , b and c ($a, b, c > 0$) [5] satisfying the formula $a^n + b^n = c^n$.

In other words: the indeterminate equation $x^n + y^n = z^n$ has no integer solution at $n \geq 3$ [6].

Fermat's theorem can be expressed in a more general way, taking into account not only natural but also negative values of “ n ”:

Theorem 1: except for $n=2$ and $n=1$ of the equation $a^n + b^n = c^n$, for any integer value of n (including negative values of n) in integers (i.e. a , b , c in full values) has no solution (Beylarov E.B.).

This theorem will be formulated in more detail below as Theorem 3.

It is clear that for $n=1$ and $n=2$ there is an infinite set of solutions satisfying the equation $a^n + b^n = c^n$ [1]:

- With $n=2$ we get Pythagorean triples $3^2 + 4^2 = 5^2$ or $5^2 + 12^2 = 13^2$
- For $n=1$ $5^1 + 7^1 = 12^1$ etc.

As we have already mentioned, integer and positive numbers a , b and c satisfying the equality $a^n + b^n = c^n$ express Pythagorean triples for $n=2$ [1]. This also means that the numbers that are solutions to the equation are the sides of a right triangle. That is, from them one can build a triangle, and this triangle will be right-angled.

So these numbers satisfy the triangle inequalities:

$$a+b>c, a+c>b, b+c>a, c-b<a, c-a<b [1]$$

Theorem 2. Arbitrary positive numbers a , b and c satisfying the equality $a^n + b^n = c^n$ (for $n > 1$) must satisfy the triangle inequalities, i.e.: $a+b>c$; $a+c>b$; $b+c>a$; $c-b<a$; The inequalities $c-a<b$ $b-a<c$ must be applied.

In other words, when $n > 1$, the numbers a, b, c satisfying the equation $a^n + b^n = c^n$ must be the sides of any triangle. For $1 < n < 2$ this triangle will be obtuse, with $n = 2$ it will be right-angled, and for $n > 2$ it will be acute-angled.

To prove it, let us consider three possible cases of expressions $a + b$ and c for integer and positive numbers a, b and c satisfying the equation $a^n + b^n = c^n$

1. $a+b=c$ 2. $a+b<c$ 3. $a+b>c$

1. For $a+b=c$, raising both sides of the equation to the n -th power, we get:

$$a^n + \dots + b^n = c^n$$

On the left side of the equation, we get the positive limits of the binomial opening between the first and last limits. If we remove the positive limits of the binomial hole on the left side, then $a^n + b^n < c^n$. This situation contradicts the equation $a^n + b^n = c^n$. That is, if $a^n + b^n = c^n$, then the first option, i.e. $a+b=c$ is impossible.

2. If $a+b<c$, then it is clear that $(a+b)^n < c^n \vee a^n + \dots + b^n < c^n$. Then, when positive binomial limits appear on the left side of the inequality between the first and last limits, the expression on the left side will be smaller.

That is, $a^n + b^n << c^n$, which contradicts the case $a^n + b^n = c^n$. Therefore, the 2nd case is also impossible.

3. Thus, the 3rd case, i.e. $a+b>c$, is the only possible one.

For the case 3.1. $0 < n < 1$, the triangle inequality is not applied; on the contrary, the following inequalities are applied (Fig. 1):

$$a+b<c; a<c-b \vee b<c-a$$

Example: $(25)^{1/2} + (49)^{1/2} = (144)^{1/2}$ $25 + 49 < 144$ etc.



Fig. 1

3.2. In the case of $n=1$, the only possible case is $a+b=c$. This is expressed by a point located at distances a and b from the end points on segment c (Fig. 2);



Fig 2.

3.3. With $1 < n < 2$, a, b, c will be the sides of an isosceles triangle (Fig.3);

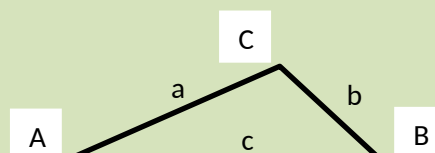


Fig. 3

3.4. With $n=2$, a , b , c will be the sides of a right triangle.

3.5. With $2 < n$, a , b , c will be the sides of an acute-angled triangle.

Let's consider several theorems and their proofs in the direction of proving Fermat's theorem:

Theorem 3: For arbitrary real numbers $0 < a < b < c$, there always exist real numbers $n > 1$ and $0 < m < 1$ such that the equations $a^n + b^n = c^n$ and $a^m = b^m + c^m$ are satisfied (Beilarov E.B.)

Proof: if we divide both sides of the equation by c^n , we get $\left(\frac{a}{c}\right)^n + \left(\frac{b}{c}\right)^n = 1$.

Here, the function $y = \left(\frac{a}{c}\right)^x$ monotonically decreases (Figure 4, graph 1), and the function $y = 1 - \left(\frac{b}{c}\right)^x$ monotonically increases (Figure 4, graph 2).

Since the domain of the functions is $(-\infty, +\infty)$, their graphs must intersect at one point. (See Fig. 4.)

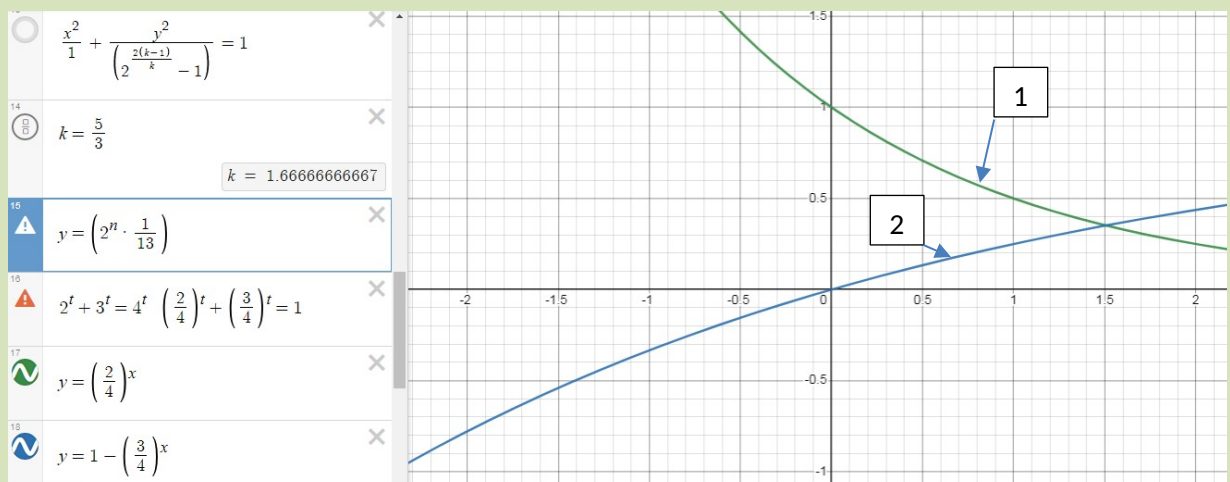


Fig. 4.

For an arbitrary triangle ABC , the abscissa of the intersection point of the graphs $y = 1 - \left(\frac{b}{c}\right)^x$ and $y = \left(\frac{a}{c}\right)^x$ will satisfy the equation $(x) a^x + b^x = c^x$.

The part of "Theorem 3" related to the equation $a^m = b^m + c^m$ is proved similarly.

Note that in the case of $0 < a < b < c$, it is impossible to get a real number "k" that satisfies the equation $b^k = a^k + c^k$. This is because a is less than b and c is greater than b . For an arbitrary value of k in the expression $b^k = a^k + c^k$, one of the expressions a^k and c^k will be positive numbers greater than b^k , and the other less than b^k .

Thus, Theorem 3 is proved.

This means that there are always corresponding numbers «n» and «m» satisfying the conditions $a^n + b^n = c^n$ for the largest side of an arbitrary triangle and $a^m = b^m + c^m$ for smaller sides.

There is no corresponding number «k» that satisfies the condition $b^k = a^k + c^k$ for the average side "b".

In our view, the truth of this theorem is one of the most important points complicating the proof of Fermat's theorem as a whole. While attempts are being made to prove that a, b, c cannot be simultaneously complete or that they cannot be coprime for natural values of n greater than 2 ($n > 2$).

From "Theorem 3" one can see that the search for such contradictions about a, b, c in the equation $a^n + b^n = c^n$, taking "n" as natural, will not give any results. The presence of a rational or irrational "n" that satisfies the condition $a^n + b^n = c^n$ for arbitrary a, b, c makes it impossible for there to be a contradiction in this direction.

Note that the correctness of "Theorem 3" requires a change in the direction of the search in relation to the SFT proof. In other words, Theorem 4 can be formulated as follows:

Theorem 4. There is no integer $n(2 < n)$ greater than 2 that satisfies the equation $a^n + b^n = c^n$ for arbitrary integers a, b, c (Baylarov E.B).

Theorem 4" fundamentally differs from "Theorem F" in the formulation of the problem. But the proof of "Theorem 4" will also be the proof of "Theorem F".

Note. Finding a general formula for "n" that satisfies the condition $a^n + b^n = c^n$ for an arbitrary triangle is a time-consuming technical task. Finding a general formula for "n" is related to the problem of finding the formula for the "logarithm of the sum" ($\log(f(x) + g(x)) = ?$). Finding the first will contribute to the solution of the second.

For equilateral triangles, the general formula for n satisfying the condition $a^n + b^n = c^n$ can be obtained as follows:

$$\frac{c}{2} : a = \sin \frac{C}{2} ; c = 2a \sin \frac{C}{2} ; \sin \frac{C}{2} = \frac{c}{2a} h = a \cos \frac{C}{2} ; \cos \frac{C}{2} = \frac{h}{a} ; \text{See Fig. 5.}$$

Since $a=b$ in an equilateral triangle, we can write the equation $a^n + b^n = c^n$ as: $a^n + a^n = c^n$

Here we get the following:

$$2a^n = c^n$$

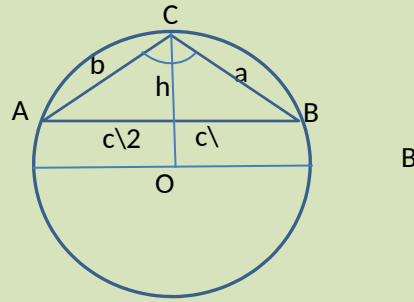


Fig. 5.

We logarithm both sides of this expression to base 2::

$$\frac{1-n}{n} = \log_2 \sin \frac{C}{2} \rightarrow \frac{1}{n} - 1 = \log_2 \sin \frac{C}{2} \rightarrow n = \frac{1}{1 + \log_2 \sin \frac{C}{2}}$$

Based on the conducted studies, the following can be noted (see Fig. 6.1 - 6.4)

6.1. C –for an equilateral triangle when it is hypogonal (Fig. 6.1.)

With a=4; b=4; c=7 , according to formula $n = \frac{1}{1 + \log_2 \sin \frac{C}{2}}$ $n=1.23861262585$
and $a^n + b^n = c^n = 11.1365097709$

6.2. The case of approaching a right triangle (Fig. 6.2.)

With a=5.74456264656; b=4; c=7 , according to formula $n = \frac{1}{1 + \log_2 \sin \frac{C}{2}}$
 $n=2.00000000002$ and $a^n + b^n = c^n = 49.0000000015$

6.3. As the triangle approaches an equilateral one, n tends to infinity (Figure 6.3).

With a=6.9; b=6.9; c=7 , according to formula $n = \frac{1}{1 + \log_2 \sin \frac{C}{2}}$

$n= 48.1728979257$ and $a^n + b^n = c^n = 5.1383255777 \times 10^{40}$

6.4. When angle C is acute, the value of n is negative (Figure 6.4).

With a=9; b=9; c=7, according to formula $n = \frac{1}{1 + \log_2 \sin \frac{C}{2}}$

$n= -2.75808748945$ and $a^n + b^n = c^n = 0.00466815930687$

6.5. with a+b=c n=1.



Fig. 6.1.



Fig. 6.2.

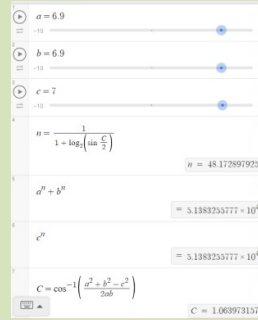


Fig. 6.3

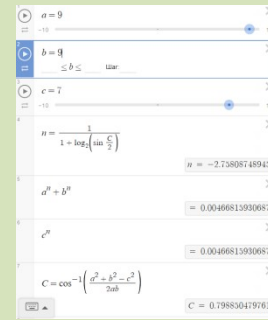


Fig. 6.4.

$$n = \frac{1}{1 + \log_2 \sin \frac{C}{2}} \quad (\text{B1})$$

the formula is true for arbitrary equilateral triangles, except for equilateral triangles. Since $\sin 30^\circ = 1/2$ in the expression $(a^n + a^n = a^n)$ for an equilateral triangle, the denominator of the expression becomes "0" and the resulting uncertainty is $1/0$.

In the following analyzes, we will show that the case of an equilateral triangle is also a limiting case for Elbe curves (Beilarov E.B.).

For equilateral triangles, we express the general formula "n" with the ratio of the sides, with the formula without an angle. If

$$a^n + a^n = c^n \quad 2a^n = c^n$$

If we divide both sides by c, we get:

$$2 \frac{a}{c} = 1$$

If we log both sides to base 2, we get:

$$1 + n \log_2 \frac{a}{c} = 0 \quad n = \frac{-1}{\log_2 \frac{a}{c}} \quad (\text{B2})$$

Let's compare formulas (B1) and (B2):

$$n = \frac{1}{1 + \log_2 \sin \frac{C}{2}} \quad (\text{B1})$$

$$n = \frac{-1}{\log_2 \frac{a}{c}} \quad (\text{B2})$$

1. For equilateral triangles, both formulas are correct (In the Fig. 7.1. the degree of "n", calculated by the formula (B2), is marked with "m" so as not to create contradictions in the Desmos calculator.).

If $a=8$, $b=8$, $c=13$,

$$n = \frac{1}{1 + \log_2 \sin \frac{C}{2}} = 1.42767460796 \quad (\text{B1})$$

$$n = \frac{-1}{\log_2 \frac{a}{c}} = 1.42767460796 \quad (\text{B2})$$

$$a^n + b^n = 38.9357584353$$

$$c^n = 38.9357584353$$

$$a^n + b^n = c^n$$

2. When $a=b$ and the numbers a , b , c do not form a triangle, since there is no angle C , formula (B1) cannot be calculated - an uncertainty is obtained

But in this case formula (B2) is calculated and $a^n + b^n = c^n$ is paid.

(In the figure 7.2. , the degree of "n", calculated by the formula (B2), is marked with "m" so as not to create contradictions in the Desmos calculator).

When $a=5$, $b=5$, $c=13$

$$n = \frac{1}{1 + \log_2 \sin \frac{C}{2}} = \text{uncertainty} \quad (\text{B1})$$

$$n = \frac{-1}{\log_2 \frac{a}{c}} = -0.725420071279 \quad (\text{B2})$$

For "n" calculated by formula (B1), we get:

$$a^n + b^n - \text{uncertainty}$$

$$c^n - \text{uncertainty}$$

For "n" calculated by formula (B2), we get:

$$a^n + b^n = 6.42801477055$$

$$c^n = 6.42801477055$$

$$a^n + b^n = c^n$$

3. It is strange that although the formula (B1) is written for equilateral triangles, it is also valid for all right triangles - Pythagorean numbers (for $n=2$).

Because with $C=90^\circ$ $\sin \frac{C}{2} = \sqrt{\frac{2}{4}} = 2^{\frac{-1}{2}}$

However, formula (B2) is not applied for non-isosceles right triangles, (In the figure 7.3. the degree of "n", calculated by the formula (B2), is marked with "m" so as not to create contradictions in the Desmos calculator).

If $a=3$, $b=4$, $c=5$ then

$$n = \frac{1}{1 + \log_2 \sin \frac{C}{2}} = 2 \quad (\text{B1})$$

$$n = \frac{-1}{\log_2 \frac{a}{c}} = 1.35691544886 \quad (\text{B2})$$

$$a^n + b^n = 25$$

$$c^n = 25$$

$$a^n + b^n = c^n$$

(B2) düsturuna görə hesablanan n üçün alırıq:

$$a^n + b^n = 11.0009233374$$

$$c^n = 8.88061816145$$

$$a^n + b^n \neq c^n$$

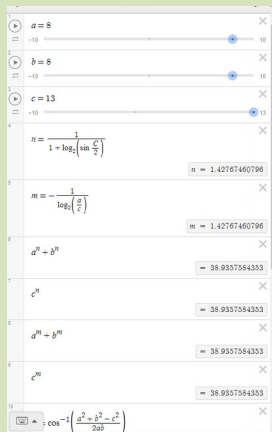


Fig. 7.1.

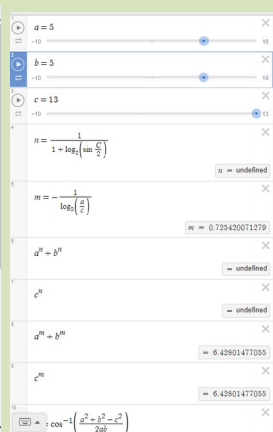


Fig. 7.2.

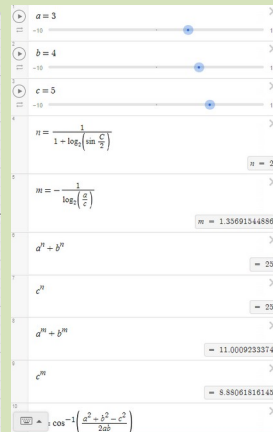


Fig. 7.3.

As odd as it may sound, it is so and it would be interesting to find out why.

In general, we can formulate "Theorem 5" for triads that can be sides of a triangle.

Theorem 5: For an arbitrary triangle with sides a , b , c , there exists a real number "n" such that $a^n + b^n = c^n$ (Beilarov E.B.)

Since this proposition is a special case of Theorem 3, it can be proved in a similar way.

Note that the Pythagorean theorem is a special case of Theorem 5.

Curves of the Elbe

Suppose a, b, c are sides of a triangle and satisfy the condition $a^n + b^n = c^n$ for $1 < n < 2$, then the center of the solution of the equation $(a/c)^n + (b/c)^n = 1$ with respect to a, b, c within a circle with a uniform radius at the origin, then on the Elbe curves (Beylarov E.B.) it will look like an ellipse, but not an ellipse (Fig. 8, graph 2 - ellipse, graph 3 - Elba).

In the case of $1 < n < 2$, the minor semi-axis of Elba curves is less than the radius of the circle, and the major semi-axis is equal to the radius of the circle (Fig. 8, graph 3).

In the case of $n=2$, we get the equation of the single circle at the beginning of the central coordinate $(a/c)^2 + (b/c)^2 = 1$ Elba coincides with the circle (fig. 8, graph 1).

The minor semiaxis of these curves is less than the radius of the circle, and the major semiaxis is equal to the radius of the circle (Fig. 8).

In the case of $2 < n$ solution of the equation $(a/c)^n + (b/c)^n = 1$ outside the circle of uniform radius at the origin of the central coordinate, the minor semiaxis is equal to the radius of the circle, the major semiaxis is greater than the radius of the circle, similar to an ellipse, but not an ellipse located on (Fig. 9, graph 1, 2 are the Elbe curves).

For clarity, in the future we will call these curves the curves of Elba (Elkhana Beylarova). In Figure 8, the 1st equation is the Elba equation, the 2nd equation is the circle equation, and the 3rd equation is the ellipse equation.

As we already mentioned, the Elbe curve - although it looks like an ellipse with equal major and minor axes, in fact it is a different figure. This can also be seen visually by writing down the corresponding formula of the ellipse with respect to the corresponding semi-axes by observing the graphs.

The Elbe curve is also not a circular arc. This can also be proven by writing the equation of a circle passing through three points.

The angles that subtend the same chord on an arc of a circle are equal, but the angles that subtend the same chord on the Elbe differ in magnitude. This is another proof that this is not an arc of a circle.

Note: a) the sides of all right-angled triangles obtained by moving the large base by the diameter of the circle, and the vertices by the circle,

change the angles adjacent to the base, but the angle at the apex ($C=90^\circ$) and n 's ($n=2$) do not change;

b) the sides and all angles of the triangles obtained by moving the vertex on the Elbe with a large support on the diameter of the circle do not change n .

For all triangles inside the Elbe, n s are fixed that satisfy the condition $(a/c)^n + (b/c)^n = 1$

For complete clarity, what has been said about Elbe curves can also be shown on the coordinate plane.

For example, to build an Elbe curve for $n=5/3$, we would do the following.

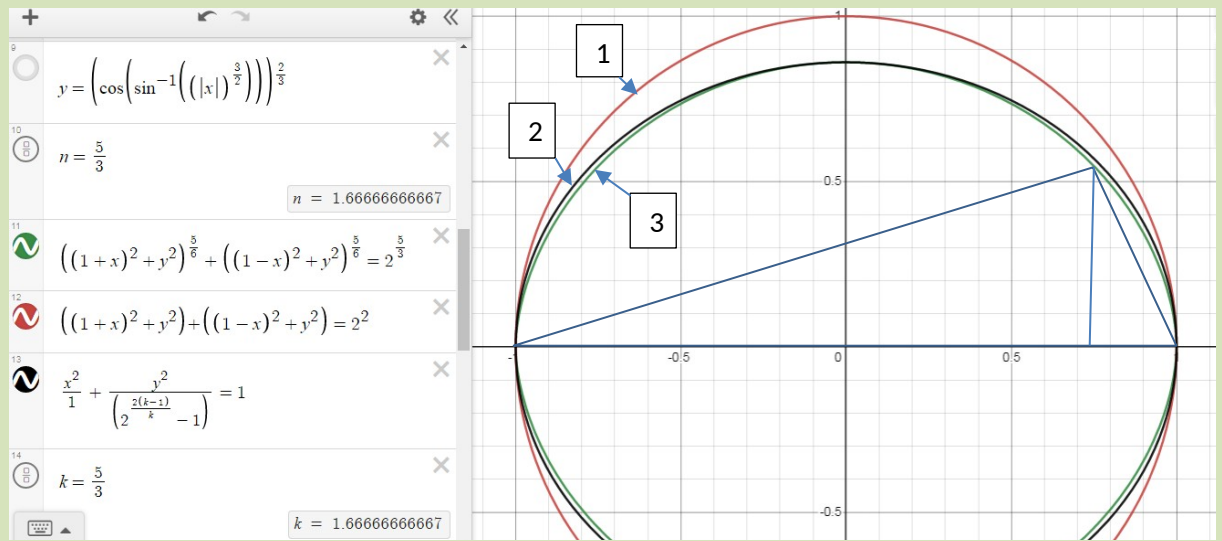


Fig. 8.

Expressing the sides of the triangle in terms of the x, y coordinates, we obtain the Elbe equation and its graph for an arbitrary n in the unit circle.

$$((1+x)^2 + y^2)^{\frac{n}{2}} + ((1-x)^2 + y^2)^{\frac{n}{2}} = 2^n$$

$((1+x)^2 + y^2)^1 + ((1-x)^2 + y^2)^1 = 2^2$ $n=2$; circle equation, extreme graph (Fig. 8, graph 1).

$((1+x)^2 + y^2)^{\frac{5}{6}} + ((1-x)^2 + y^2)^{\frac{5}{6}} = 2^{\frac{5}{3}}$ ($n=5/3$; The Elbe equation will be the innermost graph (Fig. 8, graph 3).

The curve in the middle, very close to the Elbe curve, whose semi-major axis is 1 and whose semi-minor axis is equal to the Elbe's minor semi-axis, is an ellipse graph. Its equation is shown at the bottom of the figure (Fig. 8, graph 2).

Figure 9 shows graph 1 for $n=100$, graph 2, for $n=4$, graph 3, for $n=2$ circle, graph 4 for $n=5/3$ – Elbe graphs are shown.

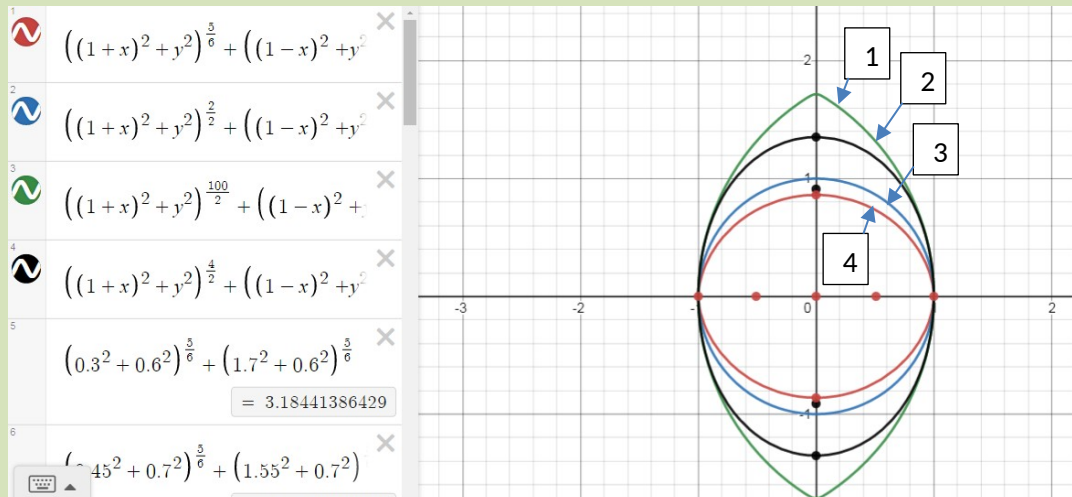


Fig.9.

So the circle is also a special case of Elbe with $n=2$.

Let us show the calculation of the minor axis of the Elbe.

Let's use the known equation $(a/c)^n + (b/c)^n = 1$.

An Elbe whose major semiaxis is 1 will have $c=2$. So,

$$(a/2)^n + (b/2)^n = 1$$

where $a=b$

$$2(a/2)^n = 1 \rightarrow (a/2)^n = 1/2 \rightarrow a/2 = (1/2)^{1/n} \rightarrow a = 2(1/2)^{1/n} \rightarrow a = 2^{(n-1)/n}$$

So, in a circle with the same radius $a=b$ - in the case of an equilateral triangle.:

$$a=b=2^{(n-1)/n}$$

In this case, the minor axis of the Elbe:

$$y_{x=0} = (a^2 - 1^2)^{1/2} = (2^{2(n-1)/n} - 1)^{1/2} \text{ (Fig. 9.)}$$

This expression can be obtained faster by taking $x=0$ from the equation $a^2 = (1+x)^2 + y^2$:

$$y_{x=0}^2 = a^2 - 1^2 \text{ here } y_{x=0} = (2^{2(n-1)/n} - 1)^{1/2}$$

In this case, the minor axis of the Elbe is $y_{x=0}$, and the major axis is $x_{y=0}$:

$$y_{x=0} = (a^2 - 1^2)^{1/2} = (2^{2(n-1)/n} - 1)^{1/2}$$

$$x_{y=0} = 1$$

The minor semiaxis of the Elbe within the unit circle, the following can be noted:

- 1) when $n=1$ is "0";
- 2) In the case of $1 < n < 2$, it is less than the radius of the unit circle, i.e. 1;
- 3) when $n=2$ it is equal to "1"
- 4) In the case of $2 < n$, it will be greater than 1 (Fig. 10).

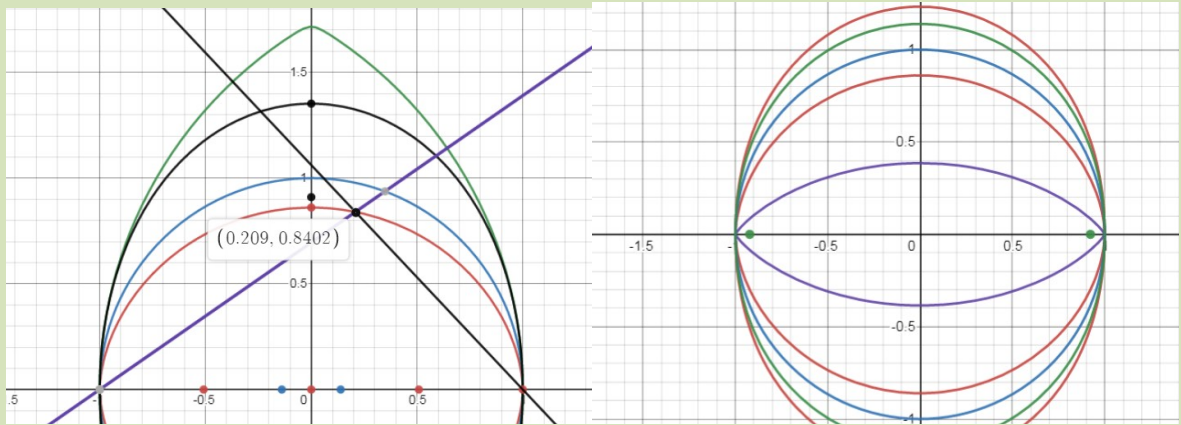


Fig.10.

Lemma 1. When dividing the sides of triangles by the largest side and multiplying by 2, since the largest side is 2, the largest value of the semiaxis on the Y axis of the Elbe curves in the positive direction approaches $3^{1/2}$ (3 in the square root) and is approximately equal to 1.73205080757 .

Proof: For the largest values of N , when the sides grow and approach the base, i.e. 2, we get an equilateral triangle. Its height will be $h_y^2 = 2^2 - 1^2 = 3$ or $h_y = 3^{1/2}$.

This value can be calculated similarly on an arbitrary circle.

Here are some important facts about Elbe curves:

1. The sides of an equilateral triangle are expressed as irrational numbers in cases where $n \neq 2$, $n \neq 1$ and $c=2$ in Arbitrary Elbe.

Because:

$$a^n + a^n = 2^n \rightarrow 2a^n = 2^n \rightarrow a^n = 2^n : 2 \rightarrow a^n = 2^{n-1} \rightarrow a = 2^{(n-1)/n}$$

Therefore, the sides of an equilateral triangle in the Elbe are irrational in all cases where the side c is complete or rational.

2. This semiaxis is the height of the corresponding equilateral triangle, and its area is equal to the difference of the squares of the side and half of the base. I mean:

$$Y_{x=0}^2 = a^2 - 1^2; \rightarrow Y_{x=0}^2 = 2^{2(n-1)/n} - 1 \rightarrow Y_{x=0} = (2^{2(n-1)/n} - 1)^{1/2}$$

This means that the vertical semiaxis of an arbitrary elbe is an irrational number.

3. The value of the semiaxis along the Y-axis of the Elbe increases and decreases depending on n .

Let's clarify the dependence of " n " on the vertical axis in Elba.

It is clear that

$$\text{tg} \alpha = Y_{x=0} \text{ or } \text{tg} \alpha = y \rightarrow \text{tg} \alpha = (2^{2(n-1)/n} - 1)^{1/2} \rightarrow \text{tg}^2 \alpha = (2^{2(n-1)/n} - 1) \rightarrow 2^{2(n-1)/n} = \text{tg}^2 \alpha + 1$$

Here, if we take the logarithm of both parts to base 2, we get:

$$\log_2 2^{2(n-1)n} = \log_2 (tg^2 \alpha + 1) \rightarrow 2(n-1)n = \log_2 (tg^2 \alpha + 1) \rightarrow 2 - 2n = \log_2 (tg^2 \alpha + 1) \rightarrow 2 - \log_2 (tg^2 \alpha + 1) = 2n \rightarrow n = 2 \backslash (2 - \log_2 (tg^2 \alpha + 1)) \text{ (v\o ya } n = 1 \backslash (1 + \log_2 \cos \alpha))$$

This shows that in the case of $tg \alpha = 1$, $n = 2$ is obtained, which is the case when the Elbe and the circle coincide.

4. The circle is a special case of the Elbe with $n = 2$.

Let us consider some special cases on this issue.

For example, consider the case $n = 5 \backslash 3$. So: $a^{5 \backslash 3} + b^{5 \backslash 3} = c^{5 \backslash 3}$. By dividing the sides of the equation by $c^{5 \backslash 3}$ and multiplying by $2n$, this triangle can be built on the diameter of a circle of the same radius.

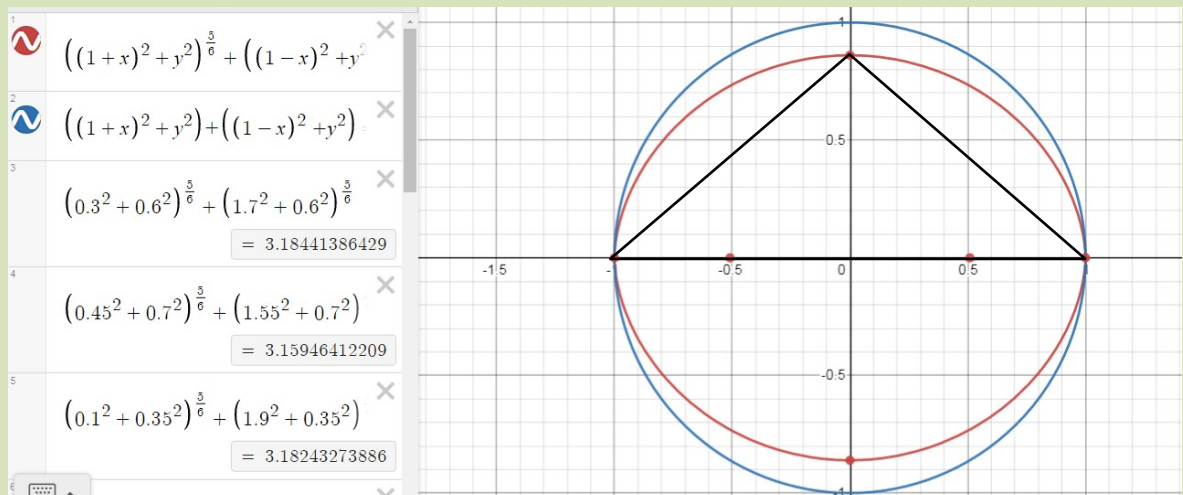


Fig. 11

Note for the triangle (Fig. 11): for a triangle that satisfies the condition $a^{5 \backslash 3} + b^{5 \backslash 3} = c^{5 \backslash 3}$ for $n = 5 \backslash 3$

$$(a \backslash c)^{5 \backslash 3} + (b \backslash c)^{5 \backslash 3} = 1$$

$$a^2 = (1+x)^2 + y^2 \quad (1) \quad \rightarrow \quad a = ((1+x)^2 + y^2)^{1/2}$$

$$b^2 = (1-x)^2 + y^2 \quad (2) \quad \rightarrow \quad b = ((1-x)^2 + y^2)^{1/2}$$

From the difference of equations (1) and (2) we get that:

$$a^2 - b^2 = 4x;$$

$$a^2 + b^2 = 2 + 2x^2 + 2y^2 = 2(1 + x^2 + y^2)$$

It would be appropriate to investigate these cases for different values of x and y in the circle, ellipse and Elbe.

$$(a \backslash c)^n + (b \backslash c)^n = 1$$

$C = 2$ on the Elbe, whose semi-major axis is equal to one. So, $(a \backslash 2)^n + (b \backslash 2)^n = 1$

where $a=b$ we get:

$$2(a/2)^n = 1 \rightarrow (a/2)^n = 1/2 \rightarrow a/2 = (1/2)^{1/n} \rightarrow a = 2(1/2)^{1/n} \rightarrow a = 2^{(n-1)/n}$$

So, in the case of an equilateral triangle $a=b$ in a circle with a uniform radius:

$$a=b=2^{(n-1)/n}$$

In this case, the smaller half of the Elbe:

$$y_{x=0} = (a^2 - 1^2)^{1/2} = (2^{2(n-1)/n} - 1)^{1/2}$$

This expression can be obtained faster by taking $x=0$ from the equation $a^2 = (1+x)^2 + y^2$:

$$y_{x=0}^2 = a^2 - 1^2 \text{ here } y_{x=0} = (2^{2(n-1)/n} - 1)^{1/2}$$

In this case, the minor semiaxis is $y_{x=0}$, and the major semiaxis is $x_{y=0}$:

$$y_{x=0} = (a^2 - 1^2)^{1/2} = (2^{2(n-1)/n} - 1)^{1/2}$$

$$x_{y=0} = 1$$

For the semi-focal distance f for a suitable semi-axial ellipse, we get:

$$f = (1 - y_{x=0}^2)^{1/2}$$

For simplicity of calculations and formulations, here, without loss of generality, we can take $c = 2$, so that the radius of the circle under consideration and the major semiaxis of the corresponding ellipse are equal to one (1).

On fig. 12 also shows the graphical differences between the Elbe curve (red) and the ellipse (green), in which the major and minor semiaxes coincide for the case $n=5/3$

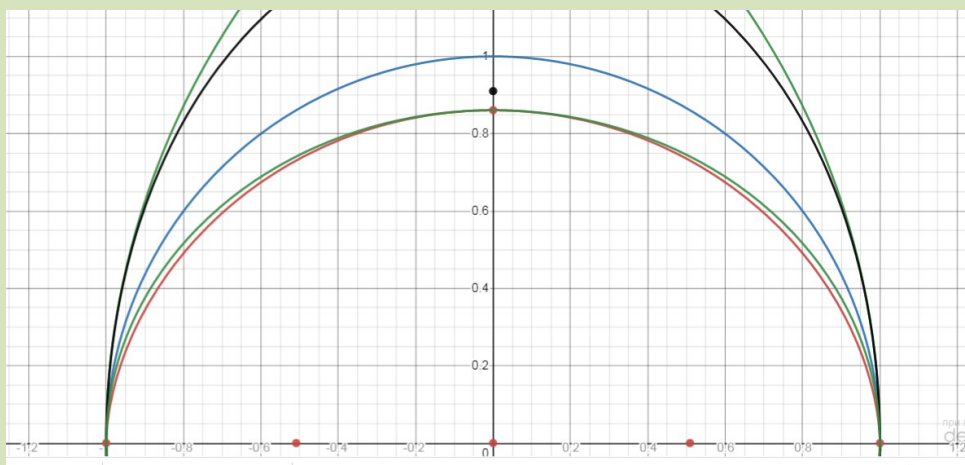


Fig.12

On fig. 13 also shows the graphical differences between the Elbe curve (blue) and the ellipse (blue), the major and minor semiaxes of which coincide with it for the case $n=10/9$.

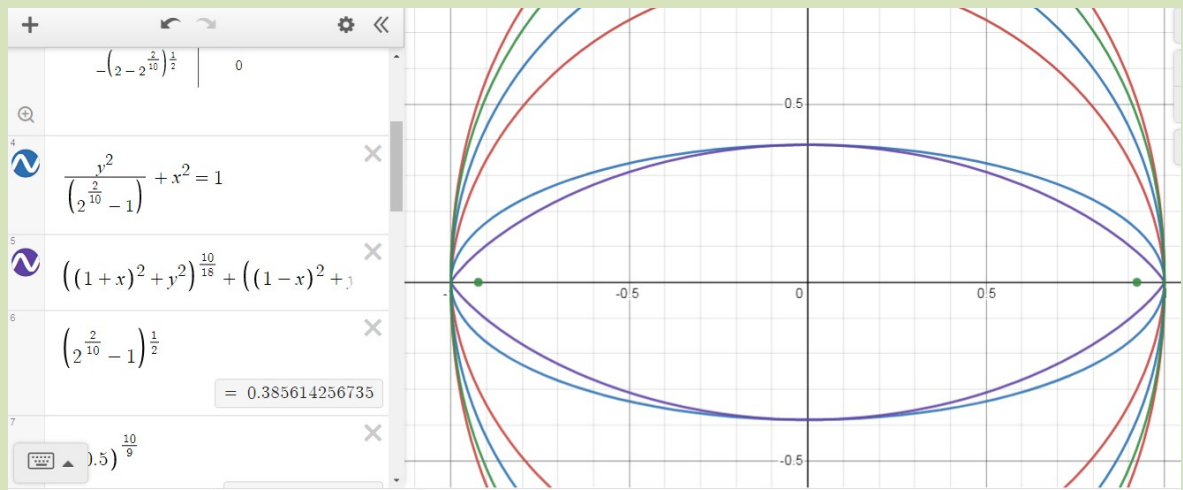


Fig.13

From a comparison of these two graphs, it can be seen that as the value of n approaches unity, the difference between the ellipse and Elbe curves is more clearly observed.

As can be seen from Figure 14, as the value of " n " increases ($n=20$ is the dark red curve in the figure), one of the numbers a and b , or both of them (if they are close or equal), approaches " c ". Of course, since the numbers " a " and " b " are less than the number " c " and the number c remains constant, the graphs will infinitely approach the Elbe (Figure 14, graph 1) with the largest " n " in the fig (where $n=100$).

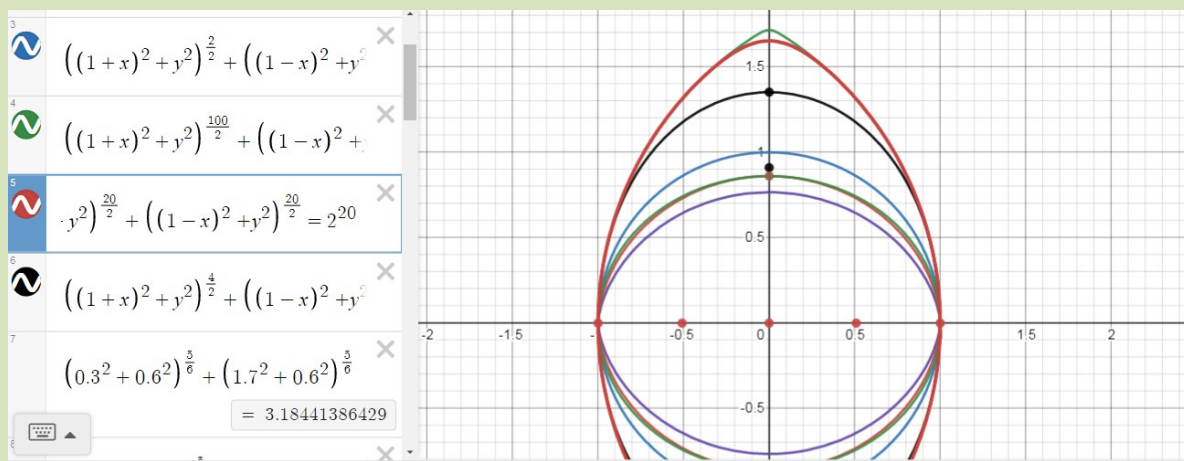


Fig. 14

Here we would like to raise an interesting question. Figure 15 shows the formulas for transforming Elbe and Ellips into a hyperbola.

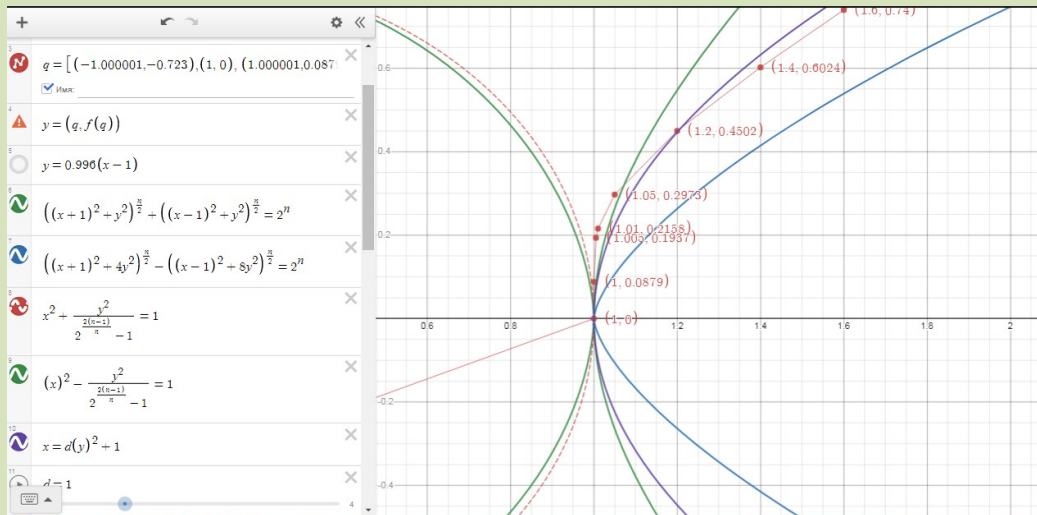


Fig 15

Depending on the coefficients of the limits y_2 in the Elba formula, the direction of the branches of the curve changes. Studying these cases will reveal interesting facts.

An interesting fact is that a circle is a special case of an ellipse with equal semi-axes. At the same time, the circle is a special case of Elbe in the case of $n=2$. In this case, it is doubtful that there is a connection between the Elbe and the ellipse. The discovery of this relation would in fact clarify many of the Elbe properties that would contribute to the solution of Fermat's theorem.

Now let's focus on a few learned facts about the circle, ellipse, and Elbe.

The study showed that:

- 1) In the case $1 < n < 2$, the Elbe curve and the ellipse with equal semi-axes are inside the same sphere, and Elbe is inside the ellipse;
- 2) in case $n=2$ Elbe, circle and ellipse overlap each other;
- 3) In the case of $2 < n$, the ellipse is inside the Elbe, and this situation continues until the value $2 < n < 4.8...$ (approximate value).
- 4) After the value $n=4.8...$ (approximate value), the ellipse again begins to move away from the Elbe, and after a certain value it completely goes beyond its limits.
- 5) It makes sense to find the logical and geometric explanation of this situation - its meaning, whether such transitions occur at higher values of n .

Although many issues related to Elbe curves have been investigated, we consider it expedient to confine ourselves to their interpretation here. In the following articles we are going to clarify these issues and our research on the proof of Fermat's last theorem.

Note: The problem of proving Fermat's triangle theorem by applying the sine theorem shows that the equation $a^n + b^n = c^n$ and the equation $\sin^n \alpha + \sin^n \beta = \sin^n \gamma$ are equally valid, and makes it possible and necessary to use the knowledge about triangles. .

Special thanks: I express my deep gratitude to my childhood friend Gasanov Ilyas Ravan, Ph.D. The studies to be published are the result of our joint activities. The mathematical expression of the proposed ideas and approaches and the mathematical solution of many of them are the result of his great efforts. His dedicated love for science was indispensable in my return to science and the publication of these studies.

To be continued.

Reference

1. E.B.Bəylərov, İ.R.Həsənov. Həndəsə. Abituriyentlərə kömək, Dərs vəsaiti. Bakı: Təhsil, 2000. 132 s.
2. E.B.Bəylərov, X.Y.Əlimərdanova. Üçbucaqlar / Riyaziyyat: İmtahana düşən çətin mövzulara dair ekspert şərhləri. Metodik vəsait. Bak: 2002. TQDK. s.169-174.
3. Paulo Ribenboim. Fermat's last theorem for amateurs. 423 p. 1999
4. İ.R.Həsənov, E.B.Bəylərov. Riyaziyyat Olimpiada Məsələləri. Ləman Nəşriyyat Poliqrafiya, Bakı:2015, 448 s.
5. Великая теорема Ферма. Jeys Trefel. Ensiklopediya. https://elementy.ru/trefil/21135/Velikaya_teorema_Ferma
6. Ивлиев Ю.А. Великая теорема ферма и современная наука/ Современные проблемы науки и образования. <https://science-education.ru/ru/article/view?id=1642> (18.12.2022)
7. Разоблачаем ! Великая теорема Ферма доказана ? <https://masterok.livejournal.com/1230401.html> (18.12.2022)